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23

An application of statistics to seismology: dispersion and modes

D.R. Brillinger

23.1 INTRODUCTION

Maurice Priestley has made substantial contributions to the analysis of time series. I mention, in particular, his work with the conceptualization of the idea of spectrum of a non-stationary time series (see Priestley 1965, his discussion of Loynes 1968 and Priestley and Tong (1973)) and with processes having discontinuous spectra (see Priestley 1962a,b; 1964). This present paper contains elements of both of these topics. Professor Priestley's work is seminal and broadly applicable. It has certainly influenced my own work.

23.2 OSCILLATIONS OF FINITE BODIES

Suppose that there is a finite body and suppose for the moment that it is linear, lying above the x-axis. Let $Y(x,t)$ denote the height at time t, of the body above coordinate x on the x-axis. In a simple case the motion could be given by

$$Y(x,t) = a\cos(kx - \omega t + \phi). \tag{23.1}$$

The Fourier transform of the complex signal associated to $Y(x,t)$, with respect to x and t, is

$$a\frac{e^{i\phi}}{(2\pi)^2}\iint e^{i(kx-\omega t)}e^{i(Kx+\Omega t)}\,dx\,dt = ae^{i\phi}\delta(\Omega-\omega)\delta(K+k)$$

with $\delta(\cdot)$ the Dirac delta function. This transform is concentrated at a point in the $\Omega - K$ plane. Here ω, Ω refer to frequency and K, k to wavenumber.

In the case of a finite body, discreteness occurs, that is for given ω, k is restricted to a discrete set of possible values,

