

*Residual analysis.*

data = fit + residual

discrepancy between data and fit

fit might be OLS or robust/resistant or ?

patterns suggest fit can be improved, e.g.  
by transformations, and there are often  
surprises

Procedure: improvement of fit by stages

*Types of residuals*

"ordinary",  $r_i = y_i - x_i^T b$

standardized,  $r_i/s\sqrt{1-h_{ii}}$

cross-validation,  $y_i - x_i^T b_{-i}$

*Uses of residual plots*

improvement of fit

identification of outliers

behavior of techniques on the data to  
portray adequacy of fit

*Types of residual plots*

residuals,  $r_i$ , vs. fitted values,  $x_i^T b$

residuals vs. explanatories,  $x_{ij}$

residuals vs. functions of the x's, e.g.  
products

residuals vs. new variables, e.g. time

$|r_i|$  vs.  $x_i^T b$

smoothed residuals vs. ...

*Some patterns*

sloping band - include linear term

curved band - add quadratic or somesuch

wedging - variability increasing  
(use weights in fitting)

- but, perhaps result of point  
density increasing (add lowess lines)

- wedging may be one-sided if  $|r|$  used

Plot of response vs. fit can be misleading

*Partial residual plots*

Plot partial residual of  $j$ -th explanatory vs. its values

i.e.  $r_i^j = y_i - \sum_{k \neq j} b_k x_{ik}$  versus  $x_{ij}$

have "removed" the linear effects of the other  $x$ 's

the slope is  $b_j$

the residuals are the  $r_i$

transform the  $j$ -th explanatory?

*Fitting by stages/one variable at a time*

Two explanatories case

$(x_1, x_2, y)$

1) fit  $y$  by  $x_1$

$y_{.1} = y - c_1 x_1$

2) fit  $x_2$  by  $x_1$

$$x_{2.1} = x_2 - d_1 x_1$$

3) fit  $y_{.1}$  by  $x_{2.1}$ , including intercept

Because of the orthogonalities this gives the one step result

$$b_0 + b_1 x_1 + b_2 x_2$$

now written as

$$b_0 + c_1 x_1 + c_2 (x_2 - d_1 x_1)$$

One can graph  $(x_1, y)$ ,  $(x_1, x_2)$ ,  $(x_{2.1}, y_{.1})$  along the way as well as residuals

Robust/resistant fitting could be used

R. D. Cook and S. Weisberg (1982). *Residuals and Influence in Regression*. Chapman and Hall.

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*The x-values.*

1. location parameter,  $p = 1$ ,  $x \equiv 1$

## 2. factor

object taking on values from a set of levels

often created via x-variables taking on the values 0, 1

e.g. for q levels set

$x^1 = 1$  for level 1 and 0 otherwise

$x^2 = 1$  for level 2 and 0 otherwise

.

$x^{q-1} = 1$  for level  $q-1$  and 0 otherwise

$x^q = 0$

provides a coding

The X-matrix is made up of 0's and 1's

`factor()`, `contrasts()`

Example - the RBC data

A: day

B: run

lm(y ~ -1 +A +B)