Spring 2013 Statistics 153 (Time Series) : Lecture Twelve

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1 Plan

So Far:

1. Trend and Seasonality
2. Stationarity
3. ARMA models

Still to come in Time Domain Techniques:

1. How to fit ARMA models to data.
2. ARIMA models
3. SARIMA models
4. Forecasting
5. Model Diagnostics and Selection

2 Recap: Fitting AR models to data

Assuming that the order p is known. Carried out by invoking the function $ar()$ in R.

1. **Yule Walker or Method of Moments**: Finds the AR(p) model whose acvf equals the sample autocorrelation function at lags $0, 1, \dots, p$. Use *yw* for method in R.
2. **Conditional Least Squares**: Minimizes the conditional sum of squares: $\sum_{i=p+1}^{n}(x_i - \mu - \phi_1(x_{i-1} - \mu) - \cdots - \phi_p(x_{i-p} - \mu))^2$ over μ and $\phi_1, \dots, \phi_p$. And σ^2 is achieved by the average of the squared residuals. Use *ols* for method in R. In this method, given data $x_1, \dots, x_n$, R fits a model of the form $x_t - \bar{x} = intercept + \phi(x_{t-1} - \bar{x}) + residual$ to the data. The fitted value of intercept can be obtained by calling $\$x.intercept$. One can convert this to a model of the form $x_t = intercept + \phi x_{t-1} + residual$. Check the help page for the R function ar.ols.
3. **Maximum Likelihood**: Maximizes the likelihood function (which is relatively straightforward to write down but which requires an optimization routine to maximize). Use *mle* for method in R. This method is complicated.

It is usually the case that all these three methods yield similar answers. The default method in R is Yule-Walker.

3 Asymptotic Distribution of the Yule-Walker Estimates for AR models

For n large, the approximate distribution of $\sqrt{n}\left(\hat{\phi}-\phi\right)$ is normal with mean 0 and variance covariance matrix $\sigma_Z^2 \Gamma_p^{-1}$ where Γ_p is the $p \times p$ matrix whose (i,j)th entry is $\gamma_X(i-j)$.

For example, in the AR(1) case:

$$\Gamma_p = \Gamma_1 = \gamma_X(0) = \sigma_Z^2/(1-\phi^2).$$

Thus $\hat{\phi}$ is approximately normal with mean ϕ and variance $(1-\phi^2)/n$.

For AR(2), using

$$\gamma_X(0) = \frac{1-\phi_2}{1+\phi_2} \frac{\sigma_Z^2}{(1-\phi_2)^2-\phi_1^2} \qquad \text{and} \qquad \rho_X(1) = \frac{\phi_1}{1-\phi_2},$$

we can show that $(\hat{\phi}_1, \hat{\phi}_2)$ is approximately normal with mean (ϕ_1, ϕ_2) and variance-covariance matrix is $1/n$ times

$$\begin{pmatrix} 1-\phi_2^2 & -\phi_1(1+\phi_2) \\ -\phi_1(1+\phi_2) & 1-\phi_2^2 \end{pmatrix}$$