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ON MAXIMUM WAVE HEIGHTS OF SEVERE SEAS

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ABSTRACT

This paper, an interim report of a continuing development effort aimed at understanding the statistics of severe seas, presents a new development of the statistical distribution of extreme wave amplitudes in which no restrictions limit applicability. This paper provides functional formulation for the extreme value solution of the general stationary case, specific expressions for the stationary Gaussian case, and a method for obtaining specific equations for non-Gaussian wave states. It is shown that extreme wave crests are independent of spectral shape in the Gaussian case, but that extreme wave heights are not. It is concluded that new advances in extreme value theory are providing a new understanding of severe seas.

INTRODUCTION

Extreme ocean waves of severe seas have a profound influence on the design and operation of offshore structures. These waves can dominate, for example, the maximum loads environmentally induced on submerged structural members and dictate the minimum elevations that platform decks must be above mean water level. A proper understanding of the statistics of extreme waves is therefore essential, particularly as the offshore oil and gas industry moves into harsher oceanic regions.

The purpose of this paper is to enhance the understanding of the maximum waves of severe seas by presenting a new development of the statistical distribution of extreme wave amplitudes. This development is based on new techniques in the field of extreme value theory and, like most previous work, treats the seaway as a stationary random process. Unlike past work, no serious restrictions are placed on the development, making the resulting expressions applicable to wave conditions of arbitrary spectral shape, i.e., to wider-band spectra of severe, chaotic seas as well as to narrow-band spectra of near-regular swell.

The new development yields functional formulation for the general stationary case and assumes the random ocean's surface to be Gaussian to permit specific formulas. Comments on obtaining specific equations for non-Gaussian wave states are offered to complete the discussion.

References and illustration at end of paper.

HISTORICAL BACKGROUND

In a classic study published in 1944-45, S.O. Rice[1] investigated the statistical properties of random noise arising from the shot effect in vacuum tubes and the thermal agitation of electrons in resistors. Although the study was aimed principally at understanding the probability distributions associated with the maxima of electrical signals and signal envelopes, by presenting expressions for the expected number of zero crossings and maxima per unit time, Rice laid a foundation upon which much of today's extreme ocean wave theory is based.

In 1952, Longuet-Higgins[2] introduced one of the first statistical distributions of extreme wave heights, based upon a sample of N available wave amplitudes and upon the assumption that the wave spectrum consisted of a single narrow frequency band. That study led to the well-known equation

$$H_{max} = 0.707\ H_s \sqrt{\ln N} \qquad \text{...........(1)}$$

where H_{max} is the expected value of the maximum wave height in a sequence of N waves, and H_s

by Sverdrup and Munk in 1947.

Derived under assumptions that limit applicability just to wave amplitudes in separate stretches of data that follow a Rayleigh distribution, Equation 1 has been used continually by ocean engineers, meteorologists and naval architects and has been demonstrated by Borgman[3] and Goda[4], among other investigators, to be a good approximation to reality for various conditions. On the other hand, other investigators, including Haring et al.[5] and Jahns and Wheeler[6], have demonstrated that observational data can also depart markedly from Equation 1.

By combining the previous works of Longuet-Higgins and Rice, Cartwright and Longuet-Higgins[7] in 1956 developed an expression for the statistical distribution of maxima in terms of the bandwidth of the wave spectrum. Unfortunately, their development is based upon the assumption of statistically independent wave amplitudes.

In 1973, Ochi[8] applied order statistics to the problem to obtain a solution for the extreme value of wave maxima. Of interest, Ochi's solution is not functionally related to the spectral bandwidth; however, the derivation assumes that the successive maxima (wave peaks) of the random process have the same probability density function and are mutually independent. As is the case with earlier work, these assumptions restrict proper application of the results to a limited range of wave states, i.e., to wave states with very narrow covariance functions only.

The development presented herein overcomes the difficulty of requiring independent wave amplitudes by applying new techniques in the field of extreme value theory.

