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Near Misses in Bingo

I. Introduction

A near miss is a special kind of failure to reach a goal, in which one is close to being successful. Near misses can arise in a variety of scenarios that involve components of both skill and luck. In games of pure chance, for example, a person that purchases a lottery ticket with the following combination (5 14 15 18 33) will experience a near miss if the actual lottery drawing turns out to be (5 14 15 18 34). Likewise, we can commonly observe near misses in games of skill, such as bowling. A near miss can be getting eleven strikes in a row, then hitting 9 pins on the last roll for a total score of 299 rather than a perfect 300.

In the case of skilled-based scenarios where we have a near miss, it can be a pretty robust indicator of future success. However, for games of pure chance, a near miss provides no information that can increase the likelihood of future success. Research also shows that in both skill-based and chance-based scenarios, players frequently interpret an occurrence of a near miss as an encouraging sign, confirming the player's strategy and raising his or her hopes for future success.

Because near-misses are ubiquitous across many different skill-based and luck-based games, there are many avenues that I can explore. For this project, I want to first explore the psychology behind near misses. I would then like to explore the frequency of near misses in bingo, a popular game of chance, and examine how the psychology of near misses may potentially affect a beginning bingo player.

II. Psychology of Near Misses

Near misses are a very interesting topic because they arise in everyday activities. In scenarios involving transportation safety and damage prevention, near misses can be taken as valuable zero-cost learning opportunities. There are many systems in place throughout the world that have improved safety through the anonymous reporting of near-miss incidences. A few examples include[1]:

- In 2005, the National Fire Fighter Near-Miss Reporting System was established, funded by grants from the U.S. Fire Administration and Fireman's Fund Insurance Company, and endorsed by the International Associations of Fire Chiefs and Fire Fighters. Any member of the fire service community is encouraged to submit a report when he/she is involved in, witnesses, or is told of a near-miss event. The report may be anonymous, and is not forwarded to any regulatory agency
- AORN, a US-based professional organization of perioperative registered nurses, has put in effect a voluntary near miss reporting system (covering medication or transfusion reactions, communication or consent issues, wrong patient or procedures, communication breakdown or technology malfunctions. An analysis of incidents allows safety alerts to be issued to AORN members.
- CIRAS (the Confidential Incident Reporting and Analysis System) is a confidential reporting system modelled upon ASRS and originally developed by the University of Strathclyde for use in the Scottish rail industry.
- In the United Kingdom, an aviation near miss report is known as an "airprox", an air proximity hazard, by the Civil Aviation Authority. Since reporting began, aircraft near misses continue to decline

Beyond near-miss reporting, studies have shown that commercial gambling systems, particularly instant lotteries and slot machines, are contrived to ensure a higher frequency of near misses than would be expected by chance alone. There are two main

[1] http://en.wikipedia.org/wiki/Near_miss_%28safety%29

reasons why near misses are significant in games of chance: they add excitement and encourage future play.

In an experiment conducted at Exeter, the researchers simulated five horses racing represented by five dots moving across a screen form a start line to a finish line. The test subjects found the close races, where the dots moved at the same fixed pace except for small increments of random forward movements, to be the most interesting and exciting. On the other hand, the test subjects found the decided races, where a horse separates early on from the pack, to be the least interesting and the worst.

In another famous experiment, researchers wanted to study how test subjects would respond to near misses in slot machines. In this experiment, the slot machines that were used had three wheels: first wheel had 70% red and 30% green logos, second wheel had 50% red and 50% green logos, and third wheel had 30% red and 70% green logos. The winning combinations were either assigned to be three red logos or three green logos, and the probability of winning in each case is 10.5%. In addition, because the wheels stop from left to right, the winning combination of three reds would have a much higher frequency of near misses than the winning combination of three greens. Thus, by prescribing the test subjects to the winning combination of three reds, we are inducing them to play on the slot machine with a higher incidence of near-misses.

Using these slot machines, and experiment was carried out on forty-four male high school students, in which they were each given 100 nickels to play with, and received 40 cents for each winning combination. The players could choose to stop at any time, and retain half of their earnings. The results showed that the test subjects who were placed in the near-miss group played significantly longer on average, suggesting that near-misses encourage future play.

III. Introduction to Bingo

A. Basic Rules of Bingo

Bingo is a game of chance played with randomly drawn numbers which players match against numbers that have been pre-printed on 5x5 matrices. The matrices may be printed on paper, card stock or electronically represented and are referred to as cards. Many versions conclude the game when the first person achieves a specified pattern from the drawn numbers. The winner is usually required to call out the word "Bingo!" which alerts the other players and caller of a possible win. All wins are checked for accuracy before the win is officially confirmed at which time the prize is secured and a new game is begun.[2]

A typical Bingo game utilizes the numbers 1 through 75. The five columns of the card are labeled 'B', 'I', 'N', 'G', and 'O' from left to right. The center space is usually marked with a "Free Space", and is considered automatically filled. The range of printed numbers that can appear on the card is normally restricted by column, with the 'B' column only containing numbers between 1 and 15 inclusive, the 'I' column containing only 16 through 30, 'N' containing 31 through 45, 'G' containing 46 through 60, and 'O' containing 61 through 75.

