5.13. (a) $n = 4$ and $p = 1/4 = 0.25$. (b) The distribution is below; the histogram is on the right. (c) $\mu = np = 1$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(X = x)$</th>
<th>0.3164</th>
<th>0.4219</th>
<th>0.2109</th>
<th>0.0469</th>
<th>0.0039</th>
</tr>
</thead>
</table>

5.14. For $\hat{p}$, $\mu = 0.49$ and $\sigma = \sqrt{p(1-p)/n} \approx 0.01576$. As $\hat{p}$ is approximately normally distributed with this mean and standard deviation, we find

$$P(0.46 < \hat{p} < 0.52) \approx P(-1.90 < Z < 1.90) = 0.9426$$

(Exact calculation gives 0.94565.)

5.21. (a) $X$, the count of successes, has a binomial distribution with mean $\mu_X = np = (1000)(1/5) = 200$ and $\sigma_X = \sqrt{(1000)(0.2)(0.8)} \approx 12.553$ successes. (b) For $\hat{p}$, the mean is $\mu_{\hat{p}} = p = 0.2$ and $\sigma_{\hat{p}} = \sqrt{(0.2)(0.8)/1000} \approx 0.01265$. (c) $P(\hat{p} > 0.24) \approx P(Z > 3.16) = 0.0008$. (Exact computation gives 0.00111; using $P(X \geq 240)$ with the continuity correction gives 0.0009.) (d) From a standard normal distribution, $P(Z > 2.326) = 0.01$, so the subject must score 2.326 standard deviations above the mean: $\mu_{\hat{p}} + 2.326\sigma_{\hat{p}} = 0.2294$. This corresponds to 230 or more successes.

5.22. (a) $M$ has a binomial distribution with $n = 30$ and $p = 0.7$, so $P(M = 20) = \binom{30}{20}(0.7)^{20}(0.3)^{10} \approx 0.1416$. (b) $P(1\text{st woman is the 4th call}) = (0.7)^3(0.3) = 0.1029$.

5.24. (a) $\binom{n}{1} = \frac{n!}{1!(n-1)!} = 1$. The only way to distribute $n$ successes among $n$ observations is for all observations to be successes. (b) $\binom{n}{n-1} = \frac{n!}{(n-1)!(1)!} = \frac{n}{(n-1)!} = n$. To distribute $n - 1$ successes among $n$ observations, the one failure must be either observation 1, 2, 3, ..., $n - 1$, or $n$. (c) $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!(n-k)!} = \binom{n}{n-k}$. Distributing $k$ successes is equivalent to distributing $n - k$ failures.

5.34. (a) $P(X \geq 23) \approx P(Z \geq \frac{23 - 20.8}{4.8}) = P(Z \geq 0.46) = 0.3248$ (with software: 0.3234). Because ACT scores are reported as whole numbers, we might instead compute $P(X \geq 22.5) = P(Z \geq 0.35) = 0.3632$ (software: 0.3616). (b) $\mu_\bar{x} = 20.8$ and $\sigma_\bar{x} = \sqrt{25} = 0.96$. (c) $P(\bar{x} \geq 23) \approx P(Z \geq \frac{23 - 20.8}{0.96}) = P(Z \geq 2.29) = 0.0110$. (In this case, it is not appropriate to find $P(\bar{x} \geq 22.5)$, unless $\bar{x}$ is rounded to the nearest whole number.) (d) Because individual scores are only roughly normal, the answer to (a) is approximate. The answer to (c) is also approximate, but should be more accurate because $\bar{x}$ should have a distribution that is closer to normal.
5.38. (a) \( \mu_X = (500)(0.001) = 0.50 \) and \( \sigma_X = \sqrt{249.75} \approx 15.8035 \). (b) In the long run, Joe makes about 50 cents for each $1 ticket. (c) If \( \bar{X} \) is Joe's average payoff over a year, then \( \mu_{\bar{X}} = \mu = 0.50 \) and \( \sigma_{\bar{X}} = \sigma_X/\sqrt{365} \approx 0.8272 \). The central limit theorem says that \( \bar{X} \) is approximately normally distributed (with this mean and standard deviation). (d) Using this normal approximation, \( P(\bar{X} > 1) \approx P(Z > 0.60) = 0.2743 \) (software: 0.2728).

Note: Joe comes out ahead if he wins at least once during the year. This probability is easily computed as \( 1 - (0.999)^{365} \approx 0.3059 \). The distribution of \( \bar{X} \) is different enough from a normal distribution so that answers given by the approximation are not as accurate in this case as they are in many others.

5.52. For each step of the random walk, the mean is \( \mu = (1)(0.75) + (-1)(0.25) = 0.5 \), the variance is \( \sigma^2 = (1 - 0.5)^2(0.75) + (-1 - 0.5)^2(0.25) = 0.75 \), and the standard deviation is \( \sigma = \sqrt{0.75} \approx 0.8660 \). Therefore, \( Y/500 \) has approximately a \( N(0.5, 0.03873) \) distribution, and \( P(Y \geq 200) = P(Y/500 \geq 0.4) \approx P(Z \geq -2.58) = 0.9951 \).

Note: The number \( R \) of right-steps has a binomial distribution with \( n = 500 \) and \( p = 0.75 \). \( Y \geq 200 \) is equivalent to taking at least 350 right-steps, so we can also compute this probability as \( P(R \geq 350) \), for which software gives the exact value 0.99517.