$2\pi \neq 1$

Statistics *215a – 11/17/03 – D.R. Brillinger*

Exploratory analysis of time series

Frequency (vs. time)-side analysis.

Spectrum analysis

One makes extensive use of sines and cosines, but that doesn't imply restriction to periodic phenomena

A *cosine wave*

$$\alpha \cos(\lambda t + \gamma),\ t=1,2,3,\ldots$$

α: the *amplitude*

λ: the *frequency*

γ: the *phase*

The *period* is $2\pi/\beta$, in units of time

The *frequency* is $\beta/2\pi$, in cycles per unit time

Consider approximating the data y_t, $t=1,\ldots,T$ by

$$\alpha \cos(\lambda t + \gamma) = \beta_1 \cos \lambda t + \beta_2 \sin \lambda t$$

with, for the moment, λ known

The *normal equations* are

$$\sum_t (\cos \lambda t)^2 b_1 + \sum_t (\cos \lambda t)(\sin \lambda t) b_2 = \sum_t (\cos \lambda t) y_t$$

$$\sum_t (\sin \lambda t)(\cos \lambda t) b_1 + \sum_t (\sin \lambda t)^2 b_2 = \sum_t (\sin \lambda t) y_t$$

Harmonic analysis (vs. synthesis)

With $i = \sqrt{(-1)}$ and remembering de Moivre’s formula,

$$\cos \lambda t + i \sin \lambda t = \exp\{i\lambda t\},$$

the *Fourier transform* of the data is

$$d^T(\lambda) = \sum_{t=1}^{T} \exp\{-i\lambda t\} y_t$$

considered as a function of all frequencies

Its amplitude,

$$|\sum_t \exp\{-i\lambda t\} y_t|^2$$

provide the so called *periodogram*.

It is useful if, for example, one wishes to approximate the data by

$$\beta_1 \cos \lambda t + \beta_2 \sin \lambda t$$

but one doesn't know λ.

This is the *hidden frequency* problem.

There may be several hidden frequencies.

One has a nonlinear least squares problem.

Examples.

Sunspots, Chandler wobble, tides, music, speech, free oscillations of the Earth, …

Discrete Fourier transform

$$d^T(2\pi s/T) = \sum_{t=1}^{T} \exp\{-i2\pi st/T\} y_t \quad , \quad s=1,\dots,T$$

This may be computed rapidly for composite values of T

Splus/R function fft()

FFTs speed up many computations

The FT provides a 1-1 unitary transformation of the y's

The *periodogram*

$I^T(\lambda) = |\sum_t \exp\{-i\lambda t\} y_t|^2 / 2\pi T$

Provides a measure of variability at frequency λ

One may have removed a measure of location, e.g. $\bar{y}$, $\hat{\mu}$

[I JUST FOUND THE HAT!!!]

Physical analogs

Newton and the prism

Discovery of hydrogen line

Filters take the form

$y_t = \sum_u a_{t-u} x_u$

Note the convolution character

$\{a_u\}$ is called the *impulse response*

Its Fourier transform

$$A(\lambda) = \sum_t a_t \exp\{-i\lambda t\}, \ 0 \le \lambda < 2\pi$$

is called the *transfer function*.

period 2π

There are:

lowpass filters (smoothers)

bandpass filters

band-elimination filters

highpass filters (roughers)

One may wish to estimate or remove:

a seasonal effect

a trend

Seasonal effect

$$s(t) = \mu + \sum_k (\alpha_k \cos 2\pi kt/K + \beta_k \sin 2\pi kt/K)$$

$k=1,\ldots,K$

For given $A(\lambda)$ filters may be realized, approximately, by FT-ing.

A bank of bandpass filters provides a decomposition of a series into frequency components

One may use a sliding window

ocean storm detection

Seismic surface waves

Cross-spectrum analysis

Cp. regression/OLS at a series of frequencies

Robust/resistant variants

Missing values (e.g. via robust method)

One may work with: spatial data, $y_{s,t}$, point process data, …

And onto Statistics 248