Figure 1: Example Bingo Card

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[2] http://en.wikipedia.org/wiki/Bingo_(U.S.)

Examining the card further, we can calculate the number of different bingo card permutations we can form using the rules stated above:

$$\#\ of\ Unique\ \text{"B"}\ Columns = 15\ \times 14\ \times 13\ \times 12\ \times 11\ =\ 360{,}360$$

$$\#\ of\ Unique\ \text{"I"}\ Columns = 15\ \times 14\ \times 13\ \times 12\ \times 11\ =\ 360{,}360$$

$$\#\ of\ Unique\ \text{"N"}\ Columns = 15\ \times 14\ \times 13\ \times 12\ =\ 32{,}760$$

$$\#\ of\ Unique\ \text{"G"}\ Columns = 15\ \times 14\ \times 13\ \times 12\ \times 11\ =\ 360{,}360$$

$$\#\ of\ Unique\ \text{"O"}\ Columns = 15\ \times 14\ \times 13\ \times 12\ \times 11\ =\ 360{,}360$$

$$Total\ \#\ of\ unique\ BINGO\ cards =\ Product\ of\ all\ five\ columns = 360{,}360^4\ \times 32{,}760$$
$$=\ 552{,}446{,}474{,}061{,}128{,}648{,}601{,}600{,}000 \approx\ \mathbf{552}\ \times\ \mathbf{10^{24}}$$

A player wins by completing a row, column, or diagonal. There are a total of 12 configurations that will complete a BINGO.

There are also additional winning possibilities on top of the 12 winning configurations stated above. Here are a few common modified versions of bingo:

- Postage Stamp: 2x2 square of marked squares in the upper-right-hand corner
- Corners: Another common special game requires players to cover the four corners
- Roving 'L': requires players to cover all B's and top or bottom row or all O's and top or bottom row
- Blackout: cover all 24 numbers and the free space

B. Simple Bingo Statistics

For this project, I would like to concentrate on bingo with the original winning conditions. In other words, a player wins only when they complete a row, column or diagonal.

To get a better understanding of the expected number of turns it takes before a single player achieves a BINGO, we observe the following distribution of turns it takes to achieve a BINGO with one player with ten thousand simulations:

Minimum	1Q	Median	3Q	Max	Mean	SD
9	36	43	49	68	42.28	9.86

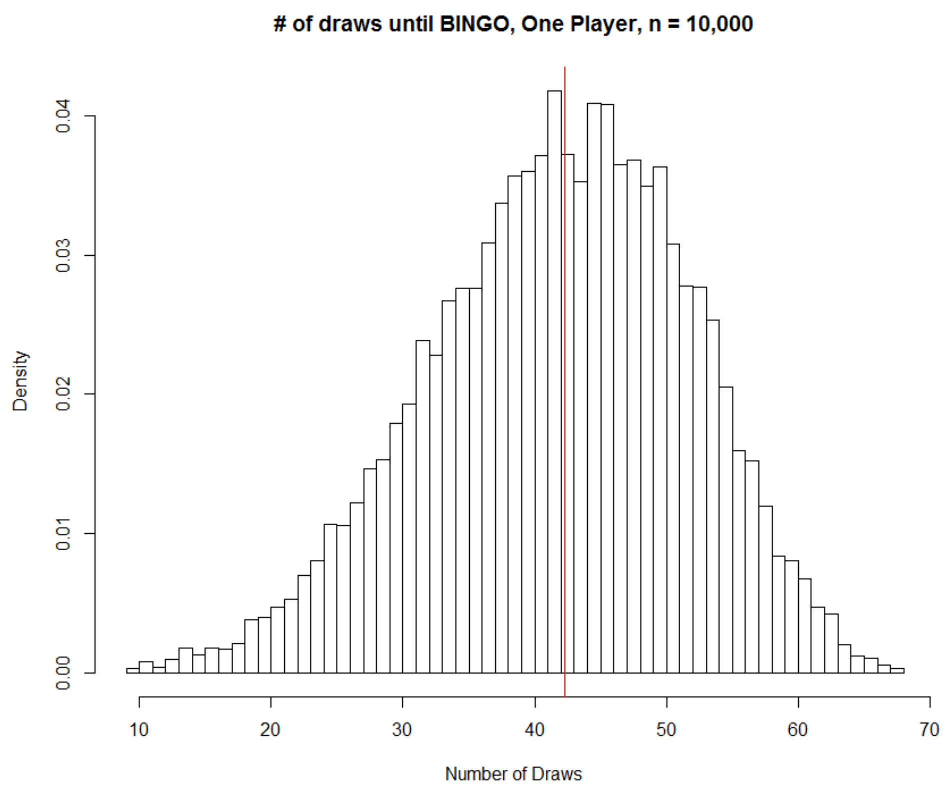

Figure 2: Ten thousand simulations of BINGO with one player. Average number of turns to achieve BINGO = 42.3 (red line). Standard deviation: 9.86.

From the results of my simulation, we see that a bingo game is slightly skewed to the right (longer games are more common). Overall, we should expect 42 to 43 numbers to be called before a single player achieves a BINGO.

