

Statistics 153 (Introduction to Time Series) Homework 1

Due on 13 February, 2013

04 February, 2013

1. Download the google trends time series dataset for the query *orkut*.
 - (a) Estimate the trend function in the data by fitting a parametric curve to the data. Provide a plot of the original data along with the corresponding trend estimate. Also provide a time plot and correlogram of the residuals. Comment on each of these plots.
 - (b) Estimate the trend by moving average smoothing. Explain reasons behind your choice of the smoothing parameter. Once again, provide a plot of the original data along with the corresponding trend estimate. Also provide a time plot and correlogram of the residuals. Comment on each of these plots.
 - (c) Eliminate the trend using differencing. What order differencing did you use and why? Provide a plot of the final detrended differenced residuals and their correlogram. Comment of each of these plots.

Comment on the relative merits and demerits of each of these three methods for dealing with trend in the context of this particular dataset.

2. Download the google trends time series dataset for the query *beach*.
 - (a) Fit a model of the form $X_t = m_t + s_t + Z_t$ to the data where m_t and s_t are deterministic trend and seasonality functions respectively and Z_t is white noise. Provide a plot of the original data along with the estimate $\hat{m}_t + \hat{s}_t$. Also provide a time plot and correlogram of the residuals. Comment on each of the plots.
 - (b) Via the method of differencing, produce residuals which have neither trend nor seasonality. What order differencing did you use? Provide a plot of the residuals and their correlogram. Comment on each of the plots.
3. While analyzing their annual sales data (number of car sales per year) for the past 50 years, a car company found that after taking three successive differences, the resulting data had a mean of 3056 and looked like white noise. If the actual data for the past four years were 2011 - 63214, 2010 - 52874, 2009 - 39981 and 2008 - 35756. What would be a reasonable forecast for sales in 2012? Explain.

4. Consider a monthly time series dataset for which we believe that the model $X_t = (at + b)s_t + Z_t$ is appropriate where s_t is a seasonal function i.e., $s_{t+d} = s_t$ and Z_t is white noise. What would be a way to difference the data in order to eliminate both trend and seasonality and why does it work?
5. Let U_1 and U_2 are uncorrelated random variables with mean zero and variance σ^2 and let ω_0 be a real number. Show that the process

$$X_t = U_1 \cos(2\pi\omega_0 t) + U_2 \sin(2\pi\omega_0 t)$$

is weakly stationary. Find its autocovariance and autocorrelation functions.