Genesis of Gamma Bursts in Neural Local Field Potentials

Priscilla (Cindy) Greenwood

U.B.C.

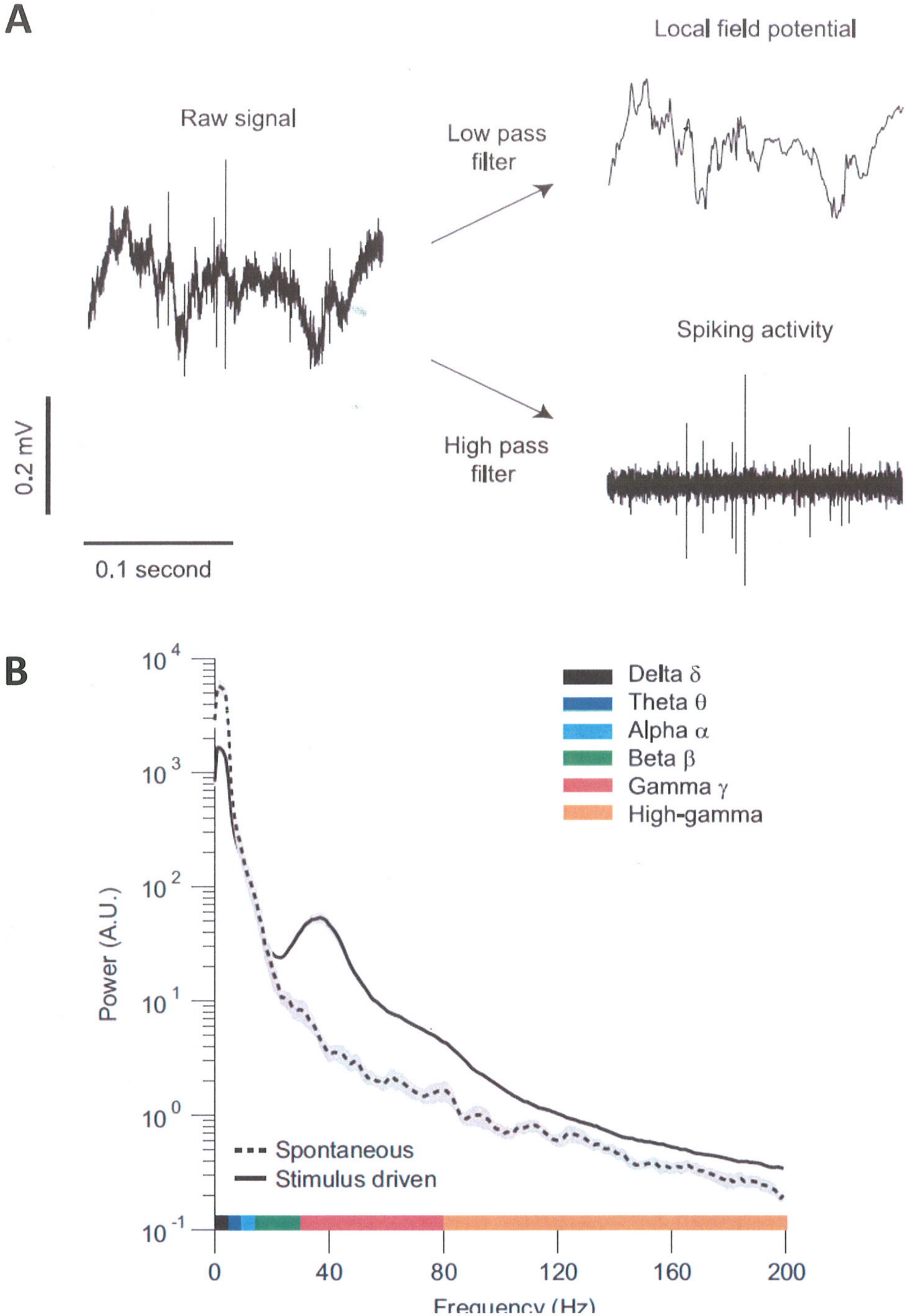
