Loève Prize 2017

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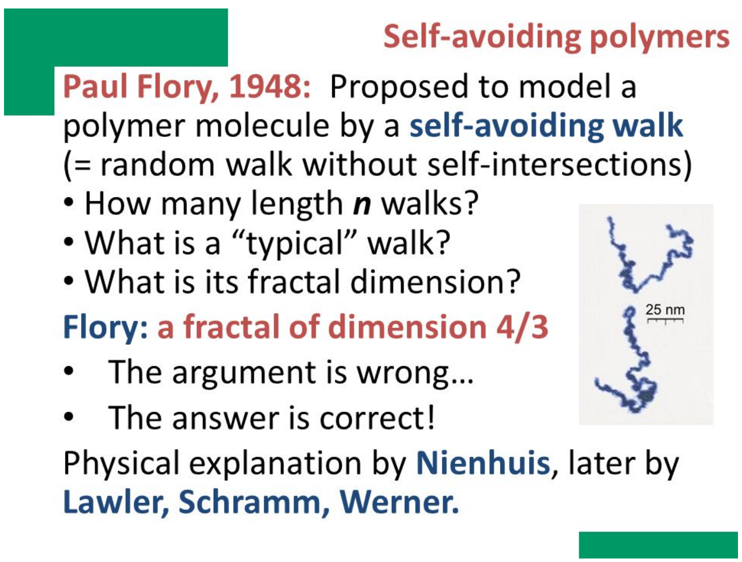
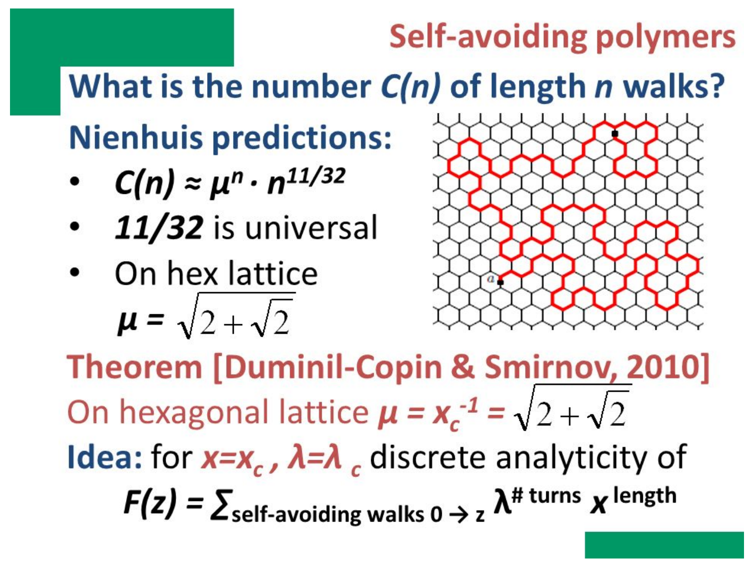
The connective constant of the honeycomb lattice equals $\sqrt{2+\sqrt{2}}$

with S. Smirnov, Annals of Mathematics 175(3) (2012, 132 citations).

This paper proves that the number c_n of self-avoiding walks (SAW) of length n on the honeycomb lattice satisfies $c^{1/n} \to \sqrt{2+\sqrt{2}}$ using (weak) discrete holomorphic observables. The idea of harvesting a weak notion of discrete holomorphicity was then used in several other papers. The SAW model is notoriously difficult, and this paper represents one of the only available result in 2D (the value $\sqrt{2+\sqrt{2}}$ was conjectured by Nienhuis in the 80's). It also constitutes a first step towards the proof of conformal invariance of the model.

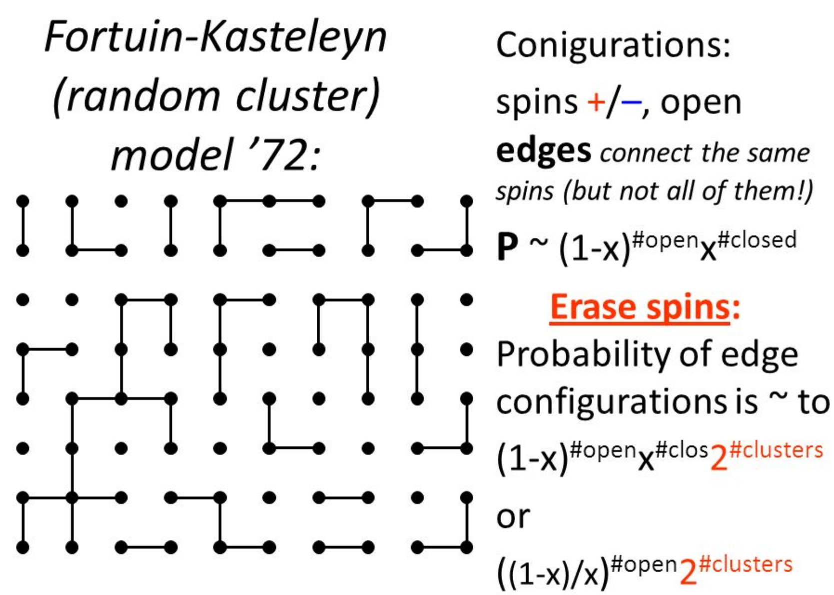




The self-dual point of the two-dimensional random-cluster model is critical for $q \geq 1$.

with V. Beffara, Probability Theory and Related Fields 153(3) (2012, 102 citations).

