Risk Measurement and Management

An in-depth look at how Wall Street professionals deal with market risk

Presented by:

Kwon Joon Chang

Christopher Lin

Tianqi Zhu

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1 Introduction

Wall Street has always been known to be a place where you can multiply your money – a place where you can cash in big on free-market capitalism to fast track your way to the American Dream. With the advent of all sorts of financial derivatives, arbitrage schemes, and the growing inclination for professionals to take on calculated risk, financial positions these days for both individuals and institutions have become increasingly complex and dangerous.

Managing portfolio risk is not only a vital part of an investment management professional, but the underlying concepts of managing risk are also constantly being utilized by veteran traders, hedge funds, and market making investors. Especially in a society where capital markets ties in heavily with the welfare and prosperity of those living in it, it is important to consider what the consequences are if risk were not well-managed.

In this paper, we are going to take a closer look at how professionals in these financial institutions measure and manage the risk of their financial positions. It is important to carefully examine the commonly used methods and determine any drawbacks that could have led to the gross underestimation of risk as seen in the past few years. The financial collapse of 2008, in which a substantial number of banks failed, is a prime example of the dangers of improper risk management when dealing with the intricacies of the markets.

We will also look at the events that led up to our current financial state, and different ways we could have potentially pre-empted or better prepared ourselves for the worst.

2 Risk Measurement

First, we must address the question: What exactly is risk? The layperson would consider risk to be an abstract notion that has a large impact in their stock market gamble: the possibility that the value of their investment would decrease due to a variety of factors. To quantitative financial experts, variance is a commonly used proxy for risk. Risk is essentially the standard deviation of return on an asset of portfolio. When examining a security, the more volatile it is (for example, a technology company's stock during times of fluctuating consumer spending) the more risky it is considered to be.

2.1 Value-at-Risk

Most financial professionals utilize a method of risk measurement called Value-at-Risk (VaR). It is a well-established industry standard risk measurement technique, and helps traders and investors prepare for the turbulence of financial markets.

Value-at-Risk is essentially a quantile of the portfolio's return distribution. It is quoted in terms of a fixed time horizon and a percentage. For example, if a 99% one-day VaR of a security is 7%, this means that it estimates for the next one-day period, there is a 99% chance that the security does no lose more than 7% of its value.

Technically speaking, we can see that the VaR equation is:

$$VaR_{\alpha} = \inf\{ x \in R : P(L > x) \leq 1 - \alpha \}$$

The VaR of a portfolio/security at the confidence level α is given by the smallest number x such that the probability that the actual loss L exceeds x is not larger than 1 – α. It acts somewhat similar to a confidence interval, and is typically based on a normal distribution such that it is easy to calculate. For the previous example, if the value of the security is $1000, then $VaR_{99\%} = 70$.

The VaR gives a threshold of your loss tolerance. In other words, the one-day VaR basically tells you, "For the most part, you will not lose more than X amount on any given day." VaR can be over any other time period (such as one week), and can also be a negative value, in which case it means that over the time horizon, the portfolio would most likely increase X amount in value.

2.2 Drawbacks of Value-at-Risk

VaR is commonly referred to as, "an airbag that works all the time, except when you have a car accident." This is because professionals use VaR to make sure their position is always within their loss tolerance. As long as someone can maintain a certain VaR, they feel comfortable saying something like: "as long as 99% of the time, my portfolio loses a maximum of $1 million, then I can tolerate this position." It creates a false sense of security because it doesn't adequately take into account the incredible downside potential if your portfolio or security encounters the 1% chance that it exceeds the loss threshold. Once the losses exceed the threshold, there is no telling what the magnitude of the loss is.

Example 2.1 To illustrate this drawback, consider two positions X_1 and X_2:

$$X_1 = \begin{cases} 1 & \text{with probability } 99\% \\ -1 & \text{with probability } 1\% \end{cases}$$

$$X_2 = \begin{cases} 1 & \text{with probability } 99\% \\ -10^{10} & \text{with probability } 1\% \end{cases}$$

Although both positions have a $VaR_{99\%} = -1$, X_2 has a much higher downside risk that is not accounted for. This shows that VaR does not distinguish between the two securities, and as long as a certain threshold is met, the position is considered "safe." This clearly creates a false sense of security, and results in the risk of a security or portfolio being grossly underestimated.

Another drawback of the VaR is that it is not sub-additive, and discourages diversification.