with

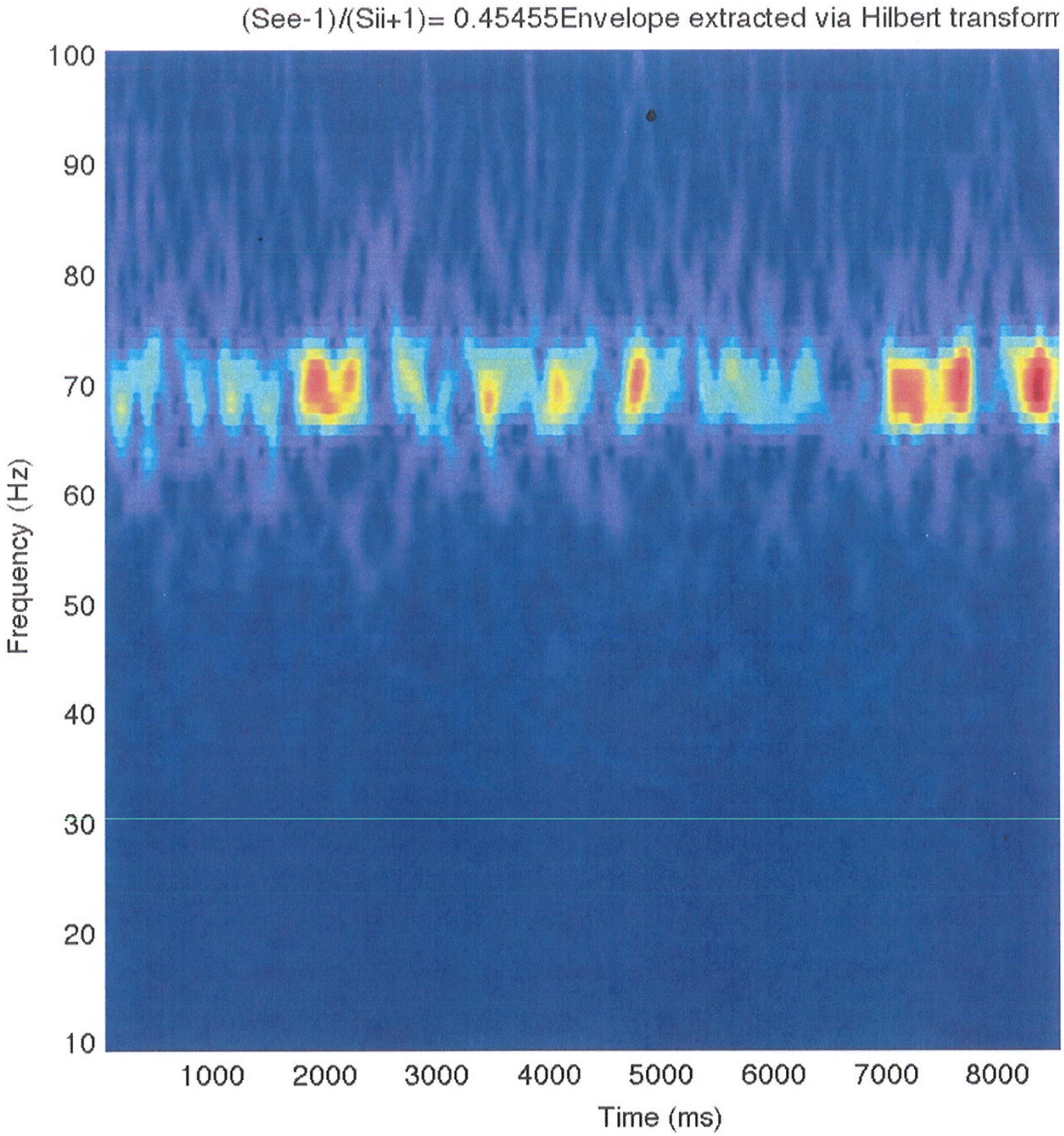
Mark McDonnell

University of Southern Australia

Lawrence Ward

U.B.C.


