Risk Aversion in Games of Chance

Imagine the following scenario:
Someone asks you to play a game and you are given $5,000 to begin. A ball is drawn from a bin containing 39 balls each numbered 1-39 and you are told its number. You are then asked to guess whether the next ball drawn, without replacement, will have a number higher or lower than the previous ball, and if you are correct, you win $1,000, but if you are incorrect, you lose $1,000. This process will be repeated until a total of 4 balls have been drawn. At this point, you have a choice between going for a fifth draw or not, but this time, the rules are slightly different:

-if your guess is correct, you double the money you had after four draws

-if your guess is incorrect, you halve the money you had after four draws

-if you decide not to go for it, you take the money you had after four draws

Would you go for the fifth draw? Why? What would it depend on, if anything? A good luck streak? The amount of money you had won at that point? The specific balls that had already been drawn?

Introduction

The manner in which human beings play lottery games tends to be distinct from the probabilistically sound method of play. In examining the above lottery game in particular, one can analyze the differences between the two approaches of play.

Data

The data for this project came from two sources. The first data set consisted of the results of 40 personal interviews with family, friends, and acquaintances. People ranging from age 15 to age 62 and from a variety of educational backgrounds and careers were interviewed. Sequences of five randomly generated numbers between 1 and 39 were used in lieu of sampling from a bin containing 39 balls. The rules of the game were explained to each participant and their responses were recorded. The second set of data was the result of running a computer simulation 10,000 times and recording the results. Using the statistical computing language R, a model was developed to simulate the game process in which the computer itself would act as the participant. In this model, random numbers were generated and the computer would 'make guesses' following the optimal strategy of play for this game.

Optimal Strategy

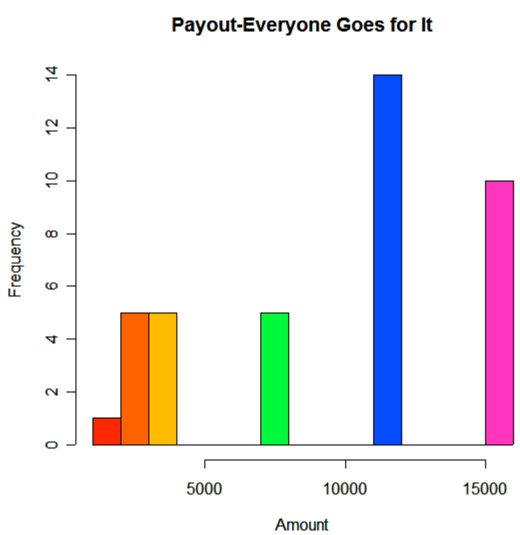
As this is not a zero-sum game, there exists an optimal strategy of play, which involves some basic probabilistic awareness on the part of the player. First, a player should keep track of the numbers that have been drawn and he should make his guess in the direction (higher or lower) that has more remaining numbers. Secondly, a player should always go for the fifth draw. The odds are, at worst, 50-50, and the expected payout is always greater than the winnings after the fourth draw. Suppose for example, that a player has gotten a 1 or 39 on his fourth draw. The player would naturally go for the fifth draw, because his odds are 100% of being correct. If, however, a player draws a number closer to 20, his odds of being correct for a fifth draw would be at worst 50-50. In this case, the player should still go for the fifth draw because the expected payout is greater than whatever amount of money the player has after four draws. For example, if the player has $8,000 after four draws and chooses to go for the fifth draw, his expected gain in payout is .5($8,000) - .5($4000) = $4,000. The player has the potential to win more money than he could lose, and so a fifth draw is always advisable. In general, players who use the optimal strategy have a higher mean payout than those who do not.

