It is well known that if a covariance matrix, Σ is decomposed by the Cholesky decomposition, such that

$$\Sigma = \mathbf{U}'\mathbf{U}$$

where $\mathbf{U}$ is an upper triangular matrix, then, given a $p \times 1$ vector x, consisting of *i.i.d.* uncorrelated random variables with $\operatorname{var} x = \mathbf{I}$, we can construct a $p \times 1$ vector x^* with $\operatorname{var} x^* = \Sigma$ by calculating

$$x^* = \mathbf{U}'x$$

The problem arises when we try to translate this to a *matrix* of random variables, where the rows of the matrix each represent an observation (similar to x in the above discussion) consisting of p values to be transformed. Since the rows of such a matrix (which we shall refer to as $\mathbf{X}$), are $1 \times p$ vectors, and not $p \times 1$ vectors, clearly some adjustment needs to be made.

First, let us consider in detail the computatation involved in calculating x^*:

$$\begin{aligned} x_j^* &= \sum_{k=1}^{p} \mathbf{U}'_{jk} x_k \\ &= \sum_{k=1}^{p} \mathbf{U}_{kj} x_k \end{aligned}$$

Translating this relationship to the rows of $\mathbf{X}$ means duplicating this relationship for each of the rows of $\mathbf{X}$:

$$\begin{aligned} \mathbf{X}_{ij} &= \sum_{k=1}^{p} \mathbf{U}_{kj} \mathbf{X}_{ik} \\ &= \sum_{k=1}^{p} \mathbf{X}_{ik} \mathbf{U}_{kj} \end{aligned}$$

Thus, the correct multiplication to transform each of the rows of $\mathbf{X}$ by $\mathbf{U}$ is:

$$\mathbf{X}^* = \mathbf{X}\mathbf{U}.$$