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CHAPTER 1

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Distinguishing Two Dimensions of Uncertainty

On April 29, 2011 Barack Obama made one of the most difficult decisions of his presidency: launch an attack on a compound in Pakistan that intelligence agents suspected was the home of Osama bin Laden. In an interview Obama described the raid as the longest 40 minutes of his life. He attributed that tension to two sources. First, he was not certain that bin Laden was actually residing in the compound. "As outstanding a job as our intelligence teams did," said the President, "At the end of the day, this was still a 55/45 situation. I mean, we could not say definitively that bin Laden was there." Second, regardless of whether bin Laden was residing in the compound, it was not certain that the operation would succeed. Asked about this concern the President cited operations by previous presidents that had failed due to chance factors, saying, "You're making your best call, your best shot, and something goes wrong – because these are tough, complicated operations."

Note that these two sources of uncertainty are qualitatively distinct. The first reflects the President's lack of confidence in his knowledge of a fact (i.e., whether or not bin Laden was residing at the compound). The second reflects variability in possible realizations of an event that is largely stochastic in nature – if the mission were to be run several times it would succeed on some occasions and fail on others due to unpredictable causes (e.g., performance of mechanical equipment, effectiveness of U.S. troops and bin Laden's defenders on a particular night).

The notion that uncertainty can take multiple forms is nearly as old as probability theory itself (Hacking, 1975). Pascal and Fermat tried to address the question of how one ought to divide the stakes of a game of chance that has been prematurely interrupted (see e.g., Devlin, 2008). Their probability theory was based on an *aleatory* conception of uncertainty involving unknown outcomes that can differ each time one runs an experiment under similar conditions. Shortly thereafter Pascal framed the choice of whether or not to believe in God as a wager with an outcome that depends on whether or not God exists. He thus advanced an *epistemic* conception of uncertainty involving missing knowledge concerning a fact that either is or is not true. Disagreement concerning the nature of uncertainty persists to this day in the two dominant schools of probability theorizing, with frequentists treating probability as long-run stable frequencies of events, and Bayesians treating probability as a measure of subjective degree of belief.

We assert that the philosophical bifurcation of uncertainty mirrors ambivalent intuitions that reside within most decision makers. Returning to the opening example, the question of whether or not bin Laden was in the compound could be construed as entailing primarily epistemic uncertainty, whereas the question of the raid's propensity to succeed could be construed as entailing primarily aleatory uncertainty. We argue that these forms of uncertainty are not mutually exclusive – indeed, the question of whether Obama's raid would succeed involved both kinds – and that there are systematic behavioral consequences of whether one form of uncertainty or another is particularly salient to a decision maker. We argue further that judgment under uncertainty entails both a (conscious or unconscious) *attribution* to epistemic and/or aleatory sources, and an assessment of one's *degree* of uncertainty corresponding to each relevant component.

In some cases, epistemic and aleatory uncertainty may relate to separate target events. For instance, the success of Obama's raid may be seen as depending on: (a) an epistemic judgment of whether bin Laden is in the compound and (b) an aleatory judgment of whether the military operation will succeed. In other cases epistemic uncertainty may qualify aleatory uncertainty concerning a given target event. For instance, an aleatory assessment that the military operation would succeed most of the time may also entail some epistemic uncertainty due to a lack of confidence in the assumptions underlying that assessment.

In this chapter we begin by characterizing cognitive differences between pure epistemic and pure aleatory uncertainty and reviewing empirical evidence that decision makers distinguish such variants. We next propose a simple two-dimensional framework that depicts how *degrees* of epistemic and aleatory uncertainty may interact in likelihood judgment when the former qualifies the latter. We then illustrate how this framework can generate novel empirical predictions, and conclude with a brief discussion.