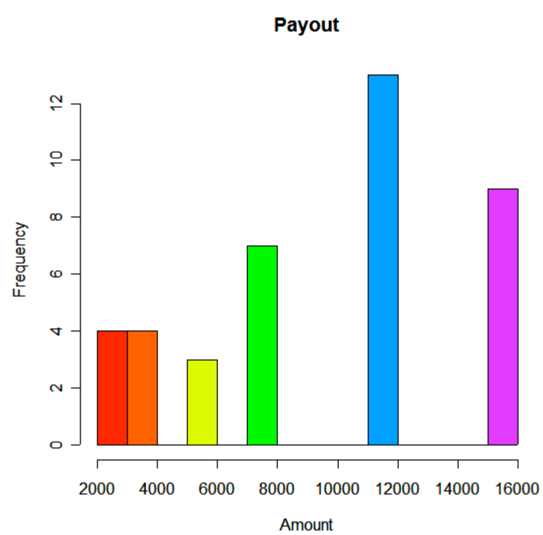
Methods

During the data collection process, in addition to recording each participant's guess responses supplemental information was also recorded, including age, profession, and reason for either going for or declining a fifth draw. In addition, the payout results for the interview data set were re-calculated to explore the alternative that everyone had gone for the fifth draw, and had made the statistically sound guess. The resulting distribution was compared with the one from the original interview data set. Later, histograms were made for both the interview and the computer data sets to compare the distribution of the end payouts. We expect the distributions to be similar, though some differences may be observed as the computer was 'playing' the optimal strategy and the human participants were not subject to that constraint. The mean, mode, and standard deviation of the payouts were calculated and compared. Attempts were made to discover if a relationship existed between a participant's choice of whether or not to go for the fifth draw and potential predictors such as the age of the participant and the number drawn fourth. If such a relationship existed within the data and a model could be fit, this would help in prediction of a new participant's choice given other supplementary information.

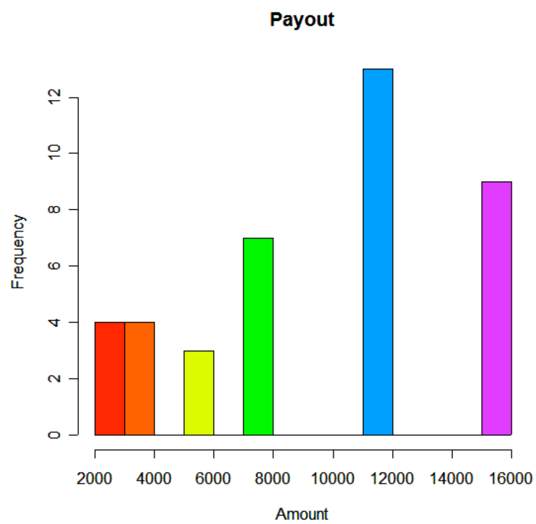
Results

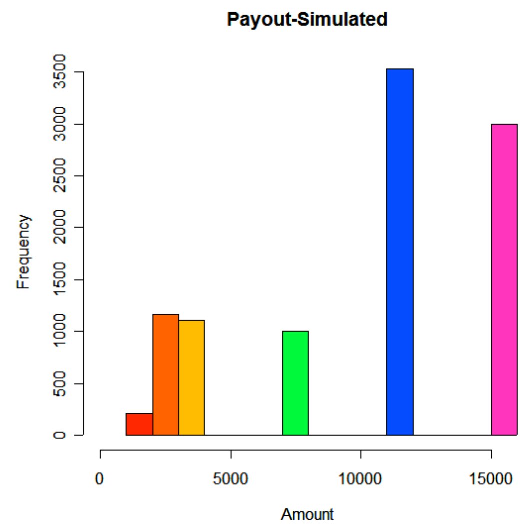
The following graphs show the differences in the payout distributions between the original interview data (on the right) and the adjusted interview data where everyone goes for the fifth draw (on the left).





The following graphs show the payout distributions for the interview data and the computer simulated data.

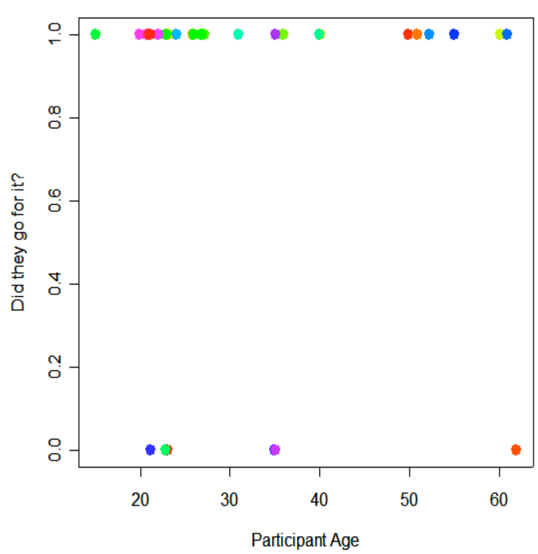




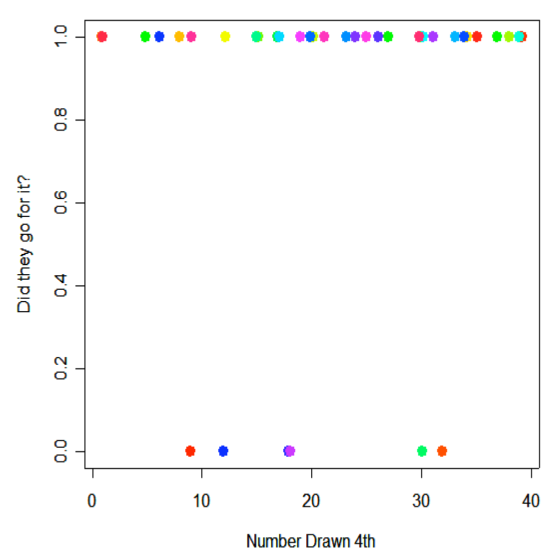
The following table compares the mean, mode, and standard deviation for each of the three distributions.