Example 2.2 Consider two positions X_1 and X_2, both given by:

$$X_i = \begin{cases} 1 & \text{with probability } 50\% \\ -1 & \text{with probability } 50\% \end{cases}$$

We can say that $VaR_{50\%}(X_i) = -1$. If both X_1 and X_2 were independent, then we can consider the combined portfolio position to be:

$$X = \frac{X_1 + X_2}{2} = \begin{cases} 1 & \text{with probability } 25\% \\ 0 & \text{with probability } 50\% \\ -1 & \text{with probability } 25\% \end{cases}$$

We can say, in this case, $VaR_{50\%}(X) = 0$. This suggests that the diversified position X is riskier than the individual position of X_i (50% of the time you will have a 0 loss compared to an increase of 1).

2.3 Expected Shortfall

To address the problem of not accounting for the magnitude of the loss given that the loss exceeds the VaR number, professionals oftentimes provide a number for the expected shortfall, which is also known as "conditional VaR." It essentially tells us that given the loss exceeds the threshold, what is the absolute expected loss? Although somewhat of a crude measurement, one way expected shortfall is determined is simply empirical: look at the historical prices, calculate their VaRs, and for the data that has an observed loss that exceeds the VaR, and take the average amount.

Technically speaking, we can consider the empirical expected shortfall equation to be:

$$s(u) = 1/k * \sum(r_i - u)$$

Where $k = k(u)$ is the number of observations that exceed u.

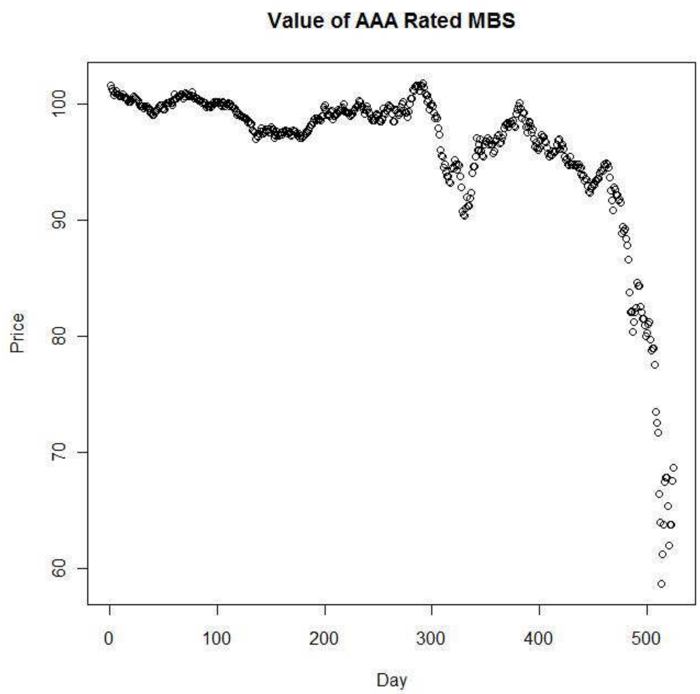
3 Application to Data

To truly illustrate the utility of VaR, as well as the shortcomings, we applied this technique to various securities, portfolios, as well as indices. We were essentially trying to mimic a real-life situation where traders would calculate the VaR and determine whether or not it would be wise to stay within a certain position.

One-day $VaR_{99\%}$ of Mortgage Backed Securities

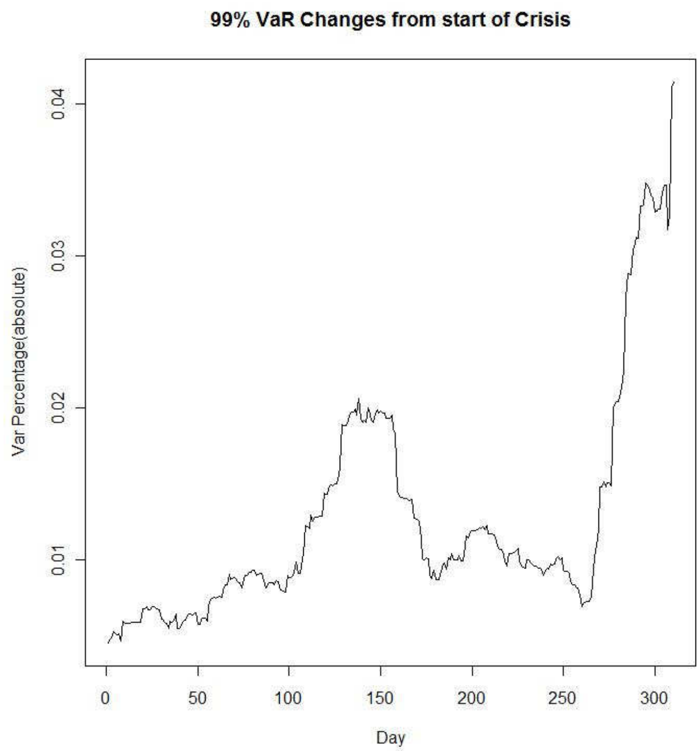
With the credit crisis being a prevalent topic, we felt it was appropriate to look at the historical data for an index of AAA Rated mortgage backed securities (Appendix A), as it is one category of assets that helped contribute to the financial collapse of 2008.