Distinguishing Epistemic from Aleatory Uncertainty

Table 1 summarizes several key features characterizing the distinction between epistemic and aleatory uncertainty that emerge from our review of the judgment and decision making literature. Pure epistemic uncertainty: (1) is represented in terms of a single case, (2) is focused on the extent to which an event is or will be true or false, (3) is naturally measured by confidence in one's knowledge or model of the causal system determining the outcome, and (4) is attributed to missing information or expertise. Pure aleatory uncertainty, in contrast: (1) is represented in relation to a class of possible outcomes, (2) is focused on assessing an event's propensity, (3) is naturally measured by relative frequency, and (4) is attributed to stochastic behavior.

	Epistemic	**Aleatory**
1. Representation	Single case	Class of possible outcomes
2. Focus of Prediction	Binary truth value	Event propensity
3. Probability Interpretation	Confidence	Relative frequency

4. Attribution of Uncertainty	Inadequate knowledge	Stochastic behavior
5. Information Search	Patterns, causes, facts	Relative frequencies
6. Linguistic Marker	"Sure," "Confident"	"Chance," "Probability"

Table 1. Distinguishing Epistemic from Aleatory Uncertainty

Several authors have proposed frameworks for distinguishing variants of uncertainty that capture some of the features listed above (e.g., Howell & Burnett, 1978; Kahneman & Tversky, 1982; Peterson & Pitz, 1988; Teigen, 1994; Rowe, 1994; Dequesch, 2004). First, several authors have distinguished case-based reasoning from class-based reasoning (features 1–3). For instance, Howell & Burnett (1978) distinguish *nonfrequentistic* events that are "for all intents and purposes unique" from *frequentistic* events that "occur in a repetitive fashion." In a similar vein Peterson & Pitz (1988) distinguish *confidence* that refers to belief that a given prediction is correct from *uncertainty* that refers to beliefs about possible values for an unknown quantity. Likewise, Gigerenzer (1994) distinguishes between single-event probabilities and relative frequencies. Second, some authors distinguish different sources to which uncertainty is attributed (feature 4). For instance, Howell and Burnett (1978) distinguish *internal* events over which people think they exert partial control from *external* events over which they think they have no control (see also Teigen, 1994).

Kahneman and Tversky's (1982) framework incorporates both of these sets of features, distinguishing between *internal uncertainty*, which is attributed to one's state of knowledge and *external uncertainty,* which is attributed to the external world. They further distinguish two forms of external uncertainty: the *singular* mode in which probabilities are viewed as propensities of a particular target event and *distributional* mode in which the event in question is seen as an instance of a class of similar cases. Note that in our view, the important attributional distinction is between one's knowledge, skill, or model of the world versus stochastic behavior that is treated as otherwise unpredictable. Thus, our "epistemic" category generally corresponds to both internal uncertainty and external-singular uncertainty, whereas our "aleatory" category generally corresponds to external-distributional uncertainty. Moreover, we see these two variants of uncertainty as not mutually exclusive.

To the extent that epistemic and aleatory uncertainty are attributed to different sources they also can be distinguished by distinct coping strategies: epistemic uncertainty can be reduced by searching for patterns or causality, whereas aleatory uncertainty cannot be reduced but can be managed by determining the relative propensities of events (Feature 5).

Finally, epistemic and aleatory uncertainty have distinct markers in natural language. New data (Fox, Ülkümen & Malle, 2011) suggest that epistemic uncertainty tends to be expressed using phrases like "I am 90% sure" or "I'm reasonably confident" whereas aleatory uncertainty tends to be expressed using phrases like “I think there’s a 90% chance” or “I’d say there’s a high probability” (feature 6).

In our view the epistemic and aleatory dimensions provide a parsimonious framework for interpreting a wide range of empirical findings. We next turn to a more detailed review of evidence for the distinctions listed in table 1.

