Statistics 153 (Introduction to Time Series) Homework 2

Due on 27 February, 2013

16 February, 2013

1. Let $\{Y_t\}$ be a doubly infinite sequence of random variables that is stationary with autocovariance function γ_Y. Let
$$X_t = (a + bt)s_t + Y_t,$$
where a and b are real numbers and s_t is a deterministic seasonal function with period d (i.e., $s_{t-d} = s_t$ for all t)
   (a) Is $\{X_t\}$ a stationary process? Why or Why not?
   (b) Let $U_t = \psi(B)X_t$ where $\psi(z) = (1 - z^d)^2$. Show that $\{U_t\}$ is stationary.
   (c) Write the autocovariance function of $\{U_t\}$ in terms of the autocovariance function, γ_Y, of $\{Y_t\}$.

2. We have seen that $\sum_{j=0}^{\infty} \phi^j Z_{t-j}$ is the unique stationary solution to the AR(1) difference equation: $X_t - \phi X_{t-1} = Z_t$ for $|\phi| < 1$. But there can be many *non-stationary* solutions. Show that $X_t = c\phi^t + \sum_{j=0}^{\infty} \phi^j Z_{t-j}$ is a solution to the difference equation for every real number c. Show that this is non-stationary for $c \neq 0$.

3. Consider the AR(2) model: $\phi(B)X_t = Z_t$ where $\phi(z) = 1 - \phi_1 z - \phi_2 z^2$ and $\{Z_t\}$ is white noise. Show that there exists a unique causal stationary solution if and only if the pair (ϕ_1, ϕ_2) satisfies all of the following three inequalities:
$$\phi_2 + \phi_1 < 1 \qquad \phi_2 - \phi_1 < 1 \qquad |\phi_2| < 1.$$

4. Consider the AR(2) model: $X_t - X_{t-1} + 0.5X_{t-2} = Z_t$ where $\{Z_t\}$ is white noise. Show that there exists a unique causal stationary solution. Find the autocorrelation function.

5. Let $\{Y_t\}$ be a doubly infinite sequence of random variables that is stationary. Let
$$X_t = \beta_0 + \beta_1 t + \cdots + \beta_q t^q + Y_t$$
where $\beta_0, \ldots, \beta_q$ are real numbers with $\beta_q \neq 0$.
   (a) Show that $(I - B)^k Y_t$ is stationary for every $k \geq 1$.

(b) Show that $(I - B)^k X_t$ is not stationary for $k < q$ and that it is stationary for $k \geq q$.

6. Let $\{Y_t\}$ be a doubly infinite mean zero sequence of random variables that is stationary. Define $X_t = Y_t - 0.4Y_{t-1}$ and $W_t = Y_t - 2.5Y_{t-1}$.

   (a) Express the autocovariance functions of $\{X_t\}$ and $\{W_t\}$ in terms of the autocovariance function of $\{Y_t\}$.

   (b) Show that $\{X_t\}$ and $\{W_t\}$ have the same autocorrelation functions.