C. Bingo Hall Sizes

Bingo starts to get a lot more interesting if we decide to add more players to the game. With more players, the prize pools increase in size, and the largest bingo halls in America can oftentimes boast a jackpot size in the neighborhood of one-hundred thousand dollars.

After sorting through Yelp and various local bingo review websites for the capacities of bingo halls across the United States, I have narrowed bingo games down to three main sizes:

1. Mega Bingo Halls & Casinos: **~1,200 players**
    a. Foxwoods Resort Casino (4,000 seats), San Manuel Indian Bingo & Casino (2,500 seats), Cherokee Tribal Bingo (2,000 seats), Penobscot High Stakes Bingo (1,800 seats), Ft. McDowell Casino and Radisson Hotel (1,700 seats), Potawatomi Bingo Casino (1,354 seats)
2. Large Bingo Halls & Casinos: **~300 players**
    a. Smaller Casinos
    b. Larger local bingo halls
3. Local Bingo Halls & Events: **~100 players**
    a. Smaller local bingo halls
    b. Retirement home games
    c. Community center games

IV. Near Misses in Bingo

After extensive searches online, I found little to no information regarding near misses in the widely played game of bingo. I thought it would be an interesting task to explore near misses in bingo games of various sizes. In particular, a near miss is when a bingo card is one square away from a BINGO when another player in the same game wins. In this case, I want to explore a few questions:

1. How many numbers should we expect to draw before a BINGO is reached, comparing across various game sizes?
2. With what frequency does a near miss occur in a bingo game, and how does it compare across games of different sizes?
3. What is the relationship between how many numbers are drawn before a BINGO and the number of near misses there are for a particular game?

Using my initial research on bingo game sizes as a range, I plan to explore the frequency of near misses for games between 100 to 1,200 players in 100 player intervals (i.e. 100, 200, 300, … , 1,200).

In order to answer the questions above, I found that simulations would be the most practical approach in finding an answer.[3] The results of my simulations, and the responses to my questions above are answered in the following sections.

D. Expected Number of Draws Before a BINGO

The expected number of draws before a BINGO is reached decreases as the number of players playing the same bingo game increases. In addition to that, the variance of draws required before a BINGO is reached decreases as the number of players increases.

[3] Specifications and code used for the simulations can be referenced in the appendix. Each game specification was run through 1000 simulations for a total of 12,000 simulations and 7.8 million unique bingo cards.

Game Size	Expected Draws Until BINGO	SD for Draws Until BINGO
100	17.075	3.882
200	15.130	3.309
300	14.205	3.071
400	13.240	2.645
500	13.210	2.939
600	12.845	2.760
700	12.555	2.629
800	12.005	2.737
900	11.950	2.575
1000	11.995	2.745
1100	11.585	2.616
1200	11.610	2.471

Table 1: Expected number of draws until a bingo is reached, with varying game sizes

Looking for closely at the games with 100, 300, and 1,200 players, we observe the following distribution of draws required to reach a BINGO:

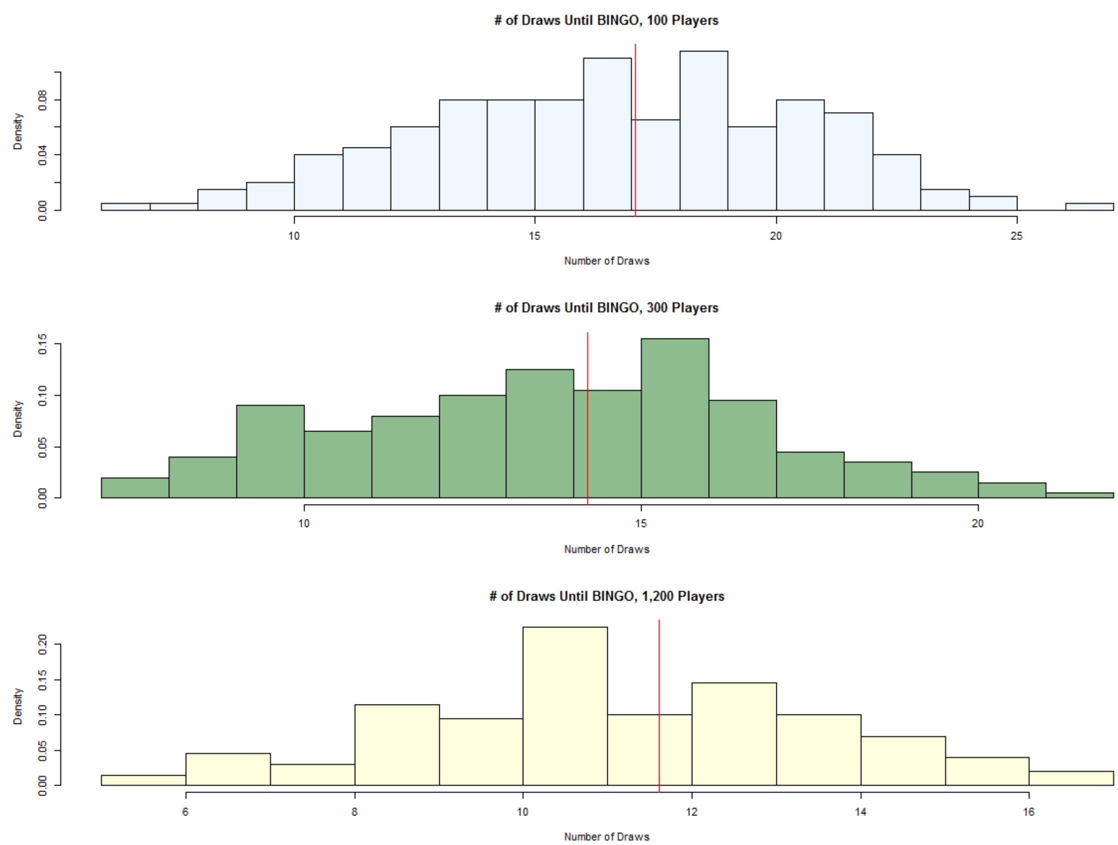

Figure 3: Distribution of draws before a BINGO was reached. Light blue = 100 player games, green = 300 player games, yellow = 1,200 player games

Comparing the histograms above, we can see that the number of draws until a BINGO is reach is distributed roughly symmetrical around the average.

Plotting this data in a scatterplot, we can see that the relationship between the expected draws until a BINGO is reached and the number of players in the game can be represented by the following power function:

$$\#\, Draws\ Until\ Bingo = 34.483\ \times (\#\, Players)^{-0.155} \qquad (1)$$

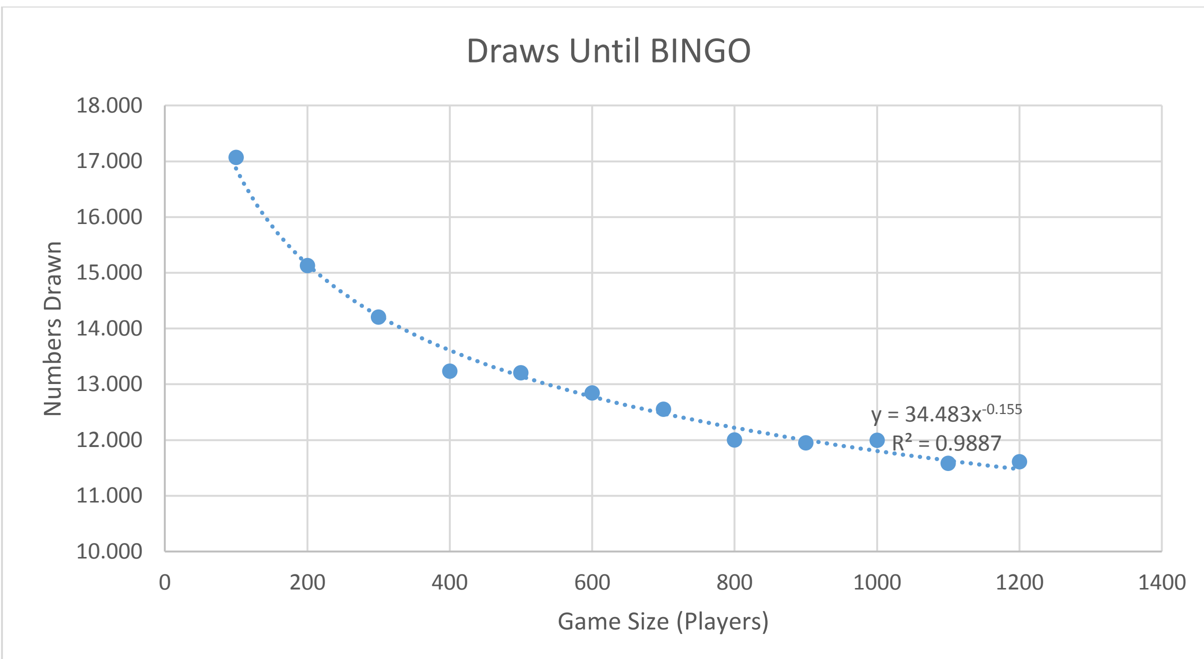

Figure 4: Expected number of draws until a bingo, across various game sizes. Each data point is the average of 1,000 simulations

E. Number and Frequency of Near Misses in Bingo Games

The expected number of near misses increases as a linear function as the number of players playing the same bingo game increases (figure 5). In addition to that, the variance of the number of near misses increases as the number of players increases.

Game Size	Expected Number of Near Misses	SD for Number of Near Misses
100	9.000	6.592
200	13.875	9.780
300	17.540	13.191
400	17.495	11.318
500	22.285	16.502
600	24.665	17.357
700	25.620	16.669
800	25.835	20.334
900	29.390	18.952
1000	34.340	27.027
1100	31.110	24.502
1200	35.015	24.885