1. Representation

Epistemic uncertainty focuses attention on a single case that may occur (or a single statement that may be true) whereas aleatory uncertainty focuses attention on classes of possible outcomes in repeated realizations of an experiment. Even young children appear to make such an intuitive distinction. In an ingenious study, Robinson et al. (2006) asked children four to six years old to predict whether an orange or green colored building block would be drawn from a bag containing both colors. Children did so by placing a tray underneath doors marked with these colors. On trials in which the block had not yet been drawn (so that two classes of events were still possible), most children placed one tray under the orange door and one under the green door. However, on trials in which the experimenter had already selected a block but not revealed it (so that one case or the other was already true), most children placed a tray under a single door, apparently making their best guess concerning the block color that had already been determined.

The present account suggests that questions about the likelihood of a single event versus the relative frequency of a class of events may prime epistemic and aleatory representations, respectively. Interestingly, several studies have found that although participants tend to be overconfident assessing probabilities that their answers to general knowledge questions are correct, they tend to be *underconfident* when later asked to estimate the proportion of items that they had answered correctly (Sniezek, Paese & Switzer, 1990; Gigerenzer, et al., 1991; Griffin & Tversky, 1992). While the particular mechanism driving this phenomenon is unclear, the pattern suggests that evaluations of epistemic uncertainty (the likelihood that I answered this item correctly) and aleatory uncertainty (the proportion of times I answered correctly) rely on distinct information, weights, and/or processes.

2. Focus of prediction

Assessment of purely epistemic uncertainty generally entails evaluation of events that are (or will be) either true or false. Assessment of purely aleatory uncertainty, in contrast, entails evaluation of the propensity of each event under consideration, on the continuous unit interval. This suggests that judgments of purely epistemic events may be more sensitive to small differences in evidence strength and therefore tend toward more extreme values (probabilities of 0 or 1) than judgments of events that also entail aleatory uncertainty. For instance, if I am confident that France is slightly larger than Spain then I should judge the probability that France is larger than Spain to be 1. However, if I am confident that Norway’s national soccer team is slightly stronger than Sweden’s, I may suppose that the probability of Norway winning a particular match is much less than 1, perhaps only 0.6. Interestingly, Ronis and Yates (1987) found that participants were more overconfident when asked to judge probabilities concerning general knowledge items (i.e., assessments of pure epistemic uncertainty) than when asked to judge probabilities of outcomes of upcoming professional basketball games (i.e., assessments of largely aleatory uncertainty). More to the point, participants expressed complete certainty for general knowledge items more than 15 times as often as they did for basketball games. Similar patterns of higher confidence and greater willingness to express certainty for general knowledge questions than for future events were obtained by Fischhoff and MacGregor (1982), Wright (1982) and Wright & Wisudha (1982).

3. Probability interpretation

Judged probability under pure epistemic uncertainty is generally interpreted as an intensional measure of confidence whereas judged probability under pure aleatory uncertainty is generally interpreted as an extensional measure of relative frequency. Thus, eliciting relative frequency may prime more aleatory thinking – and therefore more extensional thinking – than eliciting probability numbers (Kahneman & Tversky, 1996). Indeed, several studies find that the conjunction fallacy (the tendency to judge the conjunction of a plausible and implausible event as more probable than the implausible event alone) is less common when participants are asked to judge relative frequencies rather than single event probabilities (e.g., Tversky & Kahneman, 1983; Fiedler, 1988; Hertwig & Gigerenzer, 1994) or when they are presented frequentistic rather than case-specific information (Reeves & Lockhart, 1993). Likewise, the common tendency to underweight base rates when updating beliefs from case information (Kahneman & Tversky, 1973) is attenuated when features of aleatory uncertainty are made more salient. For instance, the use of base rate data increases when problems are framed as repetitive rather than unique (Kahneman & Tversky, 1979), when base rate information is presented after case information (Krosnick, Li & Lehman, 1990), when base rates vary across trials within participant (Bar-Hillel & Fischhoff, 1981), when participants are asked to think as “statisticians” (Schwarz, Strack, Hilton & Naderer, 1991) or when they are induced to think “as a scientist analyzing data” rather than “understand the individual’s personality, professional inclinations and interests” (Zukier & Pepitone, 1984).