A
Local field potential
Raw signal
Low pass
filter
Spiking activity
High pass
filter
0.2 mV
0.1 second
B
Delta δ
Theta θ
Alpha α
Beta β
Gamma γ
High-gamma
Power (A.U.)
Spontaneous
Stimulus driven
0
40
80
120
160
200


(See-1)/(Sii+1)= 0.45455Envelope extracted via Hilbert transforr
100
90
80
70
60
50
40
30
20
10
Frequency (Hz)
1000
2000
3000
4000
5000
6000
7000
8000
Time (ms)

Model of interacting populations of excitatory and inhibitory neurons

$V_E(t)$, $V_I(t)$ are potentials (or conductances, firing rates).

$$\tau_E \, dV_E(t) = (-V_E(t) + S_{EE} V_E(t) - S_{EI} V_I(t)) dt + \sigma_E \, dW_E(t)$$

$$\tau_I \, dV_I(t) = (-V_I(t) - S_{II} V_I(t) + S_{IE} V_E(t)) dt + \sigma_I \, dW_I(t)$$

i.e.

$$dV = -AV\,dt + B\,dW$$

where

$$A = \begin{pmatrix} (1-S_{EE})/\tau_E & S_{EI}/\tau_E \\ -S_{IE}/\tau_I & (1+S_{II})/\tau_I \end{pmatrix}$$

$$B = \begin{pmatrix} \sigma_E/\tau_E & 0 \\ 0 & \sigma_I/\tau_I \end{pmatrix}$$

We want: the deterministic model has damped oscillations to $(0,0)$.

Model: $dV = -AV\,dt + B\,dW$

Damped oscillations of deterministic model implies

$-A$ has eigenvalues $-\lambda \pm i\omega$

$0 < \lambda << \omega$

$$\lambda = \frac{1}{2}\left(\frac{1-S_{EE}}{\tau_E} + \frac{1+S_{II}}{\tau_I} \right)$$

$$\omega = \left(\frac{S_{EI}S_{IE}}{\tau_E \tau_I} - \frac{1}{4}\left(\frac{1-S_{EE}}{\tau_E} - \frac{1+S_{II}}{\tau_I} \right)^2 \right)^{1/2}$$

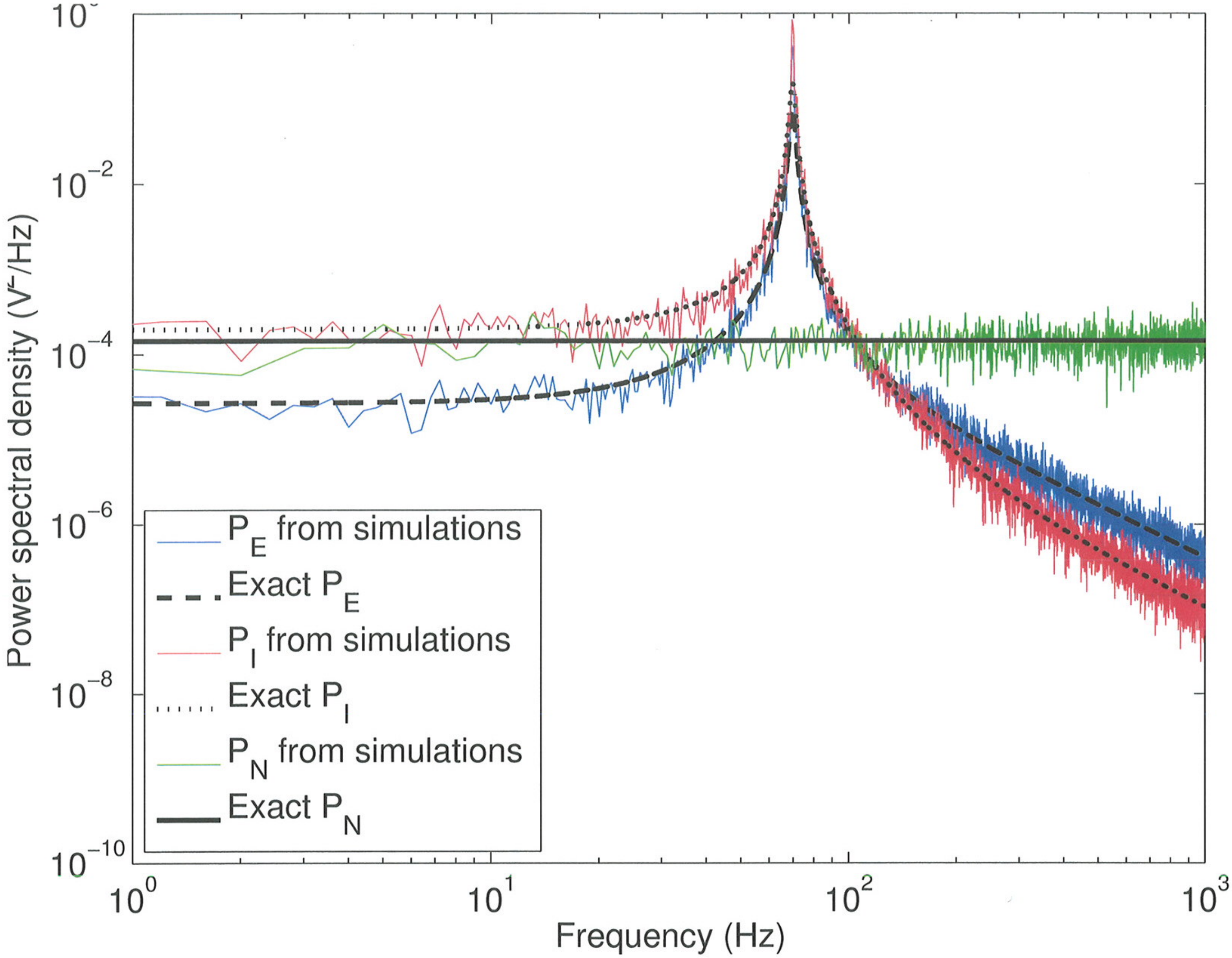
The power spectral density of $V_E(t)$ is

