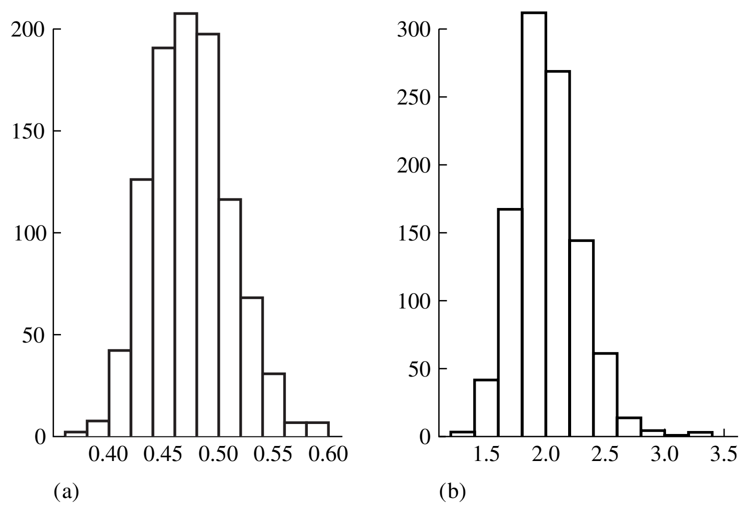

FIGURE **8.6** Histograms of 1000 simulated maximum likelihood estimates of (a) α and (b) λ.

the sampling distribution of their maximum likelihood estimates by generating many, many samples of size $n = 227$ from a gamma distribution with parameters α_0 and λ_0, forming the maximum likelihood estimates from each sample, and displaying the results in histograms. Since, of course, we don't know the true values, we let our maximum likelihood estimates play their role: We generated 1000 samples each of size $n = 227$ of gamma distributed random variables with $\alpha = .47$[illegible] and $\lambda =$ [illegible]7. For each of these samples, the maximum likelihood estimates of α and λ were c[illegible]u-lated. Histograms of these 1000 estimates are shown in Figure 8.6; we regard these histograms as approximations to the sampling distribution of the maximum likelihood estimates $\hat{\alpha}$ and $\hat{\lambda}$.

Comparison of Figures 8.6 and 8.4 is interesting. We see that the sampling distributions of the maximum likelihood estimates are substantially less dispersed than those of the method of moments estimates, which indicates that in this situation, the method of maximum likelihood is more precise than the method of moments. The standard deviations of the values displayed in the histograms are the estimated standard errors of the maximum likelihood estimates; we find $s_{\hat{\alpha}} = .03$ and $s_{\hat{\lambda}} = .26$. Recall that in Example C of Section 8.4 the corresponding estimated standard errors for the method of moments estimates were found to be .06 and .34. ■

EXAMPLE **D** *Muon Decay*

From the form of the density given in Example D in Section 8.4, the log likelihood is

$$l(\alpha) = \sum_{i=1}^{n} \log(1 + \alpha X_i) - n \log 2$$

Setting the derivative equal to zero, we see that the mle of α satisfies the following