Statistics 153 (Introduction to Time Series) Homework Five

Due on 8 May, 2013

28 April, 2013

1. Consider the following seasonal AR model:

$$(1 - \phi B)(1 - \Phi B^s)X_t = Z_t,$$

where $\{Z_t\}$ is white noise and $|\phi| < 1, |\Phi| < 1$.

   (a) Calculate the spectral density of $\{X_t\}$.

   (b) Plot the spectral density for $\phi = 0.5$, $\Phi = 0.9$, $\sigma_Z^2 = 1$ and $s = 12$.

   (c) Also plot the spectral density for the AR(1) process $(1 - 0.5B)X_t = Z_t$ and the seasonal AR(1) process $(1 - 0.9B^{12})X_t = Z_t$.

   (d) Compare and comment on the different plots.

2. The spectral density of a stationary time series $\{X_t\}$ is defined on $[-1/2, 1/2]$ by $f(\lambda) = 5$ for $1/6 \leq |\lambda| \leq 1/3$ and zero otherwise.

   (a) Evaluate the autocovariance function of $\{X_t\}$ at lags 0 and 1.

   (b) Find the spectral density of the process $\{Y_t\}$ defined by $Y_t = X_t - X_{t-12}$.

3. Consider the stationary Autoregressive process:

$$X_t - 0.99X_{t-3} = Z_t$$

where $\{Z_t\}$ is white noise.

   (a) Compute and plot the spectral density of $\{X_t\}$.

   (b) Does the spectral density suggest that the sample paths of $\{X_t\}$ will exhibit approximately oscillatory behaviour? If yes, then with what period?

   (c) Simulate a sample of size 100 from this model. Plot the simulated data. Does this plot support the conclusion of part (b)?

   (d) Compute the spectral density of the filtered process:

$$Y_t = \frac{X_{t-1} + X_t + X_{t+1}}{3}. \tag{1}$$

   How does the spectral density of $\{Y_t\}$ compare to that of $\{X_t\}$?

(e) From the simulated sample from $\{X_t\}$ in part (c), perform the averaging as in (1) to obtain a simulated sample from $\{Y_t\}$. Plot this sample. Does this plot support the spectral density plot in part (d)?

4. Without using the *arima.sim()* function in R, simulate $n = 400$ observations from the multiplicative seasonal ARMA model given by the difference equation:

$$(1 - 0.5B)(1 - 0.7B^{12})X_t = Z_t$$

where $\{Z_t\}$ is white noise. Plot the sample autocorrelation function of the simulated observations and compare it with the true acf of the process.

5. Consider the first dataset, `q1data.R`, from the second midterm. Remove the trend and seasonality by differencing first with order 52 and then a usual differencing. Call the resulting dataset $x_t, t = 1, \ldots, n$ to which a stationary model can be fit.

   (a) Estimate the spectral density of $\{X_t\}$ nonparametrically from the data $\{x_t\}$.

   (b) Fit a reasonable stationary model to $\{x_t\}$ and estimate the spectral density of $\{X_t\}$ by the spectral density of the fitted model.

   (c) Plot the two estimates of the spectral density on the same plot. Commment on the two plots.