First, we plotted out the value of these assets, and the trend from the start of this year:



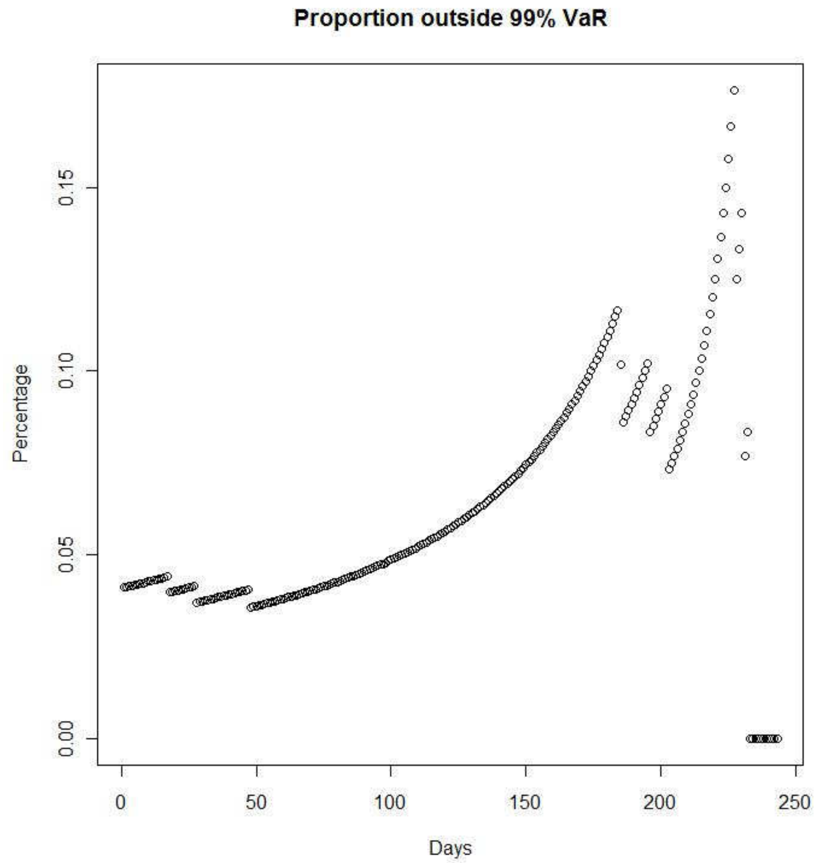
Gathering all the data, we found the VaR by taking the average and standard deviation of values in the past 10 days, recording the single-day changes in value. We then evaluated the VaR99% for the next day, and plotted out the changes in VaR over time. (The R-Source code for implementing the VaR calculation is included in Appendix B)

This is what we observed:



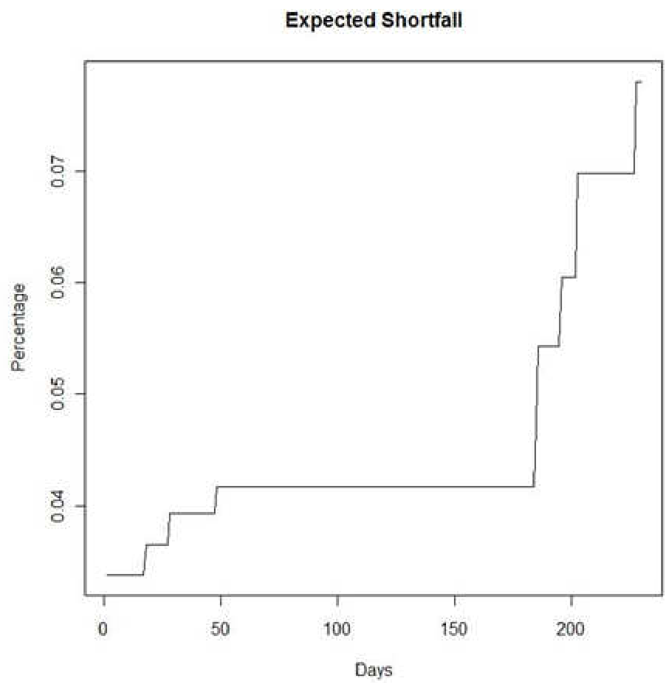
It is pretty clear that as the volatility of the MBS went up (as the fluctuation in value was noticably more significant), the VaR increased dramatically as well. As volatility went up, those who utilized the VaR braced themselves for a higher loss threshold, and would be able to either relax their position to maintain a certain VaR, or simply to realize they were taking a risk of a more significant loss within their VaR percentage. (We also did the same analysis on BBB rated MBS, which can be found in Appendix A)