$$PSD_E(f) = \frac{A_{12}^2 \sigma_I^2/\tau_I^2 + (A_{22}^2 + f^2)\sigma_E^2/\tau_E^2}{(\omega^2+\lambda^2-f^2)^2 + 4(\lambda f)^2}$$

For large f this is $\sim 1/f^2$; for small f this is $\sim$ constant.

This PSD looks like the PSD of an

Ornstein-Uhlenbeck process, except that →



Power spectral density (V²/Hz)
Frequency (Hz)
P_E from simulations
Exact P_E
P_I from simulations
Exact P_I
P_N from simulations
Exact P_N

Finding Gamma Bursts in Sample Paths of $V(t)$.

Gamma bursts occur as $V(t)$ evolves in time. The PSD cannot see them. However, sample paths of $V(t)$ have additional structure.

Using [Baxendale + G. J Math Bio 2011] we see that as λ/ω becomes small,

$$\boxed{V(t) = (V_E(t), V_I(t)) \approx \frac{\sigma}{\lambda} Q R_{-\omega t} S(\lambda t),}$$

where

$$S(t) = (S_1(t), S_2(t)), \text{ independent O.U. processes.}$$

$$R_s = \begin{pmatrix} \cos s & -\sin s \\ \sin s & \cos s \end{pmatrix},$$

$$Q \text{ is a matrix } \ni Q^{-1}(-A)Q = \begin{pmatrix} -\lambda & \omega \\ -\omega & -\lambda \end{pmatrix}$$

$$\text{e.g. } Q = \begin{pmatrix} -\omega & \lambda - A_{11} \\ 0 & -A_{21} \end{pmatrix},$$

$$\sigma = \left(\tfrac{1}{2} \operatorname{Tr}\left(Q^{-1} B B^T (Q^{-1})^T\right)\right)^{1/2}$$

The approximation:

$$dV(t) = -AV(t)\,dt + B\,dW_t$$

Let $Y(t) = Q^{-1}V(t)$:

$$dY(t) = \begin{pmatrix} -\lambda & \omega \\ -\omega & -\lambda \end{pmatrix} Y(t)\,dt + Q^{-1}B\,dW_t.$$

Let $Y(t) = R_{-\omega t} Z(t)$:

$$dZ(t) = -\lambda Z(t)\,dt + R_{\omega t} Q^{-1} B\,dW(t)$$

Let $U(t) = \frac{\sqrt{\lambda}}{\sigma} Z(t/\lambda)$:

$$dU(t) = -U(t)\,dt + R_{\omega t/\lambda} \frac{Q^{-1}B}{\sigma}\,dW(t)$$

Limit $\lambda/\omega \to 0$:

$$dS(t) = -S(t) + dW(t)$$

Unwind:

$$V(t) \approx \frac{\sigma}{\lambda} Q R_{-\omega t} S(\lambda t)$$

Gamma bursts are related to amplitude

$$S(t) = |S(t)| \left(\cos\phi(t),\ \sin\phi(t)\right)$$

where

$$\phi(t) = \arctan S_1(t)/S_2(t).$$

The approximation

$$V(t) \approx \frac{\sigma}{\sqrt{\lambda}} Q\, R_{-\omega t}\, S(\lambda t)$$

reads

$$\boxed{V(t) \approx \frac{\sigma}{\sqrt{\lambda}} Q\, |S(\lambda t)| \left(\cos(-\omega t + \phi(\lambda t)),\ \sin(-\omega t + \phi(\lambda t))\right)}$$

