Homework 4

1. Consider the effects of multicolinearity on predictions from a standard linear model.

   (a) How does multicolinearity affect the global measure trace($\Sigma_{\hat{Y}\hat{Y}}$)?

   (b) Consider predicting a new response with covariate value x_0. What is the effect of multicolinearity?

2. Consider a standard linear model with design matrix

$$X = \begin{pmatrix} -2 & -2 \\ -1 & -1 \\ 0 & 0 \\ 1 & 1 \\ 2 & 2 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}$$

   and unknown coefficients β_1, β_2.

   (a) Which linear combinations $\hat{\psi} = a_1\hat{\beta}_1 + a_2\hat{\beta}_2$ have largest and smallest variance, subject to $a_1^2 + a_2^2 = 1$?

   (b) If the columns of the design matrix are thought of as values continuous variables x_1 and x_2, then we are fitting a plane through the origin: $y = \beta_1 x_1 + \beta_2 x_2$. If $u = (u_1, u_2)$ is a direction vector, what is the estimated slope of the plane in that direction? In what direction is the estimated slope most variable? Least variable? Do the answers "make sense?"

3. Suppose that $Y \sim MVN(0, \sigma^2 I_n)$. Show that $\bar{Y}$ is independent of $\sum_{i=1}^{n}(Y_i - \bar{Y})^2$.