Furthermore, as part of our analysis, we wanted to look at how effective and accurate the VaR actually was. According to all the different VaR values we had determined over the time period, we wanted to see what proportion of days was there a single-day loss that exceeded the threshold given by the VaR. Since we had calculated a 99% VaR, we should expect that only 1% of the days had a single-day loss large enough that it exceeded the VaR. Instead, we observed:

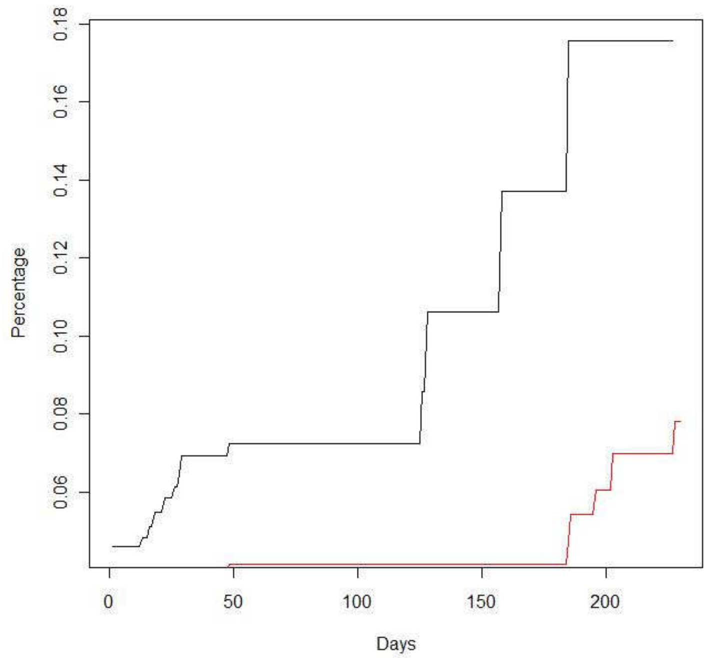


In fact, espeically since the credit crisis began and the market became distressed, the VaR was particularly inaccurate. As the crisis worsened, we could see that up to 15% of securities in this data pool had losses exceeding the 1% estimated threshold. This itself is emprical proof of the inadequecy of using the VaR as a measurement of risk. Only now, during the current volatile market conditions, the Value-at-Risk has risen so high that the proportions of single-day losses that exceed the threshold fit within the 1% parameter – simply because the threshold has become so large.

As mentioned before, we would also like to take a look at the expected loss given that we encounter the 1% chance that our losses exceed the VaR threshold. The expected shortfall then, was observed to be:



As volatility increased, we saw the expected shortfall increase as well. We see a direct correlation between the expected shortfall and the value-at-risk. To illustrate an example similar to example 2.1 in section 2.2, we compare the expected shortfalls of the AAA and the BBB mortgage-backed securities. Both have relatively similar VaRs, but dangerously different expected shortfalls.

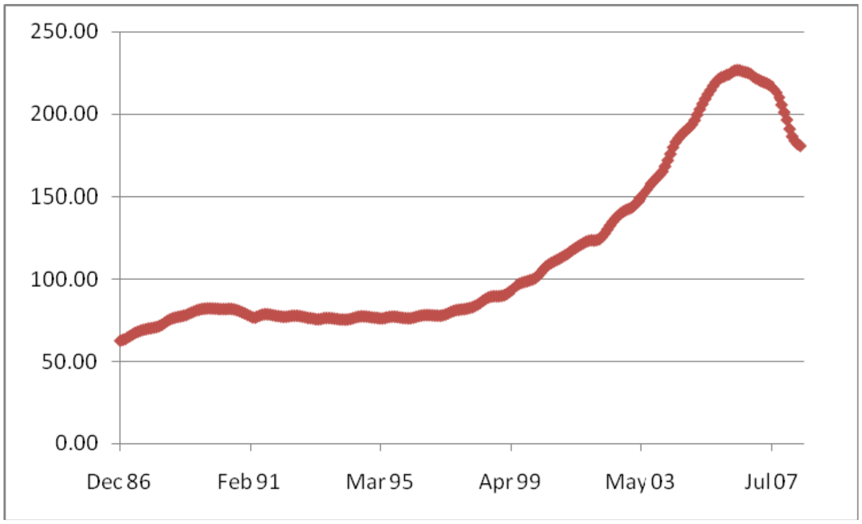


(the black line indicates the BBB expected shortfall, the red line is the AAA)

Although both have similar VaR, the magnitude of the loss is entirely different. Holding a BBB has much greater downside risk than holding a AAA security. We realize the dangers we encounter when using the VaR method, and also that the empirical expected shortfall method is also a crude form of measurement, mostly because it is very reactive method rather than a pre-emptive one.