$$k = k_n(\omega) \tag{23.2}$$

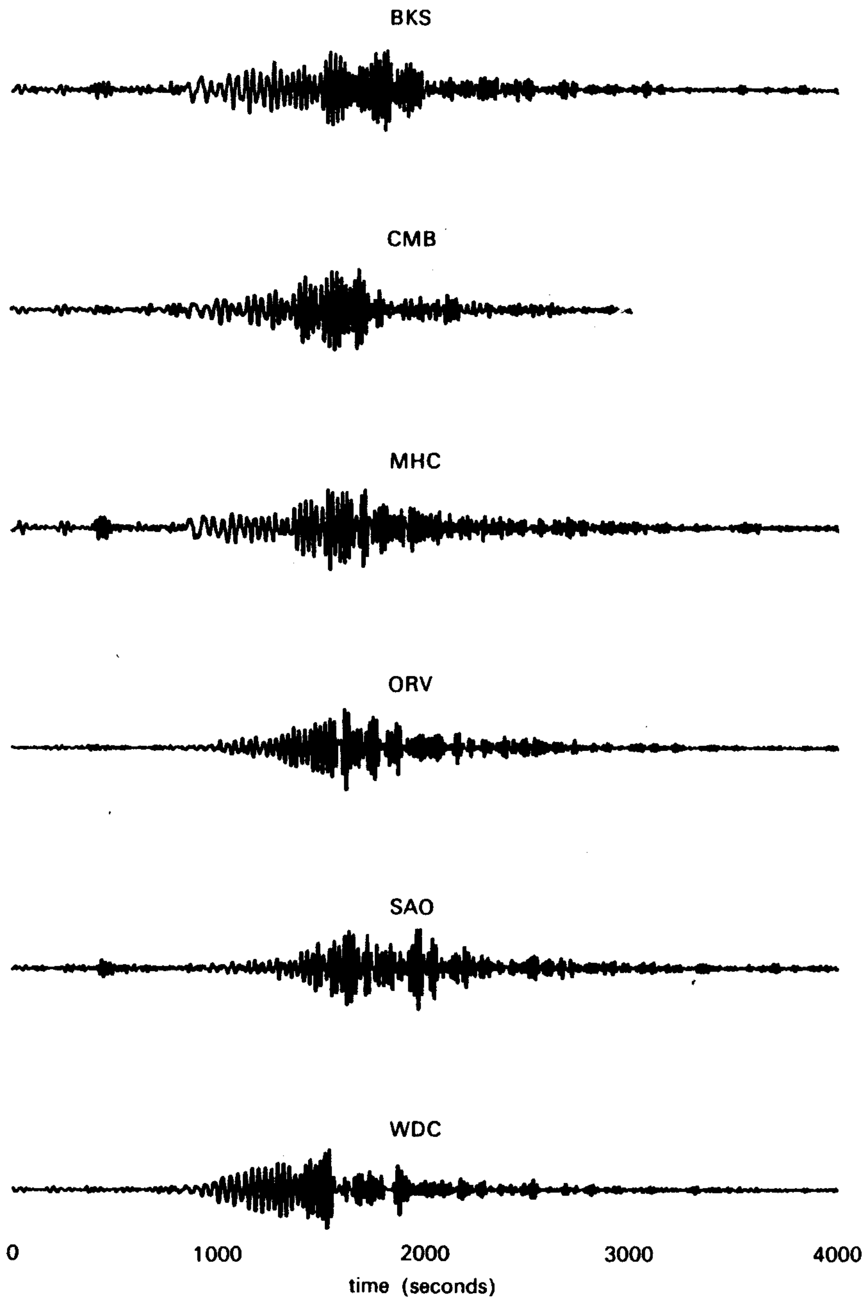

Figure 23.1. Vertical velocity records of the Armenian earthquake recorded at six stations of the Berkeley network. The concern is with Rayleigh (or surface) waves. These are seismic waves whose energy is trapped near the Earth's surface. Station locations indicated in text.

for discrete $n = 0, 1, \ldots$. The two dimensional transform of data $Y(x,t)$ of such a circumstance would lie on a sequence of curves. The allowable oscillations are referred to as modes with the case of $n = 0$ called the fundamental mode.

The (phase) velocity of the signal (23.1) is ω/k. In the case of (23.2), the ratio ω/k will generally not be constant in ω, and 'waves' of different frequencies travel with different velocities. This phenomenon is called dispersion.

Dispersion is observable in nature. One example is provided by the edge waves of oceanography. These are sea surface waves that can travel along, sidewise to the shore, in bays of certain geometry. Munk *et al.* (1964) analyze data collected in a linear array of 44 bottom pressure sensors spaced .73 km apart off the coast of southern California with Y now referring to pressure. In the above notation the data would be $Y(x_j, t)$ with $x_j = .73j$ and $j = 1, \ldots, 44$. The mass of the two-dimensional Fourier transform Munk *et al.* compute is seen to fall near curves. These curves turn out to be very close to the theoretical curves predicted by a model of the geometry of the bay concerned. A second example of the occurence of two-dimensional Fourier transforms lying close to curves is provided by the case of helioseismology. This is the subject concerned with the internal oscillations of the Sun. The cover of *Science* magazine for 6 September 1985 provides a striking example of the occurrence of such curves.

23.3 THE ARMENIAN EARTHQUAKE

The particular data that will be addressed in this paper was generated by the Armenian earthquake of 1988. This earthquake took place on 7 December at 07 hours, 41 minutes, 24.2 seconds Greenwich mean time in northern Armenia. It had an estimated magnitude of $M_S = 7.0$ and depth of 10 km. Approximately 25 000 people were killed and 19 000 injured. The signal travelled around 11 000 km along the surface of the Earth to the array of digital seismometers operated by the University of California, Berkeley in northern California.

Figure 23.1 presents records of the vertical velocity recorded at the stations BKS (Berkeley), ORV (Oroville), WDC (Whiskeytown), SAO (San Andreas), MHC (Mount Hamilton), CMB (Columbia). Their locations are scattered about northern California.

23.4 A STRUCTURAL MODEL

The frequency components of various seismic waves travel with differing velocities. Given a structural model, the dependence of frequency on velocity, and vice-versa, may be computed. Figure 23.2 presents an example of such a model. This particular model is referred to as a homogeneous layer over a homogeneous half-space. The parameters α refer to compressional velocities, β to shear velocities, ρ to densities, each for the top and lower

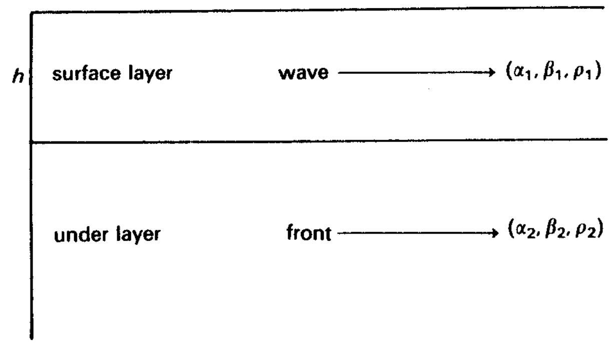

Figure 23.2. Earth model of a single layer over a half space.

layers respectively and the parameter h refers to thickness of the top layer. In the figure, the wave is envisaged as travelling from left to right.

