Statistics 215a – 9/26/06 – D. R. Brillinger

Homework 4 – due September 30

Part 4 has been corrected (from the 9/22/04) handout.

Remember the stricture, always check the dimensions of matrices.
x_i is the i-th row of X

Computing least squares quantities with an observation removed, deletion diagnostics, is an important tool.

The data will be denoted X, n by p and y n by 1.
These will be denoted X_{-i} and y_{-i} when the i-th observation, (x_i , y_i), is deleted. (This is a row vector.)
The least squares coefficients will be denoted b and b_{-i} .
The vector of residuals is r = y – Xb.
The hat matrix is $H = X(X^TX)^{-1}X^T$.
The matrix X will be assumed to be of full rank.
The dimensions of the vectors and matrices appearing are to be inferred from the expressions in which they appear.

1. Use Gauss's identity

$$(A+a^Tb)^{-1} = A^{-1} - A^{-1}a^T(I+bA^{-1}a^T)^{-1}bA^{-1}$$

to show that

$$(X_{-i}^TX_{-i})^{-1} = (X^TX-x_i^Tx_i)^{-1}$$

$$= (X^TX)^{-1} + (X^TX)^{-1}x_i^Tx_i(X^TX)^{-1}/(1-x_i(X^TX)^{-1}x_i^T)$$

2. Show that

$$b - b_{-i} = (X^TX)^{-1}x_i^Tr_i/(1-h_{ii})$$

3. Show that

$$x_ib - x_ib_{-i} = x_i(X^TX)^{-1}x_i^Tr_i/(1-h_{ii})$$

4. Show that

$y_i - x_i b_{-i} = r_i/(1-h_{ii})$