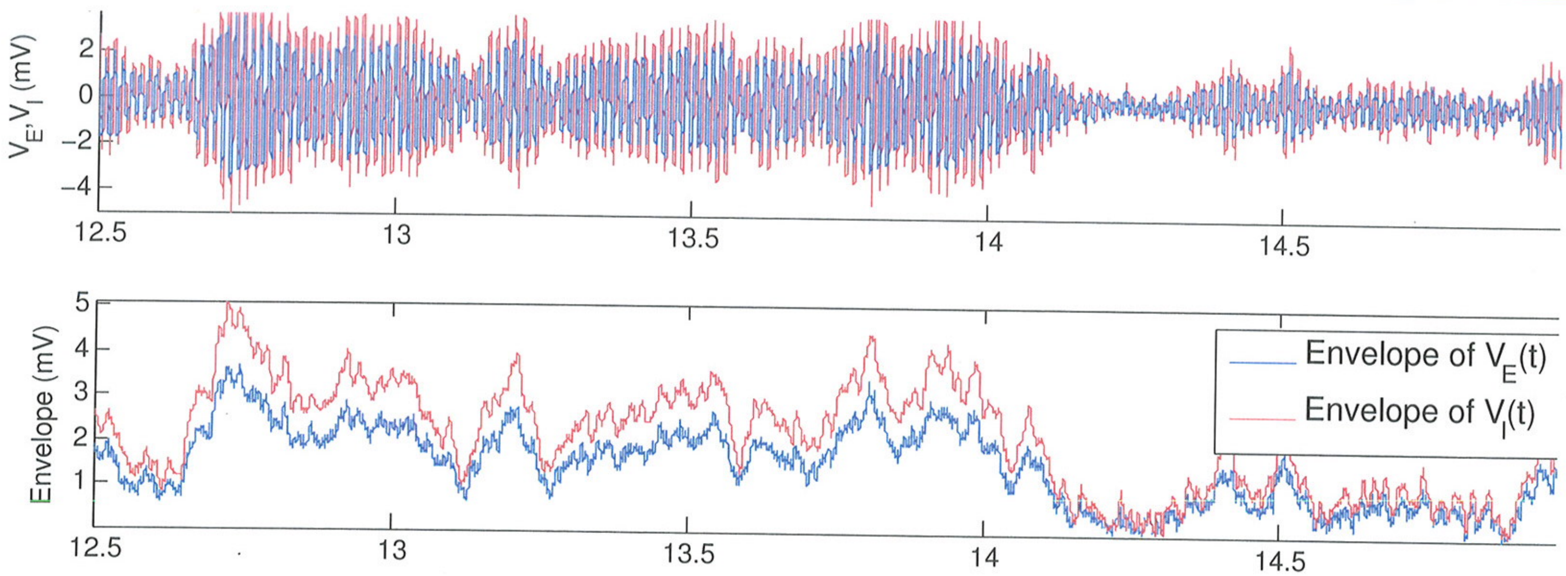
The matrix Q produces a constant phase difference between $V_E(t)$ and $V_I(t)$ of

$$2\omega / A_{11} - A_{22}.$$

The ratio of their magnitudes is

$$\left(-A_{21}/A_{12}\right)^{1/2}$$

Gamma bursts are excursions of $V_E(t)$, $V_I(t)$ away from zero.