4. Attribution of uncertainty

Aleatory uncertainty is attributed to outcomes that for practical purposes cannot be predicted and are therefore treated as stochastic (e.g., the result of a coin flip), whereas epistemic uncertainty is attributed to missing information or expertise (e.g., whether or not one has correctly answered a question on an exam) or inadequacy of one’s model of aleatory uncertainty (e.g., whether or not a financial forecast is based on valid assumptions).

One way to detect attribution of uncertainty is by observing betting behavior: People generally find it aversive to bet in situations of greater epistemic uncertainty, holding judged probability (i.e., aleatory uncertainty) constant. For example, reluctance to bet in conditions of higher epistemic uncertainty seems to underlie Ellsberg's (1961) ambiguity aversion phenomenon. Consider an urn containing 50 black balls and 50 red balls and one containing 100 red and black balls in an unknown proportion. Now consider a bet in which one must guess a color then draw a ball from one of the urns, and there is a prize for guessing correctly. Most people strongly prefer to bet on a draw from the known probability urn than the unknown probability urn regardless of color choice. Note that a draw from the known probability urn represents pure aleatory uncertainty (assuming one accepts that the draw is truly random) whereas a draw from the unknown probability urn represents a mixture of aleatory and epistemic uncertainty (because it is a random draw from an urn whose composition is unknown).

Now suppose the decision maker must choose a color from a known probability urn *after* the ball has been drawn but *before* the experimenter has revealed its color. In some sense the event has shifted from entailing pure aleatory uncertainty (a chance event about to commence) to pure epistemic uncertainty (the chance event has been resolved in an unknown way). Indeed, several authors have found that holding information constant, people prefer to guess about events or bet on them when epistemic uncertainty is less salient. For instance, Brun and Teigen (1990) found that participants preferred to guess the roll of a die, the sex of a child, and the outcome of a soccer game before the event rather than after the event. Similar demonstrations were provided by Rothbart and Snyder (1970) and Heath and Tversky (1991).

Increased salience of the epistemic dimension after an outcome is realized may also partially explain why events seem more predictable after they occurred than they did before they occurred (Fischhoff, 1975). Interestingly, this so-called "hindsight bias" is pronounced when plausible deterministic causes are cited (e.g., human skill or lack thereof), or when no causal attribution is provided, but it is virtually eliminated when the outcome is attributed to aleatory factors such as an unexpected act of nature (Wasserman, et al. 1991).

Note that a person can exhibit a high degree of epistemic uncertainty without this dimension being salient and therefore influencing betting preferences. For instance, Fox & Tversky (1995) found that the preference to bet on known probabilities rather than unknown probabilities (as with the Ellsberg urn example) diminished or disappeared in a between-subject design in which two groups each priced a bet on one of the two urns in isolation (so that epistemic uncertainty was presumably less salient). Likewise, they found that unsophisticated participants' reluctance to bet on an unfamiliar event (the future close of a particular stock) increased when they learned that more expert individuals (financial analysts and economics Ph.D. students) were also making the same choices, which presumably increased the salience but not degree of epistemic uncertainty.

Further evidence that epistemic and aleatory uncertainty entail distinct cognitive processes can be found in brain imaging data. Volz, Schubotz, and von Cramon (2004, 2005) asked participants to predict the winners of competitions between cartoon figures. Some participants learned a fixed set of rules that determined which figure would win each competition (e.g., yellow always beats blue), learning rules to different levels of proficiency, which could be seen as a manipulation of the level of epistemic uncertainty. Other participants learned that each pairing was associated with a fixed proportion of victories for one figure over another (e.g., *A* beats *B* 70% of the time), which could be seen as a manipulation of the level of aleatory uncertainty. Functional MRI identified distinctive activation for the rule-based (epistemic) task in a number of brain regions thought to subserve working memory functions.