Let x refer to distance from the origin of the earthquake to the observatory. The signal $Y(x,t)$ observed may be viewed as the motion (e.g. displacement, velocity or acceleration of the particle) at x at time t following an impulse at $x=0$ and $t=0$. Now it may be shown that the equations of motion have a solution only if a particular determinental criterion vanishes, see Bolt and Butcher (1960) and Bullen and Bolt (1985). The corresponding solution is

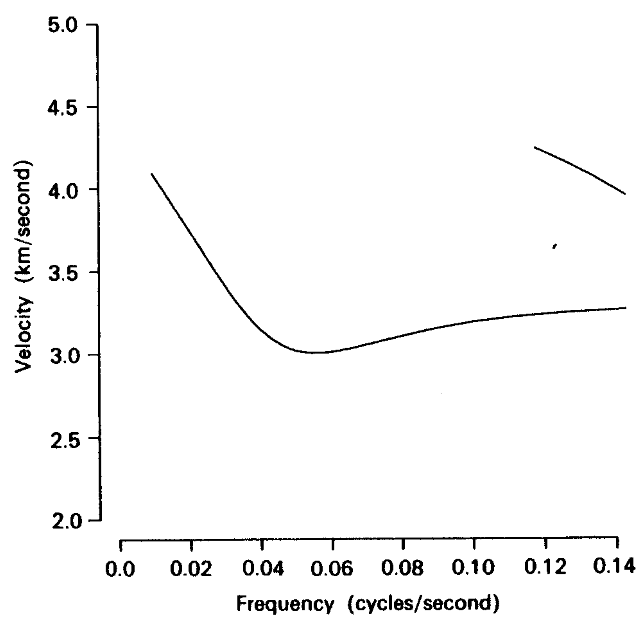

Figure 23.3. Velocities with which the frequency components particular surface waves (Rayleigh waves) travel for a specified Earth model.

called a dispersion relation. It gives the velocities with which the various frequencies travel. Higher modes correspond to further solutions. Figure 23.3 gives an example of such curves for one particular Earth model. In this case a fundamental and one higher mode occur.

23.5 DYNAMIC SPECTRUM ANALYSIS

By a dynamic spectrum will be meant a display of estimated intensity as a function of time and frequency. A naive method to derive such a display is

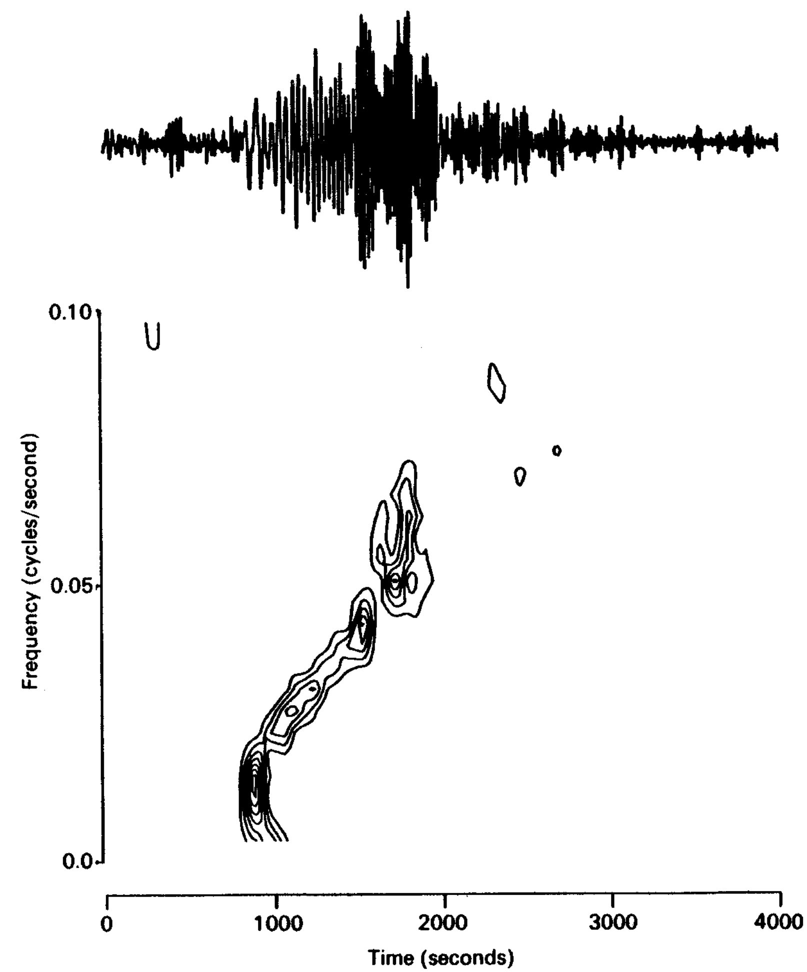

Figure 23.4. 1988 Armenian earthquake–top graph is vertical as recorded at Berkeley. Bottom graph is the estimated signal intensity as a function of frequency and time.