V_E, V_I (mV)
2
0
−2
−4
12.5
13
13.5
14
14.5
Envelope (mV)
5
4
3
2
1
Envelope of $V_E(t)$
Envelope of $V_I(t)$

The processes

$$Z(t) \equiv |S(t)|$$

and

$$\phi(t) = \arctan S_1(t)/S_2(t)$$

satisfy the system of stochastic differential equations

$$dZ(t) = \left(\frac{1}{2Z(t)} - Z(t)\right)dt + dW(t)$$

$$d\phi(t) = \frac{1}{Z(t)}\, db(t),$$

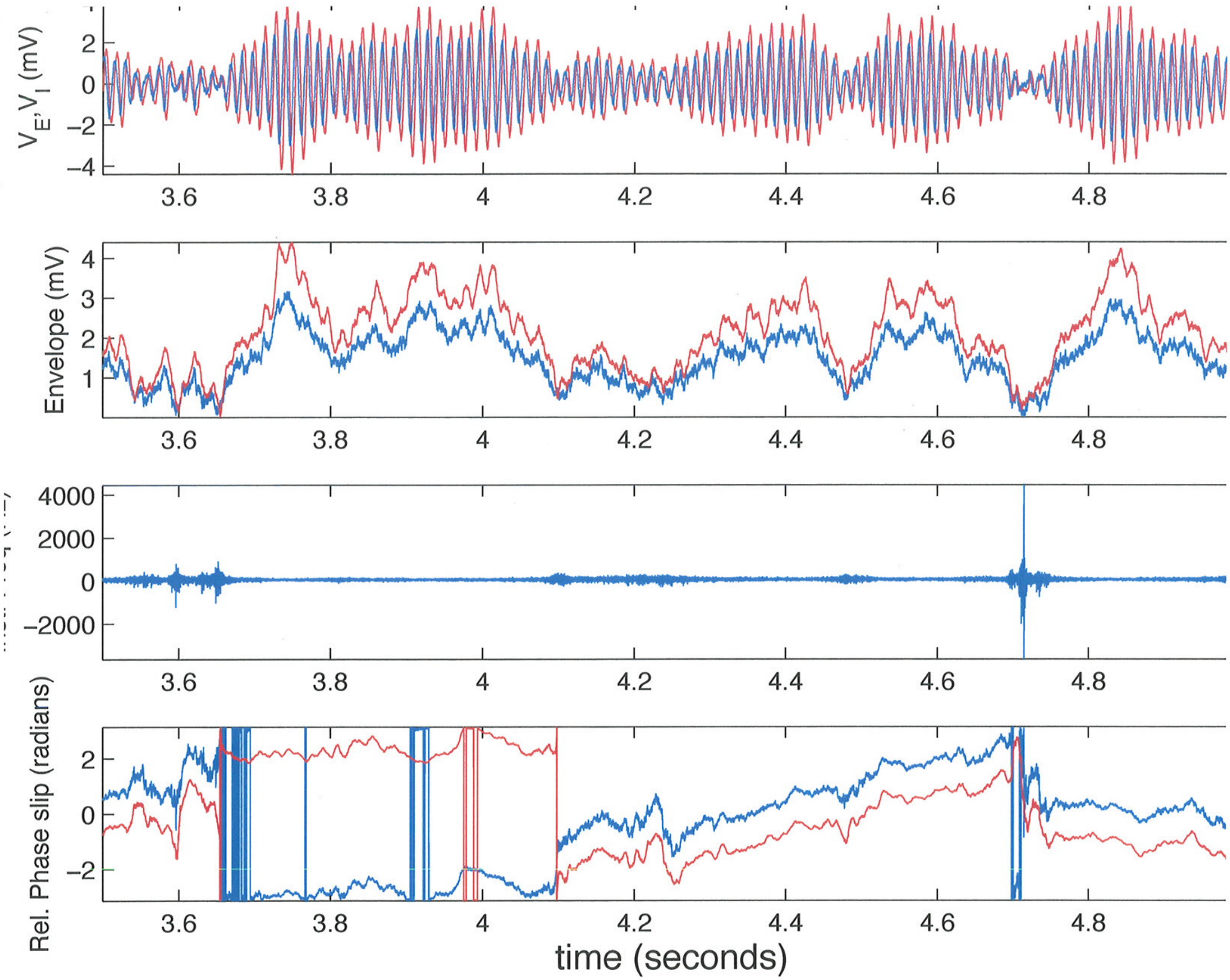
where $W(t)$ is Brownian motion

and $b(t)$ is Brownian motion run on a unit circle.