4 Risk Management and the Current Credit Crisis

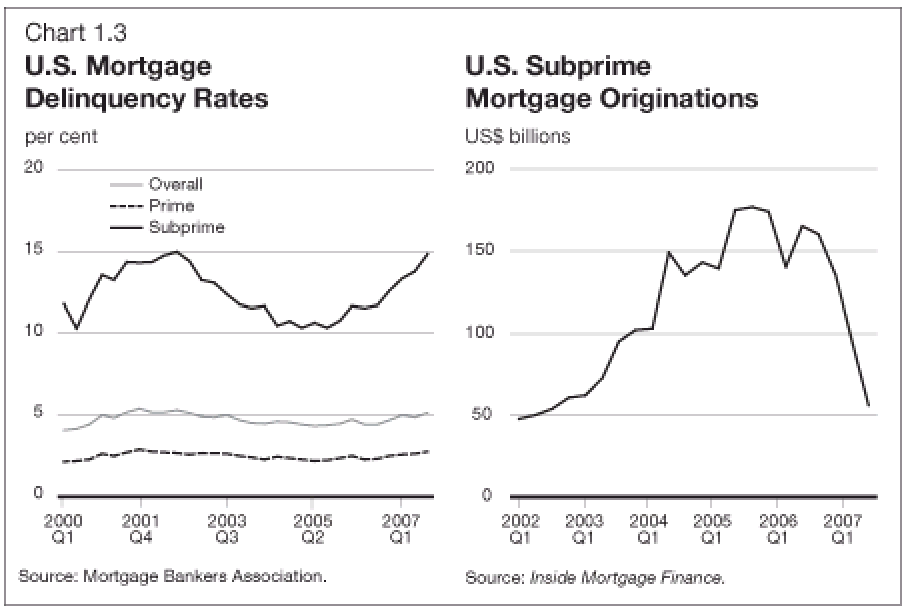
The overexpansion of credit in the US housing market did not happen overnight. The historically low Federal Funds rate between 2002 and 2005 allowed many home owners to borrow, but the real problem were rooted in the mortgage lending practices. Mortgage lenders mistakenly believed that they were taking on manageable risk during the period of low interest rates, because the values of the collateral underlying the loans were appreciating quickly. More specifically, the housing prices rapidly increased in the last decade, and the mortgage lenders relaxed their lending standards as they believed the seemingly ever-appreciating values of these homes would be viable collateral in case of default.



U.S. Real Estate Prices, 1987 to June 2008 S&P/Case‐Shiller Composite‐10 Index

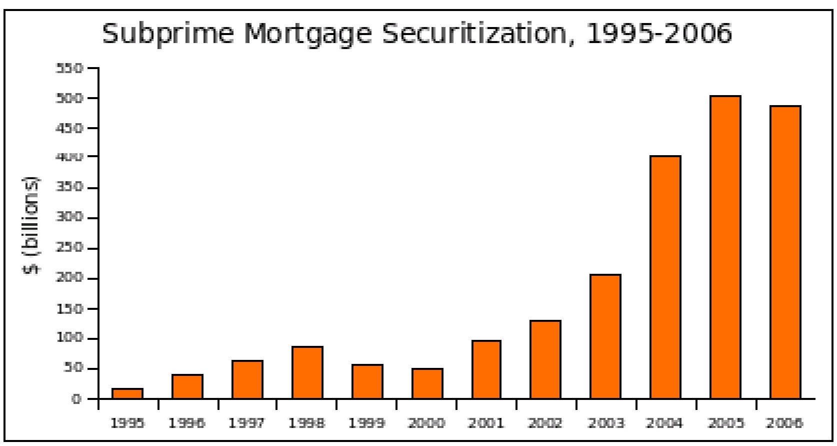
4.1 Subprime mortgages

Mortgages to borrowers with bad credit history, usually with FICO scores of less than 620, are defined to be subprime, while loans to borrowers with solid credit history are called prime. During the housing boom, number of subprime mortgage originations increased dramatically, as shown by the graph below. And once the housing boom had been instigated, mortgage lenders wanted to take advantage of this speculative bubble by attracting new borrowers to the market. And because of the rapidly increasing house prices, lenders were able to devise new ways to make new mortgages affordable even for consumers with bad credit. One way was to offer adjustable rate mortgage (ARM), in which the initial rate is fixed at a low “teaser rate” for first couple years, and then subsequently set equal to six-month LIBOR plus a fixed rate for the remaining years in the mortgage. Thus, people with bad credit had no trouble obtaining a mortgage on a house with little or no down payment and paying very little in interest payments before the adjustable rates kicked in. However, as the housing bubble burst in 2007 and the value of the collateral fell quickly, many mortgage holders were no longer able to make the interest payments once the period of teaser rates ended. Thus, foreclosures increased and house prices declined even further. The graph below shows the rise in delinquent US mortgages over time.