via a sliding Fourier analysis. Specifically, given a stretch of the time series $Y(t)$, one can compute

$$\left[\sum_v Y(t-v)\cos\omega v\right]^2 + \left[\sum_v Y(t-v)\sin\omega v\right]^2$$

with t referring to time, ω frequency and v summing over a restricted time interval. Given a contour plot of such an intensity as a function of (t, ω) one can look for ridges. In the seismological case these correspond to frequency components travelling with differing velocities.

Figure 23.4 presents part of the seismic trace of the Armenian earthquake, as recorded at the Berkeley station, and a parallel dynamic spectrum. The intensity display has the appearance of a ridge. Different frequency components do appear to be arriving at different times. The part of the seismic record analyzed, corresponds to what are called surface waves. These are waves whose energy is trapped near the Earth's surface. They provide information about the upper part of the mantle.

Dynamic spectrum analyses of earthquake records are presented in Dziewonski *et al.* (1969) and Levshin *et al.* (1972), for example.

What has usually been done, when seismologists have been concerned with fitting an Earth model, is that the parameter values of the model have been guessed until curves following the ridges of a plot like Figure 23.4 have been found. An intent of this present work is to automate that procedure.

23.6 PARAMETER ESTIMATION

When working empirically with dispersion curves and dynamic spectra, it is convenient to convert the time axis of the spectrum to velocity. This is a simple transformation $v = x/t$, the distance x to the earthquake source being known. Figure 23.5 presents the corresponding plots for the six stations of the Berkeley Digital Seismic Network. In each case there is a strong suggestion of ridges and of frequency components travelling with different velocities.

A basic problem of statistical concern is that of determining parameters of an Earth model leading to curves close to the ones suggested by Figure 23.5.

In the present case the following naive estimation procedure was employed. For station i and frequency ω, the velocity, $\hat{U}_i(\omega)$, where the intensity of the dynamic spectrum was largest, was determined. Next, $U(\omega|\theta)$ denotes the velocity at frequency ω of the fundamental mode for a particular Earth. Here θ denotes the unknown parameters. The estimate $\hat{\theta}$ is then determined by minimizing

$$\sum_{i=1}^{6} \sum_{\omega} [\hat{U}_i(\omega) - U(\omega|\theta)]^2 \qquad (23.3)$$

as a function of θ. Figure 23.6 gives the $\hat{U}_i(\omega)$ and fitted curve $U(\omega|\hat{\theta})$ for