EXTREME VALUE THEORY

As shown by the theorem of Gnedenko[9], only three different statistical distributions of maxima can exist for independent and identically distributed random variables with non-degenerate distributions in the limit. These three distributions are:

Type I: $G(x)=\exp(-\exp(-x))$ for all x

Type II: $G(x)=\exp(-(x^{\alpha}))$ $\alpha>0,\ x>0$

Type III: $G(x)=\exp(-x)^{\alpha}$ $\alpha>0,\ x\leq 0$

Classical extreme value theory provides the necessary and sufficient conditions for the above general limit laws to hold.

Leadbetter[10], in a recent work, has shown that the same three laws also apply under rather general conditions to the asymptotic intervals of time, rather than just to sampled data as considered by Gnedenko. Following Leadbetter[10], let $X(t)$ denote the statistically-stationary ocean surface displacement for the time interval $0\leq t\leq T$, and let M_T denote the extreme wave crest elevation of $X(t)$ (i.e., the maximum value of $X(t)$) during $0\leq t\leq T$ (see Figure 1). Following the continuous-case analogy to the theorem of Gnedenko, if there exist normalizing constants $a_T>0$ and b_T such that $a_T(M_T-b_T)$ has a non-degenerate limiting distribution function $G(x)$ as T approaches infinity, then

$$P\{a_T(M_T-b_T)\leq x\}\rightarrow G(x) \quad\text{.........................(2)}$$

and $G(x)$ has one of the three extreme value forms.

Application of Equation 2 allows a new understanding of maximum wave statistics.

EXTREME WAVE CRESTS-GENERAL CASE

Let $U(T)$ be the number of upcrossings at the level $u>0$ by the stationary process $X(t)$ in the interval $(0,T)$, and let μ be the statistically expected (mean) number of u-upcrossings per unit time, i.e., $\mu = E[U(1)]$.

In the general case,

$$\mu = \int_0^{\infty} zp(u,z)dz \quad\text{......................(3)}$$

where p is the joint probability density function for $X(t)$ and its derivative $X'(t)$. μ^{-1} is indicative of the mean time between consecutive upcrossings (or downcrossings) at the level u.

Using μ to scale time, let

$$\tau = T\mu \quad\text{..........................(4)}$$

be a dimensionless parameter equalling the expected number of u-upcrossings in the interval $(0,T)$.

Suppose $u=u_T = x/a_T + b_T$ with $T(1-F(u_T))\rightarrow\tau = -\ln G(x)$ as T approaches infinity, where $F(u_T)$ is the distribution function of u_T, then as shown by Leadbetter[10]

$$P(M_T\leq u_T)\rightarrow\exp(-\tau) = G(x) \quad\text{...........(5)}$$

Further, letting $\tau = \exp(-x)$, it follows that

$$P(a_T(M_T-b_T)\leq x)\rightarrow\exp(-\exp(-x)) \quad\text{.....(6)}$$

under the condition that the values of $X(t)$ are asymptotically independent in a certain sense (distributional mixing), i.e., collections of segments of $X(t)$ are at most only weakly dependent for large separations in t. (See Leadbetter[10] for details). Of importance to offshore work, this condition places no restrictions on the applicability of Equation 6 to ocean wave problems.

If $M_{T,\alpha}$ denotes a level such that

$$P(M_T \leq M_{T,\alpha}) = 1-\alpha \quad \text{.................. (7)}$$

where α is the probability, or risk, of M_T exceeding $M_{T,\alpha}$, then from Equations 6 and 7

$$M_{T,\alpha} \doteq \zeta/a_T + b_T \quad \text{.................... (8)}$$

where

$$\zeta = [\ln 1/(\ln \frac{1}{1-\alpha})] \quad \text{.................. (9)}$$

Equation 8 defines the maximum wave crest amplitude during the interval (0,T) with risk of exceedance α for stationary conditions and is applicable to both Gaussian and non-Gaussian wave states. Consequently, provided the normalizing constants a_T and b_T (or x/a_T+b_T) can be evaluated, which requires a formulation for the expected number of upcrossings at the arbitrary level u, Equation 8 is very important for both design and operational problems.