4.2 Securitization

What exacerbated the credit crisis was the securitization of the mortgage loans. The financial institutions originating these loans did not hold it in their portfolios; instead, the subprime mortgages were securitized and sold off to investors as asset-backed securities (ABS), or more specifically, mortgage-backed securities (MBS). The graph below shows the increasing level of securitization leading up to the bursting of the housing bubble. Thus, the originators were more interested in originating subprime loans for the purpose of packaging and securitizing them so that it can be sold for a profit to investors, instead of analyzing whether the borrowers will be able to make the promised payments. So naturally the quality of the subprime loans was destroyed and the flood of defaults ensued.



4.3 Risk Management

Rating agencies, financial institutions, investors and regulators all grossly underestimated the risk of mortgage-backed securities. The risk measures used are largely based on historical performance. Value-at-risk measure, one of the most common methods of measuring risk, estimates risk based on historical data anywhere from 2 to 100 years. And while historical data provide useful information in extrapolating risk estimation for the future, it cannot be used as a stand-alone model for risk management. Value-at-risk measure used by investors of mortgage-backed securities may have maintained a “safe” VaR level in maintaining mortgage-backed securities in their portfolios, but they failed to take into account the asymmetrical downside to investing in mortgage-backed securities.

One of the lessons from past financial crisis is that correlations increase in stressed market conditions. Using standard value-at-risk techniques to estimate correlation and risk from past data and assuming that those estimates to hold in midst of stressed markets will lead to a gross underestimation of the inherent risk. Because the risk subprime loans were securitized into diversified portfolios, everybody believed that the risks were diversified away. However, as the housing bubble burst, the level of defaults rose rapidly and the value of the mortgage-backed securities plummeted. Using value-at-risk techniques did not prepare investors from the subprime crisis because it did not take into account what exactly will happen to their position when the value-at-risk threshold is met. Historical data cannot predict appropriate risk because it does not account for potential extreme losses. And to top it off, many of the financial institutions holding onto the risky mortgage-backed securities were highly levered, using large amounts of debt to buy the subprime products, so once the value of the risky securities dropped significantly, they experienced amplified losses.

Thus, the risk management models in place today are not sufficient to appropriately measure risk of financial assets that are negatively skewed. In the next section, we look at some of the issues that must be addressed in order to come up with a more appropriate risk measure.

5 Issues with Current Risk Management Models

Two important issues that need to be issued in future risk management models are the asymmetric magnitude of losses due to extreme events and dependence between securities. We address the issue of asymmetrical negative losses first.

5.1 Extreme Events

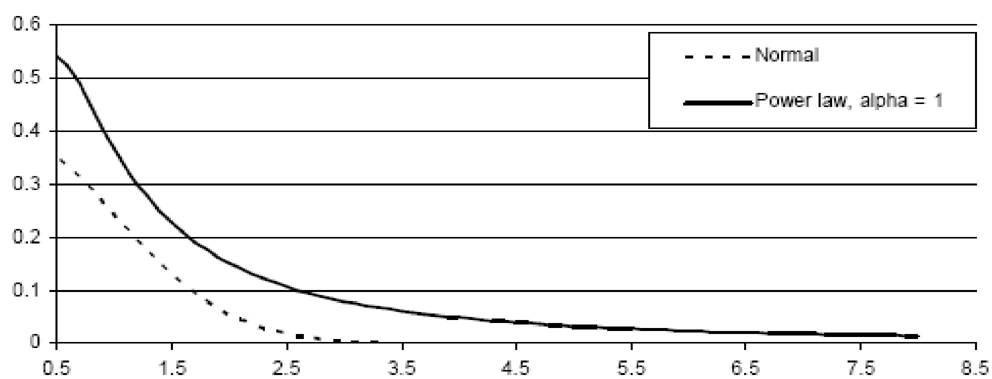
Value-at-risk technique measures risk under the assumption that asset returns are normally distributed. However, this is not consistent with empirical data. For example, consider the 12% drop in the stock market that occurred in 1929. Assuming that the returns in the stock market are normally distributed, the normal distribution predicts that a single-day loss of this magnitude occurs once every 10^{30} years. But only 58 years later, the stock market dropped by 20% in a single day on a day now referred to as “Black Monday”. As we can conclude, asset returns are non-normal and assuming normal distribution in calculating risk will underestimate the inherent risk in the markets and securities.