5. Information search

If epistemic uncertainty is attributed to inadequate knowledge, then it should be reducible by searching for information that allows one to predict outcomes more accurately. Aleatory uncertainty, in contrast, cannot be reduced once one determines the relative frequency of possible outcomes. For example, consider a “two-armed bandit” environment where the decision maker chooses between two buttons, each of which sometimes delivers a reward and sometimes does not. An epistemic mindset would suggest varying one’s choices (i.e. an exploration strategy) to determine the pattern governing the sequence of rewards. In contrast, an aleatory mindset would suggest that after one determines which button delivers a reward more often, one should select that button on every trial (i.e., an exploitation strategy). Indeed, several recent studies have suggested that the well-documented tendency toward probability matching (choosing options in proportion to their relative probabilities of success) reflects a search for patterns in random sequences, whereas tendency toward maximizing (choosing the option that offers the highest probability of success) reflects an understanding that no such patterns exist in random sequences. For instance, Gaissmaier and Schooler (2008) find that probability matchers are more likely than maximizers to identify and take advantage of predictable patterns that they later encounter in outcome sequences that are not random. Likewise, Schul, Mayo, Burnstein, and Yahalom (2007) found that participants were more likely to maximize if they were told that allocation of a target was done through a chance process than if they were told that it was done by another person with an incentive to deceive.

6. Linguistic markers

If individuals intuitively distinguish cognitive concepts this should be reflected in natural language (Hutchins, 1996). The notion that epistemic and aleatory uncertainty might have markers in natural language was anticipated by Teigen (1988, pp. 33–34) and later empirically validated by Fox, Ülkümen and Malle (2011). These authors show that speakers and listeners appear to distinguish “internal mode” statements (e.g., “I am 80% sure that…” or “I am reasonably confident that…”) that express epistemic uncertainty from “external mode” statements (e.g., “I think there is an 80% chance that…” or “I believe there is a high probability that…”) that express aleatory uncertainty. For instance, they found that speakers place more weight on singular information (e.g., strength of an impression that an episode occurred) when completing internal stems (“I am ____ % sure that…”) and more weight on distributional information (e.g., relative frequencies) when completing external stems (“I think there is a ____% chance that…"). Meanwhile, these authors found that listeners more often associate internal language with singular reasoning (e.g., how two teams match up) and uncertainty attributed to the speaker's mind (e.g., uncertainty about a couple's decision to have children) whereas they more often associate external language with distributional reasoning (e.g., how often two teams have won in the past) and uncertainty attributed to stochastic processes (e.g., uncertainty about a couple's fertility).

A Simple Framework

A simple framework for integrating varying degrees of aleatory and epistemic uncertainty represents them as orthogonal dimensions. The first dimension represents *information* contained in the decision maker's subjective probability distribution over possible outcomes (i.e., the degree to which the decision maker differentiates probabilities over possible events). For the simple binary case in which the decision maker considers a target event and its complement, this can be interpreted as judged probability of the target event, as depicted on the vertical axis in figure 1. Note that as information decreases, aleatory uncertainty increases. Aleatory uncertainty can be measured by entropy, which is defined over a set of *n* possible events into which the state space is partitioned. For a binary partition of the state space, the entropy measure, $H = -\sum_{i=1}^{n} p_i log_2 p_i$, takes on a minimum value of 0 when the target event or its complement is certain and a maximum value of 1 when the target events and its complement are deemed equally likely.

The second dimension in this framework represents the level of subjective *knowledge* of the decision maker concerning the events in question. As subjective knowledge decreases, epistemic uncertainty increases. Subjective knowledge can be measured using a likert scale (e.g., "How knowledgeable do you feel concerning your judgment of this event?"), and is depicted as the horizontal axis in figure 1.