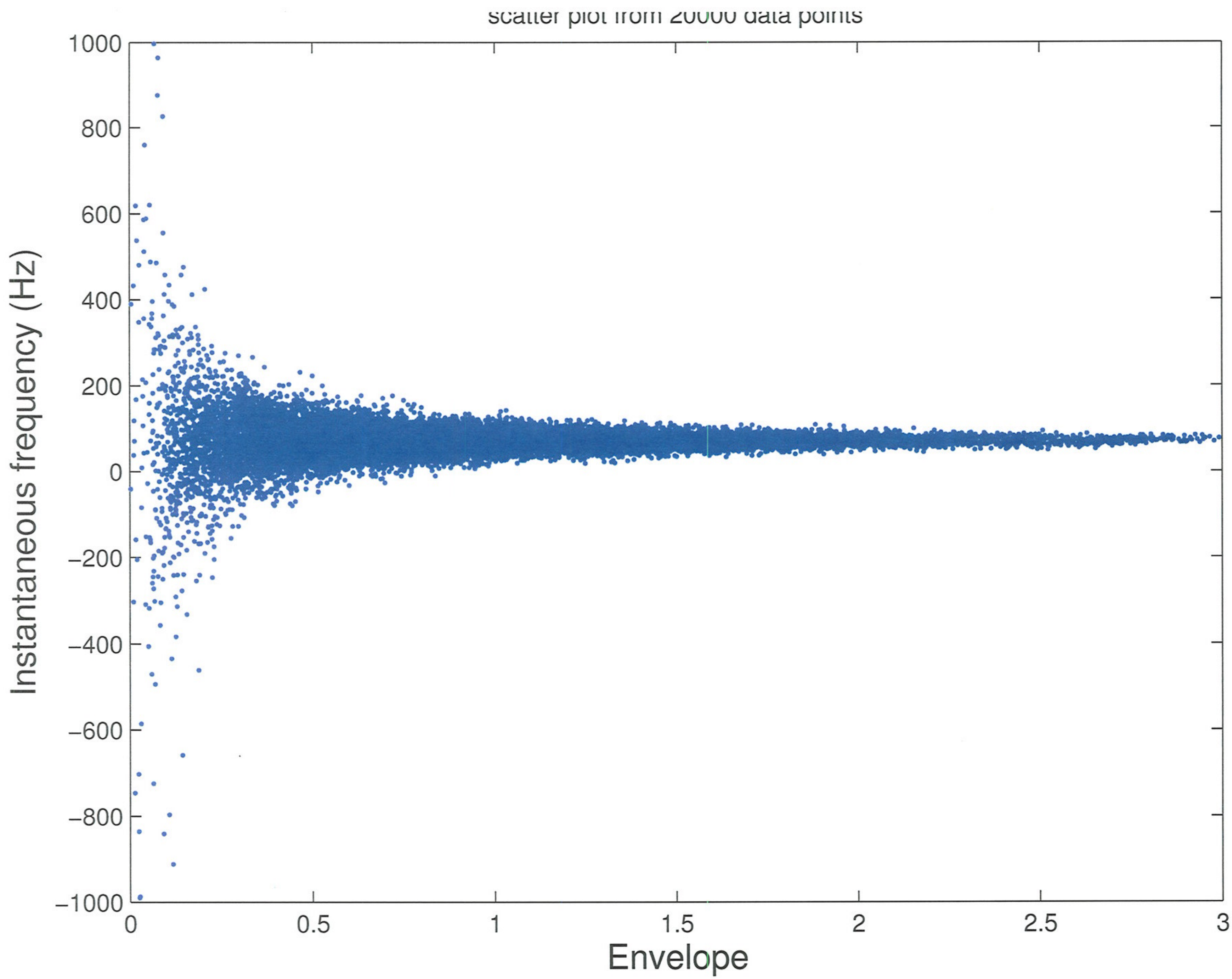
In $V(t) \approx \frac{\sigma}{\sqrt{\lambda}} Q |S(\lambda t)| \left(\cos(-\omega t + \phi(\lambda t)), \sin(-\omega t + \phi(\lambda t))\right)$

they run slowly, at rate λ, relative to ω.

Phase-slip of $V(t)$ relative to the rotation $R_{-\omega t}$ moves quickly (peaks of "instantaneous frequency") when $Z(t)$ is near zero, i.e. between excursions of the O.U. process $S(t)$.



V_E, V_I (mV)
Envelope (mV)
Rel. Phase slip (radians)
time (seconds)


scatter plot from 20000 data points
Instantaneous frequency (Hz)
1000
800
600
400
200
0
−200
−400
−600
−800
−1000
0
0.5
1
1.5
2
2.5
3
Envelope