5.2 Power Law

So instead of using normal distribution, we can use a power law distribution at the tails of the distribution. A random variable X with distribution F follows a power law with exponent α if:

$$(1 - F(x)) \sim x^{-\alpha}$$

Variable with power law distributions scales according to α, the exponent. In addition, relative to normal distributions, the power law distributions have fatter tails than the tails of normal distributions, as shown by the graph below. Since the tails measure the probability of extreme events, power law distributions will give a more appropriate measure of risk of extreme losses than normal distributions.



5.3 Default Contagion Model

The second issue that needs to be addressed is the issue of dependence between securities. Default events are dependent on each other because the default subjects are affected by common macro-economic shocks. In addition, there is a complex network of relationships between firms, government, and financial institutions, and in distressed market conditions, a market contagion can create a chain of events that lead to bursting of the speculative bubble. And this certainly was the case for the bursting of the housing bubble. So, we consider the following default contagion model that will take into account the complex dependence of securities.

1. Suppose all firms start out financially healthy and that the first default arrives according to a Poisson process with intensity λ, the conditional default rate.
2. At the first default time T_1, we update the intensity according to a non-negative function Λ such that the new intensity is:

$$\lambda_t = \lambda + \Lambda_1(t - T_1)$$

3. Thus, the new λ_t takes into account the subject at default, state of the economy, and any other information that is available at T_1. And we can continue to update λ_t in this manner at every point in time where a default occurs.
4. We can see that as the number of defaults increase, the likelihood of defaults increases as well. This is because of the existence of default correlations, and in stressful market conditions, this correlation becomes even higher. This is the domino effect where the collapse of the housing market led to a broad collapse of the financial markets.

6 Conclusion

Value-at-risk technique for measuring risk is a standard practice on Wall Street today. However, because value-at-risk method does not appropriately measure the risk of extreme events due to the assumption that asset returns are normally distributed, it provides a false sense of security. Leading up to the current credit crisis, risk managers, investors, and rating agencies alike did not account for the asymmetrical magnitude of large losses in rare events that is not accounted for in the tails of normal distributions. In addition, investors did not properly measure the level of correlation between default rates in the different parts of the United States. Mortgage-backed securities investors believed that they were protected because the underlying mortgages were geographically diverse, but default correlations proved to be much higher than estimated in the distressed market conditions.

Thus, going forward, we need to improve on the risk management models by having a risk measure that will quantify risk on a monetary scale, be sensitive to large losses, encourage diversification, and take into account the variety of macro-effects that occurs in our dynamic and constantly changing economy. And if institutions can properly manage risk by addressing these issues, we can avoid another damaging financial collapse in the future.

Sources:

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2. John C. Hull. *Credit Crunch of 2007: What Went Wrong? Why? What Lessons Can be Learned?*. University of Toronto, 2008.
3. Kay Geisecke, Thorsten Schmidt, and Stefan Weber. *Measuring the Risk of Large Losses. Journal of Investment Management*, Vol.6, No.4, (2008), pp.1-15
4. Kay Giesecke, Lisa R. Goldberg. *Forecasting Extreme Financial Risk.* 2005.
5. Paul Embrechts. *Extreme Value Theory: Potential and Limitations as an Integrated Risk Management Tool.* Swiss Fedeal Institute of Tehnology, Zurich.
6. Joshua V. Rosenberg, Til Schuermann. *A General Approach to Integrated Risk Management with Skewed, Fat-tailed Risks.* Federal Reserve Bank of New York, 2005.