Table 2: Expected number of near misses per game, with varying game sizes

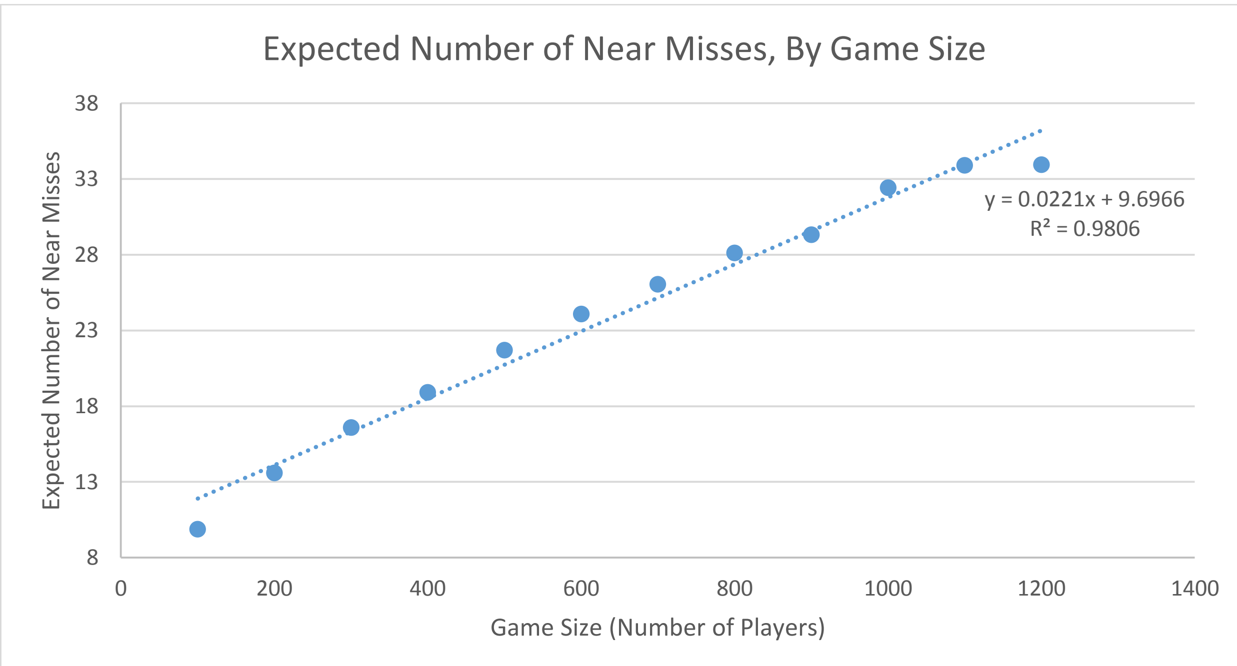

Figure 5: Expected number of near misses in a game of bingo, across various game sizes. Each data point is the average of 1,000 simulations

There is, however, a problem with comparing the number of near misses across games of different sizes. If games are larger, then we should expect to see more near misses, just due to the fact that there are more players playing the game. In order to correct for increasing game sizes, we simply divide the expected number of near misses by the number of players playing in each game to determine the expected frequency or probability of a near miss occurring across bingo games of different sizes. Now, we observe the following:

Game Size	Expected Near Miss Frequnecy	SD for Near Miss Frequency
100	0.093	0.066
200	0.069	0.049
300	0.059	0.044
400	0.044	0.028
500	0.045	0.033
600	0.041	0.029
700	0.036	0.024
800	0.032	0.025
900	0.033	0.021
1000	0.034	0.027
1100	0.028	0.022
1200	0.029	0.021