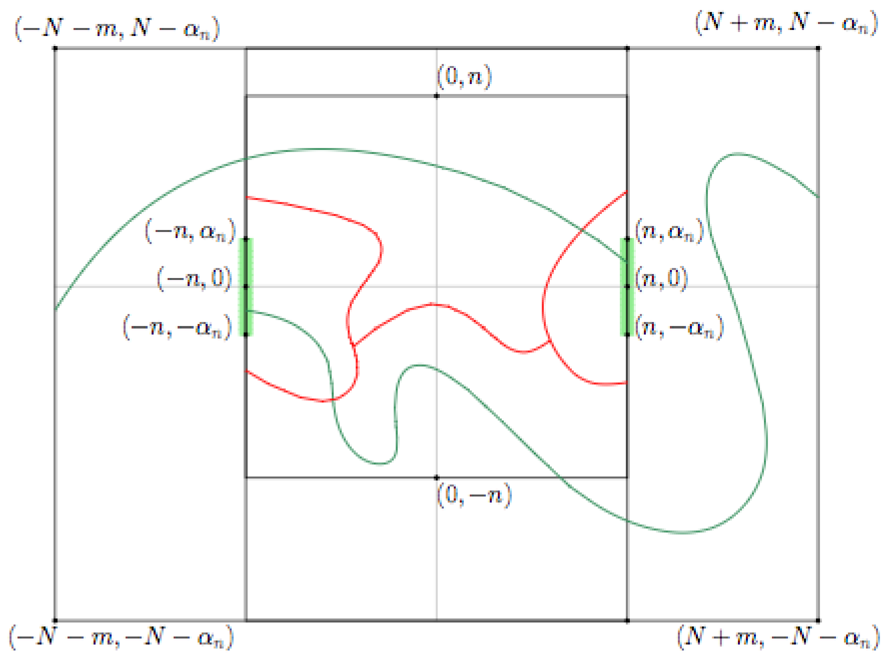
This paper provides the first rigorous computation of the critical point p_c for the random-cluster model (RCM). This long standing question goes back to the introduction of the model, where duality relations were used to predict non-rigorously the value of p_c. The proof is based on a new Russo-Seymour-Welsh result and sharp threshold theorems coming from the theory of boolean functions. These techniques and the results of the paper were used to prove many new theorems in the field.



Connection probabilities and RSW-type bounds for the FK Ising model

with C. Hongler and P. Nolin, Communications in Pure and Applied Math. 64(9) (2011, 56 citations)

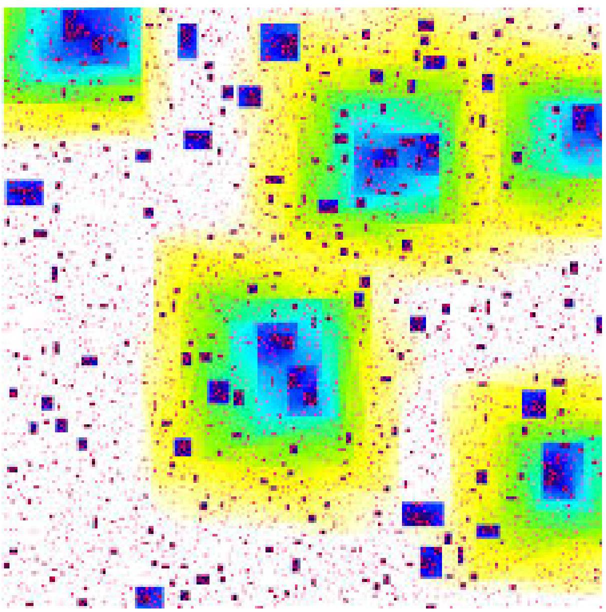
This paper contains the first example of a RSW-type theory for percolation models with dependency. The result was later used in a variety of subsequent papers, including the proof that the Glauber dynamics of the critical Ising model mixes in polynomial time, and the conformal invariance of interfaces in the critical Ising model (this result was proved in a joint paper with D. Chelkak, C. Hongler, P. Nolin and S. Smirnov gathering 90 citations).

