3. Let $X_n$ be an irreducible chain with transition matrix $P$. Let $Y_n$ be the jump chain $Y_n = X(T_n)$ where $T_0 = 0$ and

$$T_{n+1} = \min\{m > T_n : X_m \neq X(T_n)\}.$$

(a) Show that $Y_n$ is Markov, and write its transition matrix $Q$ in terms of $P$.

(b) Show that $(Y_n)$ is recurrent iff $(X_n)$ is recurrent.

(c) Assuming recurrence, find the relation between the $P$-invariant measure and the $Q$-invariant measure.

(d) Deduce that, on an infinite state space, it is possible for $(Y_n)$ to be positive-recurrent while $(X_n)$ is not.

4. Let $X_n$ be a finite irreducible chain with transition matrix $P$. Fix a subset $A$ of $S$. Define a transition matrix $Q$ on $A$ by

$$q(i,j) = p(i,j)/\sum_{k \in A} p(i,k).$$

Suppose $Q$ is irreducible. In the case where $P$ is reversible, find a simple explicit formula for the stationary distribution $\pi^*$ of $Q$ in terms of $P$ and its stationary distribution $\pi$. Give an example to show that the formula may not hold in the non-reversible case.