Table 3: Expected probability of near misses per game, with varying game sizes

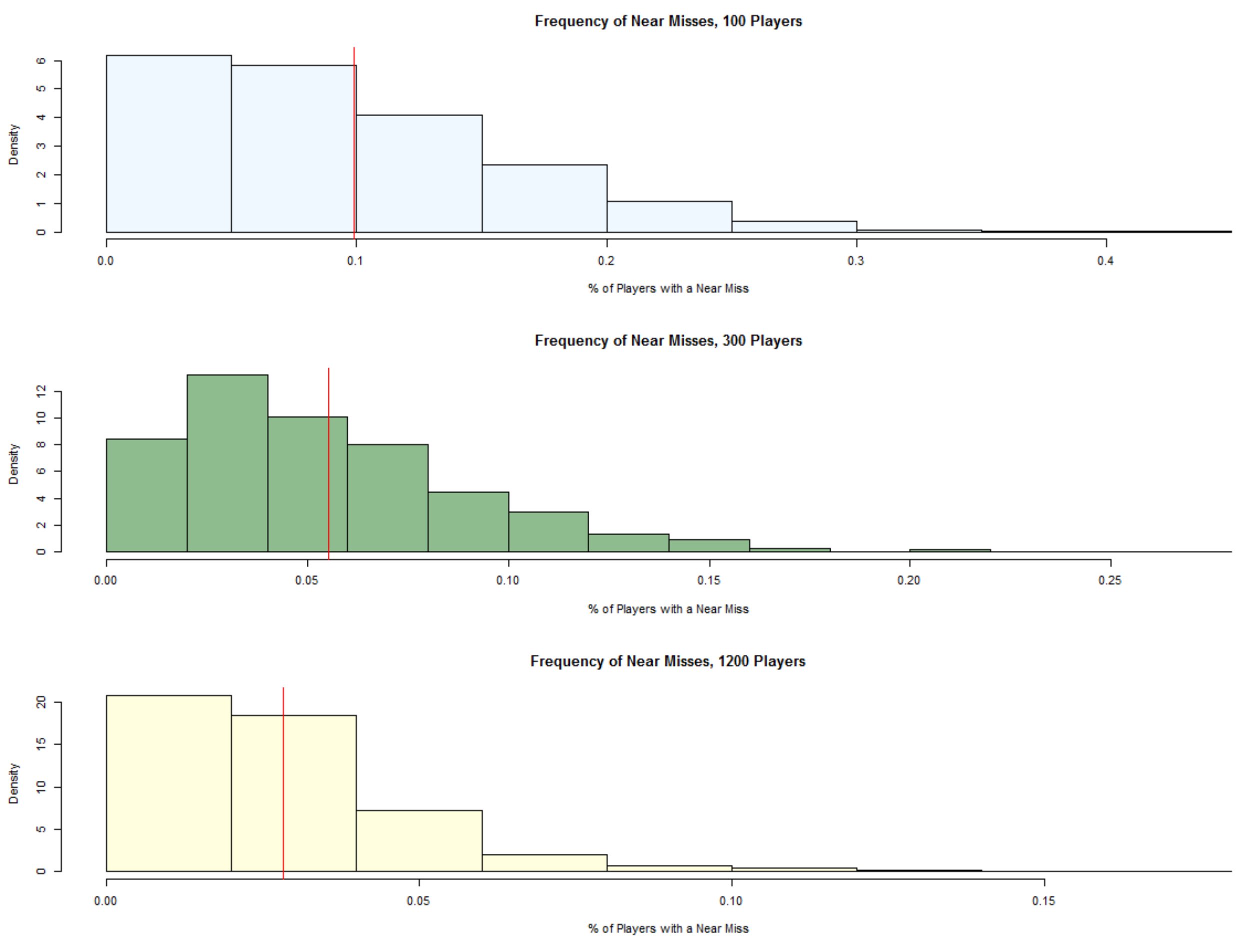

Figure 6: Distribution of frequency of near misses per bingo game. Light blue = 100 player games, green = 300 player games, yellow = 1,200 player games

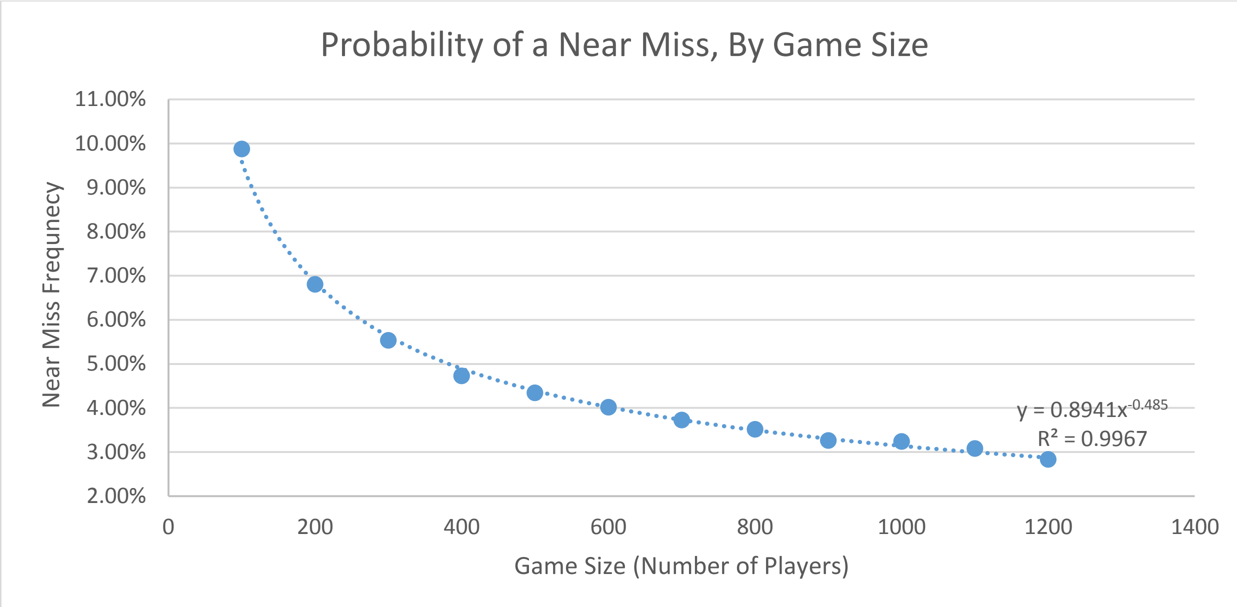

Figure 7: Expected probability of a near miss in a game of bingo, across various game sizes. Each data point is the average of 1,000 simulations

Interestingly, we observe that the expected probability of a near miss decreases as the number of players playing the same bingo game increases. In addition to that, the variance of the probability of a near miss decreases as the number of players increases. Once again, we are able to model the relationship between the probability of a near miss and game size:

$$Probability\ of\ Near\ Miss = 0.8941\ \times (\#\ Players)^{-0.485} \qquad (2)$$

F. Probability of a Near Miss, Given Number of Draws for BINGO

Lastly, I want to explore the probability of a near miss, given the number of draws it took to win a game of bingo, regardless of the size of the game. This will help us get a better understanding as to why smaller games of bingo have a higher probability of obtaining near misses.