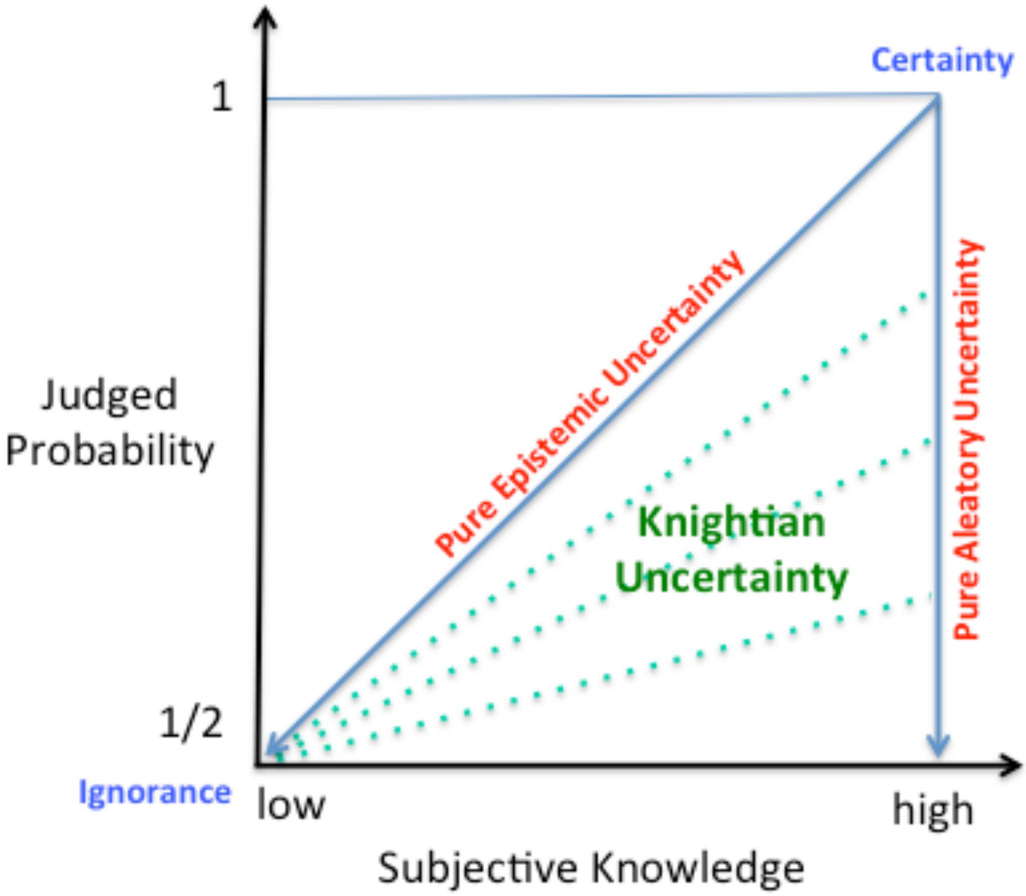

Figure 1. A Two-Dimensional Framework for Characterizing Uncertainty When Evaluating a Target Event Against its Complement

Pure aleatory uncertainty entails complete confidence in one's assessment of the probability distribution over possible outcomes (the vertical line at high subjective knowledge), and is the domain of decision under risk. Knightian uncertainty, in which the decision maker does not know the precise probability distribution over possible outcomes, includes everywhere in which there is some degree of epistemic uncertainty (i.e., west of the pure aleatory uncertainty line). Ignorance occurs at the point of maximum epistemic uncertainty (minimum subjective knowledge) and maximum aleatory uncertainty (maximum entropy), the lower left-hand corner of figure 1. Certainty occurs only in the absence of either form of uncertainty (i.e. maximum subjective knowledge and judged probability of 1 for the event or its complement), the upper right-hand corner of figure 1. The *x*-axis defines points at which the decision maker assesses probability to be equal for all events in the partition (i.e., the "ignorance prior" probability = ½ for the simple binary case), with confidence in that assessment increasing as one travels east.

