

Homework 2

1. Suppose that in the standard linear model, the design matrix X contains a column of ones. Show that the sum of the residuals from the least squares fit equals 0.
2. Suppose that the independent variables in a standard least squares model are replaced by rescaled variables $u_{ij} = k_j x_{ij}$ (for example, centimeters are converted to meters.) Show that \hat{Y} does not change. Does $\hat{\beta}$ change? (Hint: express the new design matrix in terms of the old one.)
3. In order to estimate two parameters θ and ϕ a number of independent measurements are taken, each having errors with mean zero and variance σ^2 :
 - (a) n observations have mean θ .
 - (b) m observations have mean $\theta - \phi$.
 - (c) m observations have mean $\phi - \theta$.

Set this up in the form of a standard linear model. What are the least squares estimates of θ and ϕ ? Find $Var(\hat{\phi})$, $Cov(\hat{\phi}, \hat{\theta})$, and $Var(\hat{\phi} - \hat{\theta})$. What is the estimate of σ^2 ?

4. For a linear model with $\Sigma_{ee} = \sigma^2 V$, where V is a known positive definite matrix, we have seen how to form the generalized least squares estimates. How could σ^2 be estimated?