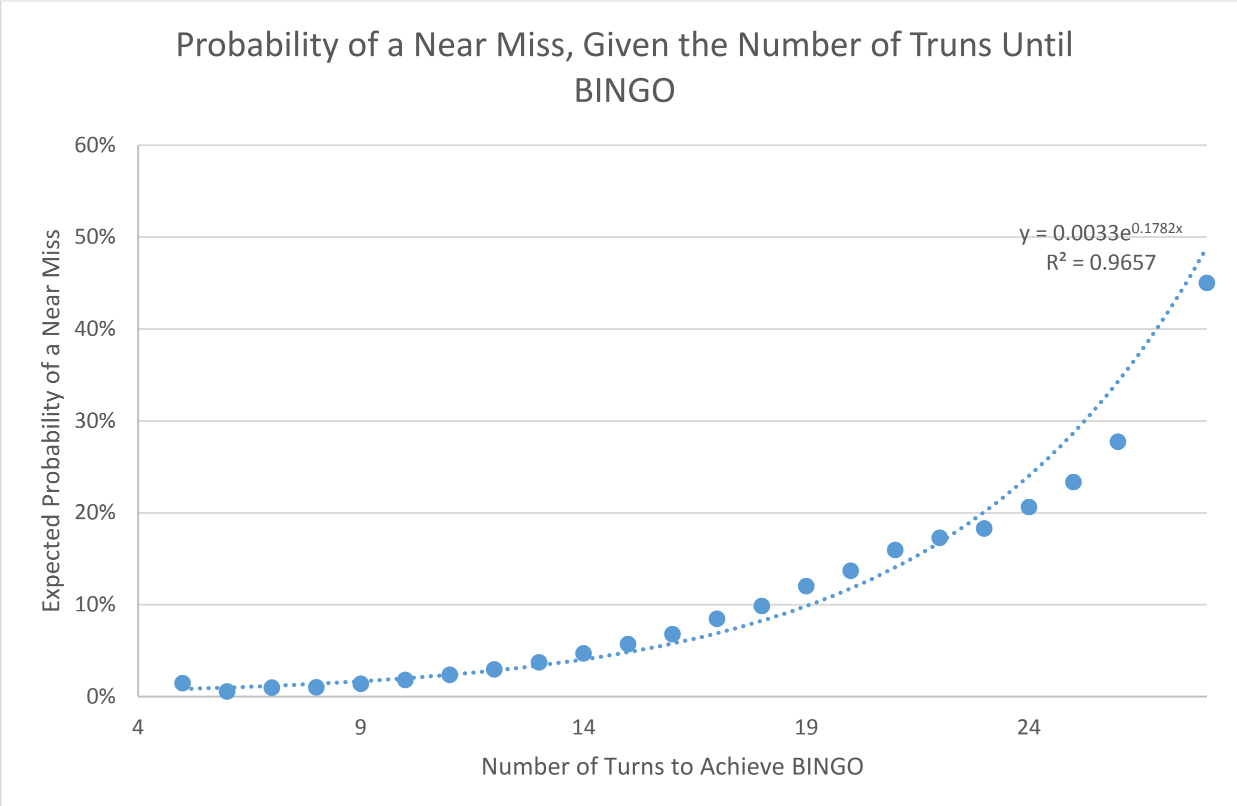

Figure 8: Expected probability of a near miss in a game of bingo, as a function of the number of turns to achieve a BINGO, regardless of game sizes. The data points are an aggregate of 12,000 simulations.

As we can see, in figure 8, the probability of getting a near miss increases exponentially (3) as a function of the number of turns to achieve BINGO.

$$Probability\ of\ Near\ Miss = 0.033\ \times e^{0.1782(\#\ Draws\ Until\ BINGO)} \tag{3}$$

Piecing the results of my simulation together, we saw earlier that the smaller the bingo game is, on average we should expect more draws until a bingo is reached. Looking at the results of the graph above, it reaffirms our earlier analysis on the expected number of draws until a BINGO is reached as well as the expected frequency of BINGOs in a particular bingo game, using data across all game sizes.

V. Conclusion

After considerable research, we have significant evidence to suggest that humans tend to respond positively to near misses in games of both skill and chance, as near misses are interpreted as an encouraging sign. In games of chance, near misses are widely believed to encourage future play, even though the probability of winning remains constant from trial to trial.

After running simulations across bingo game sizes of 100 players up to 1,200 players, we were able to determine that the number of near misses in a bingo game increases linearly as a function of the number of players in the bingo game. This means that as the size of the bingo game increases, the frequency of near misses decreases in a manner that can be represented by a power function.

In the United States, we see that smaller games of bingo are much more popular. Whether this may be a result of convenience (it's hard to find a lot of players as well as venues for larger games), the fact is that players will win and get near misses more often in smaller games than larger games – which may add a greater factor of excitement than just a large jackpot number. If you want to play bingo for fun, then I would highly suggest playing in smaller games as you'll have the thrill of getting closer to a win more often than in the larger games.

