Practice Midterm 1 solutions.

1.. Take a year as 365 days. Each other student has chance 364/365 to have a different birthday than you. So the chance that all 26 other students have birthdays different from yours is $(364/365)^{26} \approx 0.931$. So the chance that someone has the same birthday as you is $1 - 0.931 = 6.9\%$.

Comment. (a) The answer $26/365 = 7.1\%$ is wrong – you can't add, because the events are not mutually exclusive.

(b) This is <u>different</u> from "the birthday problem", which asks for the chance that some two people have the same birthday.

(c) Taking 365.25 days in a year would be more precise, but the difference is negligible. The calculation is sensible provided your birthday isn't February 29. If your birthday is February 29, the calculation is $1 - (1460/1461)^{26} \approx 1.8\%$.

2. If there are N families then there are $1 \times 0.15N + 2 \times 0.3N + 3 \times 0.25N + 4 \times 0.2N = 2.3N$ children. Of these, $0.75N$ are in 3-child families. So chance $= \frac{0.75N}{2.3N} = 32.6\%$.

Comment. Can't do this via Bayes formula.

3.

		x			
		0	1	2	3
y	0	0	3/20	6/20	1/20
	1	1/20	6/20	3/20	0

(b) $E(XY) = 1 \times 6/20 \ + 2 \times 3/20 = 0.6$.

4. (a) No, because 7 is a "red" number.

(b) If you win at least once then you are up by \$18; if you don't win then you are down \$18. There is negligible chance then I am up or down as much as 18, so

P(you gain more than me) $\approx$ P(you win at least once) $= 1 - (37/38)^{18} = 38.1\%$.

Comment. Normal approximation isn't relevant here.

5.. Let D be the difference "number in sample know Jackson - number know Kane". Then $D = \sum_{i=1}^{100} X_i$ where

$$\begin{aligned} X_i &= 1 \quad \text{if i'th student knows Jackson but not Kane} \\ &= -1 \quad \text{if i'th student knows Kane but not Jackson} \\ &= 0 \quad \text{otherwise} \end{aligned}$$

So $P(X_i = 1) = 21\%, P(X_i = -1) = 12\%, P(X_i = 0) = 67\%$. We can now calculate

$$EX = 0.09 \ \ \text{s.d.}(X) = \sqrt{0.33 - 0.09^2} = 0.567$$

$$ED = 9.0 \ \ \text{s.d.}(D) = \sqrt{100} \times 0.567 = 5.67$$

Since D has approximately Normal distribution

$$P(D < 0) \approx \Phi(\frac{-0.5 - 9.0}{5.67}) = \Phi(-1.67) = 4.7\%$$

using the discreteness correction.