Pure epistemic uncertainty (e.g., concerning general knowledge items) is most naturally measured by confidence or subjective knowledge. However, when forced to quantify this using a probability measure, one's feeling of confidence may be used as a proxy for evaluating likelihood (represented by the solid diagonal line in f igure 1). Thus, when the decision maker feels completely ignorant, her judged probabilities will tend toward 1/*n* (1/2 in a two-alternative forced choice paradigm), when the decision maker feels completely knowledgeable, her judged probabilities will tend toward 1 (and 0), and when the decision maker feels moderately knowledgeable, she will assess intermediate probabilities.

When uncertainty is partly aleatory, even a confident decision maker may assess probabilities less than 1 (e.g., an expert may believe that the stronger team would win a particular soccer match only 80% of the time). Moreover, as confidence in one's knowledge diminishes toward ignorance, belief will generally converge toward the ignorance prior probability (1/2 in the case of two teams). Indeed, some authors have documented a bias toward the ignorance prior probability that is more pronounced as self-rated knowledge decreases (Fox & Clemen, 2005; See, Fox, & Rottenstreich, 2006).

Points northwest of the "pure epistemic uncertainty" line represent less coherent assessments in which judged probability is high but subjective knowledge is low. The extreme case in the northwest corner, in which one believes that an event will definitely occur but feels completely ignorant about the topic, might be deemed the point of blind faith or superstition.

Some Implications

While the framework described in this chapter is a conceptual model, it does suggest some testable hypotheses concerning the relationship between judged probability, subjective knowledge, and evidence strength. A few examples follow.

First, judged probabilities will generally be more extreme (i.e. closer to 0 or 1) as subjective knowledge increases. Moreover, for pure epistemic uncertainty there should be a closer correspondence between ratings of knowledge and judged probabilities than for mixed uncertainty. For instance, confidence in my knowledge that Oslo is further west than Venice should correspond closely with my judged probability that Oslo is further west. In contrast, for partly aleatory uncertainty the relationship between subjective knowledge and judged probability should be attenuated. For instance, I may be highly confident in my knowledge that Norway has a slightly better soccer team than Sweden yet judge the probability of a victory by Norway to be only 60%.

Second, when both forms of uncertainty are present, the impact of a particular balance of evidence (e.g., the perceived relative strength of two teams) on judged probability will be amplified as subjective knowledge increases. Stated another way, for mixed uncertainty, judged probability will reflect an interaction between subjective knowledge and balance of evidence. This can be seen in the dotted lines in figure 1 that represent contours in which evidence ratios are held constant, but judged probabilities diverge as subjective knowledge increases.

Third, because epistemic uncertainty is represented as a binary (true/false) event and aleatory uncertainty is represented as a continuous propensity, we speculate that sensitivity of judged probabilities to differences in evidence strength will generally be greater for pure epistemic than mixed uncertainty, holding subjective knowledge level constant. Thus, if I am fairly certain that Paris is about 20% larger than London (pure epistemic uncertainty) then I may judge the probability that Paris is larger to be close to 1. However, if I feel fairly confident that one of two teams is 20% stronger than another (mixed uncertainty) I may judge the probability that the stronger team wins a match to be much less than 1.

These hypotheses can be formalized using an extension of support theory (Tversky & Koehler, 1994) that incorporates reliance on ignorance prior probabilities (see Fox & Rottenstreich, 2003). A fuller account of this model is beyond the scope of the present chapter.

Concluding Remarks

The notion that decision makers might distinguish variants of uncertainty is a theme that has been visited in the judgment and decision making literature from time to time in recent decades (e.g., Howell & Burnett, 1978; Kaheman & Tversky, 1982; Keren, 1987; Teigen, 1994). Nevertheless, most investigators continue to treat uncertainty as a unitary construct that can be measured by subjective probabilities with little regard for differences in information processing across its variants. In this chapter we enumerate several features that distinguish epistemic and aleatory dimensions of uncertainty, and we provide evidence from previous studies that decision makers intuitively differentiate these dimensions. We assert that these two forms of uncertainty are not mutually exclusive and propose a simple framework that incorporates both. We look forward to future research that refines, formalizes, and tests implications of this framework.

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