Statistics 215a – 9/26/04 – D. R. Brillinger

Ordinary Least Squares (OLS).

$(\mathbf{y}-\mathbf{X}\beta)'(\mathbf{y}-\mathbf{X}\beta)$

$= (\mathbf{y}-\mathbf{Xb}+\mathbf{X}(\mathbf{b}-\beta))'(\mathbf{y}-\mathbf{Xb}+\mathbf{X}(\mathbf{b}-\beta))$

$= (\mathbf{y}-\mathbf{Xb})'(\mathbf{y}-\mathbf{Xb}) + (\mathbf{y}-\mathbf{Xb})'\mathbf{X}(\mathbf{b}-\beta) + (\mathbf{b}-\beta)'\mathbf{X}'(\mathbf{y}-\mathbf{Xb}) + (\mathbf{b}-\beta)'\mathbf{X}'\mathbf{X}(\mathbf{b}-\beta)$

$= (\mathbf{y}-\mathbf{Xb})'(\mathbf{y}-\mathbf{Xb}) + (\mathbf{b}-\beta)'\mathbf{X}'\mathbf{X}(\mathbf{b}-\beta)$

if $\mathbf{X}'\mathbf{Xb} = \mathbf{X}'\mathbf{y}$ (normal equations)

$(\mathbf{y}-\mathbf{X}\beta)'(\mathbf{y}-\mathbf{X}\beta)$ is a minimum at **b** satisfying normal equations

Weighted Least Squares (WLS).

W: *n* by *n*, symmetric, non-negative definite, known up to a constant

$Min_\beta\ (\mathbf{y}-\mathbf{X}\beta)'\mathbf{W}(\mathbf{y}-\mathbf{X}\beta)$

$\mathbf{b} = (\mathbf{X}'\mathbf{W}\mathbf{X})^{-}\mathbf{X}'\mathbf{W}\mathbf{y}$

Reduces to OLS if write

$\mathbf{R} = \mathbf{W}^{1/2}\mathbf{X}$, $\mathbf{s} = \mathbf{W}^{1/2}\mathbf{y}$

$\mathbf{R}'\mathbf{R}\mathbf{b} = \mathbf{R}'\mathbf{s}$

If $\mathbf{W} = \mathbf{U}'\Lambda\mathbf{U}$, $\mathbf{W}^{1/2} = \mathbf{U}'\Lambda^{1/2}\mathbf{U}$ and $(\mathbf{W}^{1/2})'\mathbf{W}^{1/2} = \mathbf{W}$

Example.

$\mathbf{W} = \mathrm{diag}\{w_i\}$

$(\mathbf{y}-\mathbf{X}\beta)'(\mathbf{y}-\mathbf{X}\beta) = \Sigma_i^n\, w_i\, (y_i - \mathbf{x}_i'\beta)^2$

$\mathbf{x}_i'$ is the *i*-th row of $\mathbf{X}$