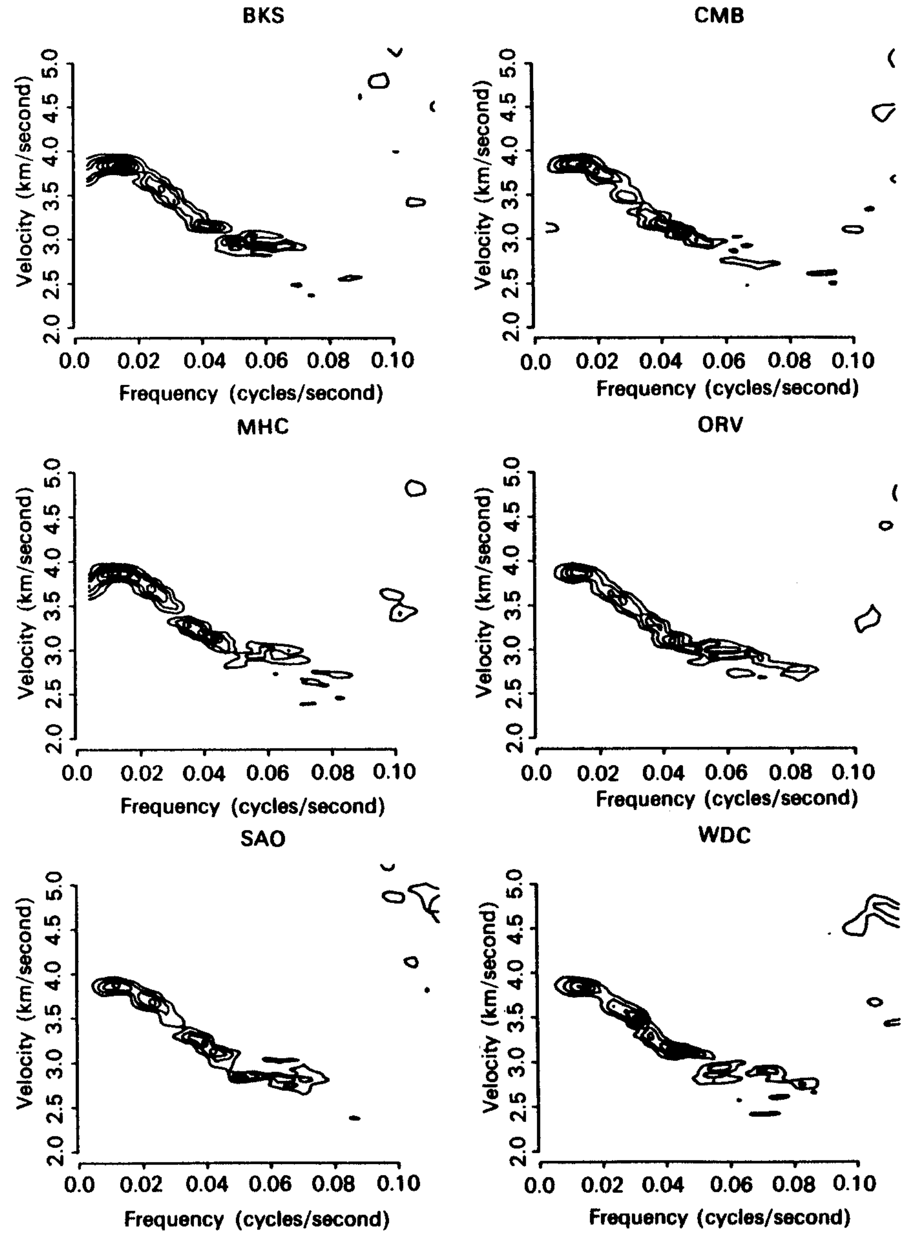

Figure 23.5. Estimated signal intensities as a functions of frequency and velocity.