	Mean	Mode	SD
Interview Data	$10,025	$12,000	$4,549
Adjusted Int. Data	$10,100	$12,000	$4,914
Simulated Data	$10,665	$12,000	$4,811

The following plots show the possible explanatory variables ‘age’ (of participant) and ‘number drawn 4th’ against the choice of going for the fifth draw.



Corr = -0.0555



Corr = 0.0736

Discussion

In looking at the comparison of the payout distribution from the original data with that from the adjusted data, it is important to first note the difference in scales. Upon realizing this difference, one can say that the distribution shifts toward higher values when all of the players choose to go for a fifth draw. The mean of the adjusted distribution is slightly greater than that of the original distribution. This is especially interesting since of the six participants that chose not to go for a fifth draw, several would not have guessed correctly had they used probability to determine in which direction to guess. Despite this fact, however, the adjusted distribution still tends toward the higher values.

Although finding a model for prediction of a participant's choice to go for the fifth draw could have been useful and interesting, there did not appear to be a linear relationship between the choice and possible covariates age and the number drawn fourth. From the plots showing the covariates vs. the response, it is clear that the range of covariate values for participants who did and did not go for the fifth draw overlapped quite a bit. In addition, the correlation between each predictor and the response was very low in each case, indicating that there exists little dependence of the response on the predictors.

One of the main objectives of this project was to examine the differences between the probabilistically-sound method to play the lottery game and the methods that people use in practice. These differences are typically the result of the human tendency toward risk aversion. Naturally, some people are more inherently risk averse than are others, and the results of the project demonstrate this. Although many participants chose to take a fifth draw, some participants abstained for fear of losing the money that they had already won. These people demonstrated the human tendency to be more sensitive to loss than to gain; although the expected value of winnings after a fifth draw was greater than that after only four draws, several people wanted only to avoid the risk of losing any money.

As our methodology included asking participants why they did or did not go for the fifth draw, we can examine some of their responses to look for similarities, exhibition of sound reasoning, etc. For those that decided to go for the fifth draw, the most common reasons were:

- I had good odds and it's free money
- I'm on a good winning streak
- I wanted to get rich and I'm just playing the probabilities
- I had a 2 to 1 chance of winning a lot of money
- The expected value is higher if I go for it
- My fourth number made the choice easy

The last reason given above was common for people who got 1, 2, 38, 39, and other such very low or very high numbers as getting such a number gave them a better chance to double the money. Another common answer was that the participant would win some money no matter what, and since they were given money to begin the game, they will always come out ahead. Only one person, a statistics student, calculated the expected values in order to make a decision about going for the fifth draw and, as expected, such calculations led him to go for it. Now we examine some of the reasons given by participants who did not go for the fifth draw:

- I've had bad luck so far
- I don't want to risk losing the money I have already won
- I'm guaranteed to get the money I have won so far, and my fourth number is less than optimal
- I'm risk averse
- I'm poor and I need this money

Here, as in the responses for participants who went for it, we see that some people made their decision based on belief in lucky or unlucky streaks, which is clearly not statistically sound. However, for those that did not go for the fifth draw, the most common reasons have to do with being risk averse: wanting to keep the money they had already won because they need it or because they simply do not want to risk it. One person opted not to go for the fifth draw because his fourth number was 21, which didn't give him the best odds for doubling the money.

It is important to note that the results of the project may have changed had the setup of the game been altered. Different amounts of money and/or a different number of draws could have resulted in more participants forgoing a fifth draw.

Deficiencies in Our Study

There are several methodological deficiencies present in our study. First, a sample size of n = 40 is fairly small and may have led to some bias or inconclusive results. In addition, participants were not randomly selected. In fact, the participant group consisted of friends, family, and acquaintances of the investigators, which led to the sample having an age range skewed toward the younger generation. Many of the participants were Berkeley students who may have had experience in the study of probability. Furthermore participants knew that they would not be receiving their winnings and this may have affected their playing strategy or lack thereof. Lastly, the sampled population may not have been inclined toward gambling, i.e. were a lottery company to want to collect information about the expected payout of one their lottery games, they would likely want to sample people who would actually buy a lottery ticket.

Conclusion

Given that the optimal strategy is to go for the fifth draw, it is interesting to see that some people, albeit a small percentage, did not go for it. This study made it apparent that in an attempt to predict the payout distribution for a lottery game such as this, one must always take into account the human psychology.