Minimizing this with respect to π gives the optimal portfolio

$$\pi_{\text{opt}} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

For example, if the investments are equally risky, $\sigma_1 = \sigma_2 = \sigma$, then $\pi = 1/2$, so the best strategy is to split her total investment equally between the two securities. If she does so, the variance of her return is, by Theorem A,

$$\operatorname{Var}\left(R\left(\tfrac{1}{2}\right)\right) = \frac{\sigma^2}{2}$$

whereas if she put all her money in one security, the variance of her return would be σ^2. The expected return is the same in both cases. This is a particularly simple example of the value of diversification of investments.

Suppose now that the two securities do not have the same expected returns, $\mu_1 < \mu_2$. Let the standard deviations of the returns be σ_1 and σ_2; usually less risky investments have lower expected returns, $\sigma_1 < \sigma_2$. Furthermore, the two returns may be correlated: $\operatorname{Cov}(R_1, R_2) = \rho\sigma_1\sigma_2$. Corresponding to the portfolio $(\pi, 1 - \pi)$, we have expected return

$$E(R(\pi)) = \pi\mu_1 + (1 - \pi)\mu_2$$

and the variance of the return is

$$\operatorname{Var}(R(\pi)) = \pi^2\sigma_1^2 + 2\pi(1 - \pi)\rho\sigma_1\sigma_2 + (1 - \pi)^2\sigma_2^2$$

Comparing this to the result when the returns were independent, we see the risk is lower when the returns are independent than when they are positively correlated. It would thus be better to invest in two unrelated or weakly related market sectors than to make two investments in the same sector. In deciding the choice of the portfolio vector, the investor can study how the risk (the standard deviation of $R(\pi)$) changes as the expected return increases, and balance expected return versus risk.

In actual investment decisions, many more than two possible investments are involved, but the basic idea remains the same. Suppose there are n possible investments. Let the portfolio weights be denoted by the vector $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$. Let $E(R_i) = \mu_i$, $\operatorname{Cov}(R_i, R_j) = \sigma_{ij}$ (so, in particular, $\operatorname{Var}(R_i)$ is denoted by σ_{ii}), then

$$E(R(\pi)) = \sum \pi_i\mu_i$$

and

$$\operatorname{Var}(R(\pi)) = \sum_{i=1}^{n}\sum_{j=1}^{n} \pi_i\pi_j\sigma_{ij}$$

The investment decision, the choice of the portfolio vector π, is often couched as that of maximizing expected return subject to the risk being less than some value the individual investor is willing to tolerate. Some investors are more risk averse than others, so the portfolio vectors will differ from investor to investor. Equivalently, the decision may be phrased as that of finding the portfolio vector with the minimum risk subject to a desired return; there may well be many portfolio choices that give the