EXTREME WAVE CRESTS-GAUSSIAN CASE

It has been assumed so far that X(t) is statistically stationary. In this section, it is assumed further that X(t) is also Gaussian with a zero mean.

As shown by Rice[1], the mean number of u-upcrossings per unit time for a Gaussian process can be estimated by

$$\mu = (\lambda_2^{1/2}/\sigma) \exp(-u^2/2\sigma^2) \quad \text{................ (10)}$$

where

$$\sigma = C(0) = \int_0^\infty S(f)df \quad \text{.................... (11)}$$

$$\lambda_2 = -C''(0) = \int_0^\infty f^2 S(f)df \quad \text{............... (12)}$$

S(f) equals the spectral density at frequency f.

If C(r)ln r approaches zero as r approaches infinity, the results of Leadbetter[10] are applicable to the discussion and it follows from Equations 4, 8 and 10 that

$$\zeta/a_T + b_T = \sigma\sqrt{2(\ln T_o + \zeta)} \quad \text{......... (13)}$$

where

$$T_o \equiv T\sqrt{\lambda_2}/\sigma = N \quad \text{.................... (14)}$$

with N being the number of wave cycles within time T.

Rewriting Equation 13,

$$M_{T,\alpha} = \sigma\sqrt{2(\ln N + \zeta)} \quad \text{........... (15)}$$

i.e., the maximum crest amplitude during the interval (0,T) with risk of exceedance α for Gaussian stationary conditions. The corresponding mean of the large sample approximation to the distribution M_T is given by $\zeta = \gamma = 0.5772$ . . . (Euler's number).

As shown by Equation 15 when expressed in terms of the number of wave cycles, the extreme crest elevation during time T is independent of spectral bandwidth. Although equivalent in form to formulation by Cartwright and Longuet-Higgins[7] and Ochi[8], Equation 15 represents to the authors' knowledge the first extreme value solution for the Gaussian case in which successive wave crests are not assumed to be independent. Indeed, Equation 15 is derived under the assumption that C(r)ln r approaches zero as r approaches infinity, a condition on the covariance function which always should be met by the random seaway.

Equation 15 is an exceedingly useful expression for offshore engineering work due to its easy application to ocean wave problems.

EXTREME WAVE HEIGHTS-GAUSSIAN CASE

The field of ocean waves is, to a certain degree, unique among the physical sciences since it is one of the few fields concerned with values of successive crests and troughs (i.e., heights). Most branches of physical sciences, at least those that attract statisticians, usually are concerned just with peak values (i.e., amplitudes). Thus, extreme value theories presently in existence are geared to crest-to-mean statistics only and are not particularly well suited for attacking the general crest-to-trough problem.

The extension from extreme wave amplitudes to extreme wave heights, however, is trivial for very narrow-band conditions. In this case, the extreme wave height H_T during the interval (0,T) is approximated by

$$H_T = 2 M_T \quad \text{.......................... (16)}$$

which implies that the expected value of the large sample approximation to the distribution of H_T is

$$H_{T,\alpha} = 2 M_{T,\alpha} \quad \text{....................... (17)}$$

or from Equation 15

$$H_{T,\alpha} = 0.707 H_s \sqrt{(\ln T_o + \zeta)} \quad \text{........... (18)}$$

taking $H_s = 4\sigma$ to be the usual approximation to significant wave height.[4]

Equations 16-18 are based on the assumptions that the maximum crest and the minimum trough are equal and occur successively, which tends to be the case for very narrow-band conditions. For wider-band conditions, however, the maximum crest and the minimum trough tend to occur at some interval apart. Indeed, as the bandwidth of the wave spectrum increases, the probability of the maximum crest, minimum trough and maximum wave height occurring during the same wave cycle diminishes. Equation 17 is therefore not necessarily correct for wave conditions with non-narrow bandwidths and should be replaced by the more general expression

$$H_{T,\alpha} \leq 2\, M_{T,\alpha} \quad \text{.......................(19)}$$

applicable to stationary Gaussian cases of arbitrary bandwidth.