VI. Appendix

The following two screenshots show the code that I used for the simulations. Here I show the simulations of 100 player games as an example, with the other simulations having the exact same structure.

```
#######################################################################
#########################################################100 Players
#######################################################################

#Number of simulations set to 1,000
n100=1000

#Generate bingo boards
bingo100=array(NA,c(5,5,n100,100))


#Assign randomly permuted numbers into each space, subject to the bounds of the column
for(i in 1:n100){
  for(j in 1:100){
    bingo100[,1,i,j]=sample(1:15,5)
    bingo100[,2,i,j]=sample(16:30,5)
    bingo100[,3,i,j]=sample(31:45,5)
    bingo100[,4,i,j]=sample(46:60,5)
    bingo100[,5,i,j]=sample(61:75,5)
  }
}

#free space
bingo100[3,3,,]=100

bingo100a=bingo100

#The following lines of code runs through each game of bingo and replaces a square
#with 100 if that number has been called out. The games of bingo end when one of the
#boards has a winning combination
counter100=array(0,c(n100,100))
for(i in 1:n100){
  for(j in 1:100){
    tempboard = bingo100[,,i,j]
    while(sum(tempboard[1,])!=500 & sum(tempboard[2,])!=500 & sum(tempboard[3,])!=500 &
            sum(tempboard[4,])!=500 & sum(tempboard[5,])!=500 & sum(tempboard[,1])!=500 &
            sum(tempboard[,2])!=500 & sum(tempboard[,3])!=500 & sum(tempboard[,4])!=500 &
            sum(tempboard[,5])!=500 & sum(tempboard[1,1],tempboard[2,2],tempboard[3,3],
                                          tempboard[4,4],tempboard[5,5])!=500 &
            sum(tempboard[1,5],tempboard[2,4],tempboard[3,3],
                tempboard[4,2],tempboard[5,1])!=500 ){
      tempboard[tempboard==randomdraws[i+1000,counter100[i,j]]]=100
      counter100[i,j] = counter100[i,j] + 1
    }
    bingo100[,,i,j]=tempboard
  }
}
```

```
#Find which player took the shortest amount of move to reach BINGO in each game
min100=rep(0,times=n100)
for(i in 1:n100){
  min100[i]=min(counter100[i,])
}

#Re-simulate each game, stopping each game when a BINGO is reached.
#The BINGO is reached at the counter number found above
for(i in 1:n100){
  for(j in 1:100){
    tempboard=bingo100a[,,i,j]
    for(k in 1:min100[i]){
      tempboard[tempboard==randomdraws[i+1000,k]]=100
    }
    bingo100a[,,i,j]=tempboard
  }
}

#Go through the re-simulated games in order to determine how many boards contain a near miss
nearmiss100=rep(0,n100)
for(i in 1:n100){
  for(j in 1:100){
    tempboard=bingo100a[,,i,j]
    if(sum(tempboard[1,])!=500 & sum(tempboard[2,])!=500 & sum(tempboard[3,])!=500 &
         sum(tempboard[4,])!=500 & sum(tempboard[5,])!=500 & sum(tempboard[,1])!=500 &
         sum(tempboard[,2])!=500 & sum(tempboard[,3])!=500 & sum(tempboard[,4])!=500 &
         sum(tempboard[,5])!=500 & sum(tempboard[1,1],tempboard[2,2],tempboard[3,3],
                                       tempboard[4,4],tempboard[5,5])!=500 &
         sum(tempboard[1,5],tempboard[2,4],tempboard[3,3],
             tempboard[4,2],tempboard[5,1])!=500 ){
      if(sum(tempboard[1,1:4]) == 400 || sum(tempboard[1,2:5]) == 400 ||
           sum(tempboard[2,1:4]) == 400 || sum(tempboard[2,2:5]) == 400 ||
           sum(tempboard[3,1:4]) == 400 || sum(tempboard[3,2:5]) == 400 ||
           sum(tempboard[4,1:4]) == 400 || sum(tempboard[4,2:5]) == 400 ||
           sum(tempboard[5,1:4]) == 400 || sum(tempboard[5,2:5]) == 400 ||
           sum(tempboard[1:4,1]) == 400 || sum(tempboard[2:5,1]) == 400 ||
           sum(tempboard[1:4,2]) == 400 || sum(tempboard[2:5,2]) == 400 ||
           sum(tempboard[1:4,3]) == 400 || sum(tempboard[2:5,3]) == 400 ||
           sum(tempboard[1:4,4]) == 400 || sum(tempboard[2:5,4]) == 400 ||
           sum(tempboard[1:4,5]) == 400 || sum(tempboard[2:5,5]) == 400 ||
           sum(tempboard[1,1],tempboard[2,2],tempboard[3,3],tempboard[4,4]) == 400 ||
           sum(tempboard[5,5],tempboard[2,2],tempboard[3,3],tempboard[4,4]) == 400 ||
           sum(tempboard[1,5],tempboard[2,4],tempboard[3,3],tempboard[4,2]) == 400 ||
           sum(tempboard[5,1],tempboard[2,4],tempboard[3,3],tempboard[4,2]) == 400
      ){
        nearmiss100[i] = nearmiss100[i] + 1
      }
    }
  }
}
```