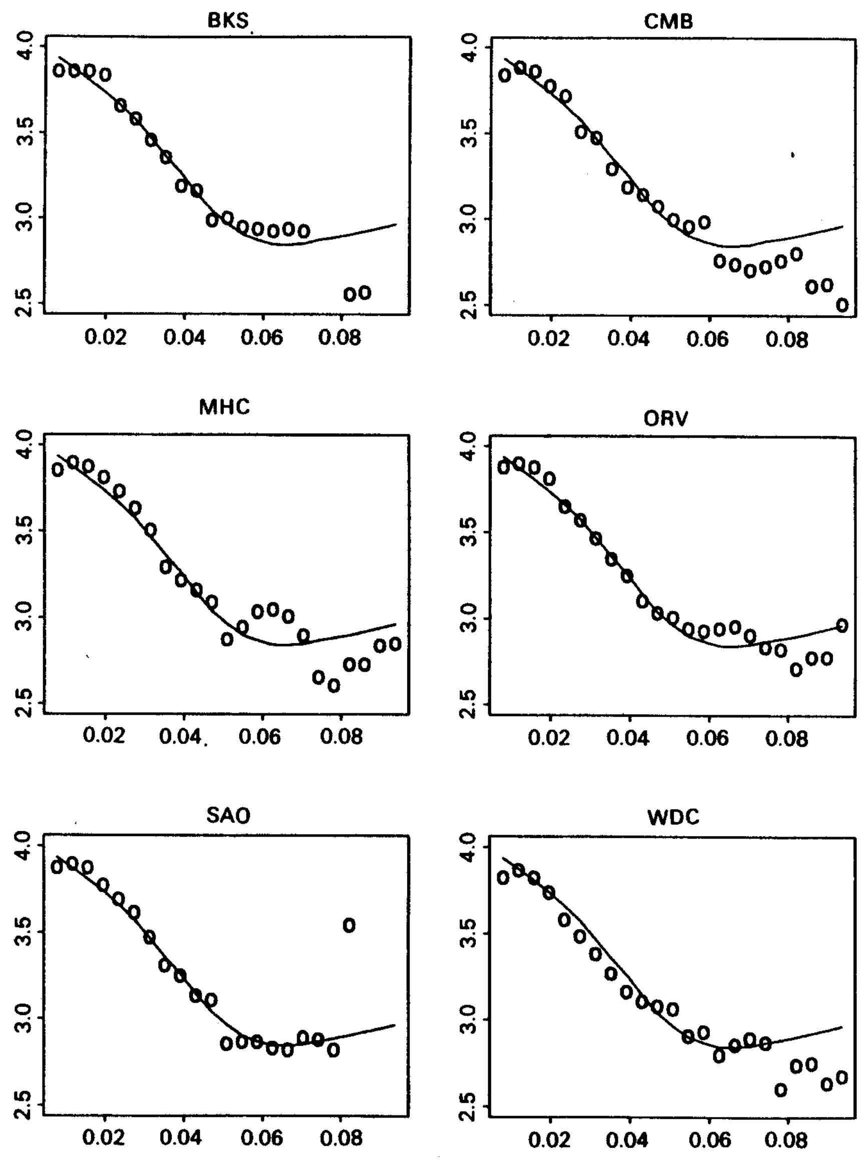

Figure 23.6. For given frequency, the '0' indicates the velocity at which the intensity was largest for the given station. The curve is the result of the fitting. The same curve is plotted for each station. The vertical axis is velocity in *k*/sec; the horizontal axis is frequency in cycles/sec.

the six stations. The fits appears reasonable, particularly at the lower frequencies where the signal to noise ratio is greatest.

In the model the ratio of densities, ρ_2/ρ_1 was taken to be 1.2, a figure derived from independent studies. The estimated parameter values are as follows:

$$\hat{h} = 22.61\,\text{km}$$
$$\hat{\alpha}_1 = 4.94\,\text{km/sec}$$
$$\hat{\beta}_1 = 3.88\,\text{km/sec}$$
$$\hat{\alpha}_2 = 6.62\,\text{km/sec}$$
$$\hat{\beta}_2 = 4.62\,\text{km/sec.}$$

Uncertainty is estimated via the jack-knife, events are dropped in turn from the criterion (23.3). Approximate 95% confidence intervals, taking the traces to be independent and the errors to be normal of constant variance, are

$$10.11 < h < 50.56$$
$$4.48 < \alpha_1 < 5.06$$
$$3.23 < \beta_1 < 4.65$$
$$1.17 < \alpha_2 < 37.37$$
$$1.01 < \beta_2 < 21.26.$$

23.7 DISCUSSION

The example of this paper shows the usefulness of the concept of spectrum for a nonstationary signal. It further illustrates the novel case where a two-dimensional Fourier transform is neither continuous, nor concentrated at points, but rather lies on curves.

The present approach makes no specific use of higher modes. In any case these may not have been excited in the present event. An alternate procedure, making use of higher modes and other data sets, is under development in joint research with B.A. Bolt.

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24

On periodogram-based spectral estimation for replicated time series

P.J. Diggle and I. Al-Wasel

24.1 INTRODUCTION

Figure 24.1 shows time series consisting of measurements of the concentration of luteinizing hormone (LH) in blood samples taken at intervals of 5 minutes from each of 8 apparently healthy men. LH is secreted in a pulsatile manner involving complicated feed-back mechanisms in the endocrine system (Lincoln *et al.* 1985). Endocrinologists are interested in characterizing the frequency characteristics of this pulsatile process. The pattern of variation over time is complex, and spectral analysis is a natural technique to use in an attempt to characterize the contributions to the overall variation from different frequency ranges (Murdoch *et al.* 1985). Clearly, the sampling regime limits the range of frequencies which can be detected. Figure 24.2 shows a second set of data, taken from the same subjects, but in which each series consists of measurements from blood samples taken at intervals of 1 minute. In this second set of data, the objective is to discover whether there are any high-frequency patterns of variation superimposed on the low-frequency effects which are clearly visible in the first set of data. With this in mind, the low-frequency variation has been filtered out by subtracting from the original data a weighted 7-point moving average with weights proportional to 1, 3, 6, 7, 6, 3, 1.

Although spectral analysis is a very highly developed methodology, almost all of this development has been in the context of a single, long time series $\{x_t : t = 1, \ldots, n\}$. See, for example, Priestley (1981). This perhaps reflects the origins of the subject in signal processing and the physical sciences. However the usefulness of time series methodology is becoming more widely accepted in the biomedical sciences, where replicated experiments are the rule rather than the exception.

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Maurice B. Priestley

Developments in Time Series Analysis

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