Appendix A

CMBS INVESTMENT GRADE AAA INDEX from 1/1/2008 to Present

CMBS INVESTMENT GRADE BBB INDEX from 1/1/2008 to Present

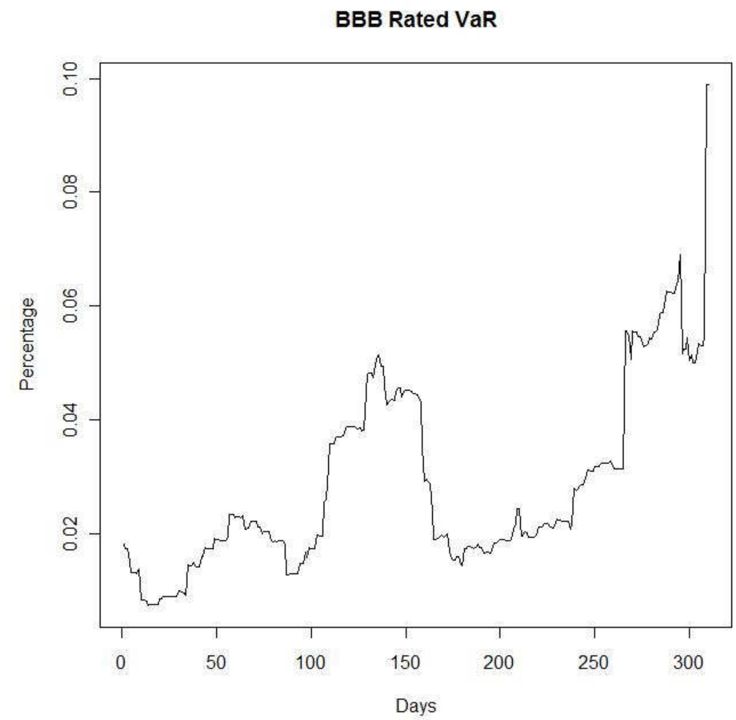
Appendix B

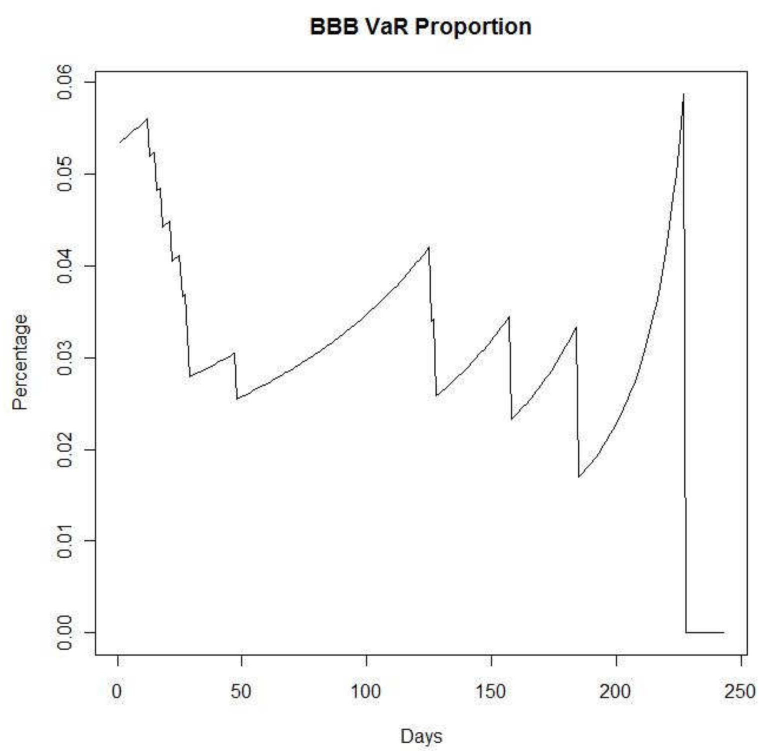
Source R Code:

```
var = function(percent=0.01,x,days=400,horizon=90){

diff = (days-horizon):(days-1)

      for(i in (days-horizon):(days-1) )
      {

            diff[(i-days+horizon+1)] = (x[i+1] - x[i])/x[i]

      }


count = qnorm(percent,mean(diff),sd(diff))

return(count)
}

ztest = function(percentage=0.01,z,days=400,horizon=90){

TEST = days:(length(z)-1)

for( i in days:(length(z)-1)){


stat = (z[i+1]- z[i])/z[i]

TEST[(i-days+1)] = (stat < var(percentage,z,i,horizon))

}

return(sum(TEST)/length(TEST))

}

vartest = function(horizon=90,sdate=281,x)
{

jb = sdate

y = jb:(length(z)-1)
```

```
for(i in jb:(length(z)-1)){
y[i-jb+1] = ztest(.01,z,i,horizon)

}
return(y)
}
varplot = function(horizon = 30, set = z){
j = 200:509
for(i in 1:310) j[i] = var(horizon=30,x = set,days=j[i])
return(j)
}
expsf = function(percentage=0.01,z,days=400,horizon=90){
TEST = days:(length(z)-1)
for( i in days:(length(z)-1)){

stat = (z[i+1]- z[i])/z[i]
TEST[i-days+1] = NA
if ((stat < var(percentage,z,i,horizon))) TEST[i-days+1] = stat
}
return(mean(TEST,na.rm = TRUE))
}
j = 281:510
for (i in 1:length(j)) j[i] = expsf(z=z,horizon = 30,days = j[i])
```

Appendix C


BBB Rated VaR
Percentage
0.02
0.04
0.06
0.08
0.10
0
50
100
150
200
250
300
Days


BBB VaR Proportion
Percentage
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0
50
100
150
200
250
Days