

Differentiating  $f(\theta)$  with respect to  $\theta_j$  and setting the result equal to 0 produce

$$\sum_{i=1}^p x_{ij} y_i = \sum_{i=1}^p \sum_{k=1}^q x_{ij} x_{ik} \theta_k.$$

If we let  $y$  denote the column vector with entries  $y_i$  and  $X$  denote the matrix with entry  $x_{ij}$  in row  $i$  and column  $j$ , these  $q$  normal equations can be written in vector form as

$$X^t y = X^t X \theta$$

and solved as

$$\hat{\theta} = (X^t X)^{-1} X^t y.$$

In the method of least absolute deviation regression, we replace  $f(\theta)$  by

$$h(\theta) = \sum_{i=1}^p \left| y_i - \sum_{j=1}^q x_{ij} \theta_j \right|.$$

Traditionally, one simplifies this expression by defining the residual

$$r_i(\theta) = y_i - \sum_{j=1}^q x_{ij} \theta_j.$$

We are now faced with minimizing a nondifferentiable function. Fortunately, the MM algorithm can be implemented by exploiting the convexity of the function  $-\sqrt{u}$  in inequality (3.2). Because

$$-\sqrt{u} \geq -\sqrt{u^n} - \frac{u - u^n}{2\sqrt{u^n}},$$

we find that

$$\begin{aligned} h(\theta) &= \sum_{i=1}^p \sqrt{r_i(\theta)^2} \\ &\leq h(\theta^n) + \frac{1}{2} \sum_{i=1}^p \frac{r_i^2(\theta) - r_i^2(\theta^n)}{\sqrt{r_i^2(\theta^n)}} \\ &= g(\theta | \theta^n). \end{aligned}$$

Minimizing  $g(\theta | \theta^n)$  is accomplished by minimizing the weighted sum of squares

$$\sum_{i=1}^p w_i(\theta^n) r_i(\theta)^2$$