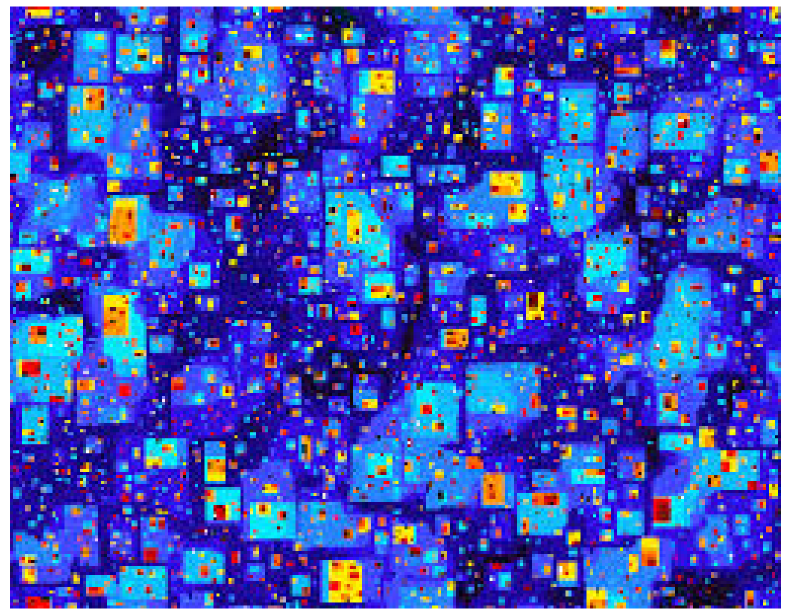


The sharp threshold for bootstrap percolation in all dimensions

with J. Balogh, B. Bollobs and R. Morris, Transaction of the American Mathematical Society 354 (2012, 118 citations).

This paper answered a long standing question on the sharpness of the phase transition for a famous monotonic cellular automaton, called bootstrap percolation. The techniques (combining Probability Theory and Combinatorics) developed in the proof were later used to treat universality questions for more general monotonic automata.





[Random Currents and Continuity of Ising Model's Spontaneous Magnetization]

with M. Aizenman and V. Sidoravicius, Communications in Mathematical Physics 334 (2015, 42 citations).

This paper provides the first rigorous proof that the Ising model undergoes a continuous phase transition in 3D, and constitutes as such one of the only results on critical 3D systems coming from statistical physics. The proof relies on a geometric analysis of a graphical representation of the Ising model. Such point of view is now adopted in several papers studying the Ising model.

Continuity of the Phase Transition for Planar Random-Cluster and Potts Models with $1 \leq q \leq 4$

with V. Tassion and V. Sidoravicius, Communications in Mathematical Physics 349 (1), (2017, 36 citations).

This paper develops a renormalization scheme for crossing probabilities in the critical random cluster model. Combined with the weak holomorphicity of parafermionic observables, one can prove that the phase transition of the random-cluster model with cluster-weight $q \in [1, 4]$ is continuous. This answers half of a conjecture by Baxter about the behavior of the 2D Potts models. Since then, the renormalization techniques have been used in other models to prove a dichotomy result: either RSW-type estimates are valid, or there is exponential decay of correlations in the disordered phase.

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Discontinuity of the phase transition for the planar random-cluster and Potts models with $q > 4$

with M. Gagnebin, M. Harel, I. Manolescu, V. Tassion arXiv preprint arXiv:1611.09877 (7 citations)

This paper completes Baxter's conjecture on the behavior of the 2D Potts model by proving that the phase transition of the random-cluster model is discontinuous as soon as the cluster-weight q is strictly larger than 4. The techniques used in this paper are quite different from the one used in the companion paper [5]. In this paper it was justified rigorously the Bethe Ansatz for the associated 6V model. Together with probabilistic techniques, this allowed to compute explicitly the correlation length of the model at criticality, thus confirming a prediction of Baxter.

Sharp phase transition for the random-cluster and Potts models via decision trees.

with A. Raoufi and V. Tassion, May 2017.

This paper provides a general theory of sharpness of lattice spin models that applies to a wide variety of models. The proof uses the theory of randomized algorithms coming from computer science. As a direct application, the paper answers a long-standing conjecture about exponential decay of correlations for the random-cluster model and the Potts model. The techniques also extend to treat continuum percolation models, and two papers were published subsequently to this one, proving conjectures about sharpness of the phase transition for Voronoi and Poisson-Boolean percolation.

A new proof of the sharpness of the phase transition for Bernoulli percolation and the Ising model

with V. Tassion, Comm. Math. Phys. 343(2) (2015, 26 citations).

This paper provided a completely new derivation of an important result in percolation theory dating from the 80. In addition to be extremely short (only 3 pages for Bernoulli percolation) and elementary, the proof exploits a new characterization of the critical point, which can be also used to study other models.