It follows that Equation 18 should be replaced by

$$H_{T,\alpha} \leq 0.707\, H_s \sqrt{\ln T_o + \ln[1/\ln(1/(1-\alpha))]} \quad \text{....................................(20)}$$

utilizing Equation 9. The inequality of Equation 20 depends upon the spectral shape of the wave state, with the difference between Equations 18 and 20 increasing as the bandwidth increases. The nature of this inequality and the corresponding variation in risk are presently under investigation by the authors.

While the maximum crest amplitude is independent of spectral bandwidth, it is clear from Equation 20 that the maximum wave height is not. Consequently, it is desirable to express offshore problems in terms of wave amplitudes, rather than wave heights, and thus avoid this problem. In engineering analyses using deterministic design waves, for example, it is suggested that all parameters determining surface profiles be based on design maximum crest elevations.

EXTREME WAVE CRESTS-NON-GAUSSIAN CASES

For severe seas and other types of wave conditions in which nonlinear processes are important, the seaway can become clearly non-Gaussian. To illustrate, Haring et al.[5] analyzed 376 hours of storm records collected on the Continental Shelf and found the distributions of surface deviations about the mean to be non-Gaussian. Jahns and Wheeler[6] showed that substantial deviations from the Gaussian assumption also can occur in shallow water. Thus, to understand the statistics of extreme maxima for these important wave conditions, non-Gaussian cases must be addressed.

For an important class of non-Gaussian processes, it is possible to estimate extreme maximum values in the same manner described in the previous sections. Let $X(t)$ be non-Gaussian, and suppose $Y(t) = g(X(t))$ is Gaussian with a zero mean for some increasing function $y = g(x)$ with $g(0) = 0$ and an inverse $x = g^{-1}(y) = h(y)$, so that $X(t) = h(Y(t))$. (As an example, if $g(x) = \ln(1+\beta x) = y$, then $h(y) = [\exp(y)-1]/\beta = x$.) Letting M_T^Y be the maximum value of $Y(t)$ in the interval $(0,T)$, it follows that $M_T^Y = g(M_T^X)$ and $M_T^X = h(M_T^Y)$. All comments made under the Gaussian assumption obviously apply to M_T^Y. Hence, replacing σ with σ^Y,

$$M_{T,\alpha}^X = h[\sigma^Y \sqrt{2(\ln N + \zeta)}] \quad \text{................(21)}$$

Actual expressions of $g(x)$ for extreme wave statistics are not considered known at the present by the authors; however, plausible functional forms could be discerned by plotting wave data on normal probability paper.

Equation 21 as well as other approaches are being given serious consideration to gain further insight into the statistical nature of severe seas.

SUMMARY AND CONCLUSIONS

This paper has attempted to clarify existing knowledge of ocean wave statistics by offering a new development of the statistical distribution of extreme wave amplitudes. In so doing, the following has been concluded:

1. The statistical distribution of wave maxima follow a Type I distribution in the limit, with a_T and b_T (or x/a_T+b_T) being determined from consideration of the expected number of upcrossings at some arbitrary level.

2. The maximum wave crest amplitude during time T and with risk of exceedance α is given for the general stationary case by Equation 8. Of importance to offshore work, there are no known restrictions that limit applicability of Equation 8 to ocean wave problems.

3. The maximum crest amplitude for the stationary Gaussian case is given by Equation 15. Equation 15 represents to the authors' knowledge the first extreme value solution for Gaussian wave conditions in which successive crests are not assumed to be independent. Consequently, the equation is applicable to wave states of arbitrary spectral shape, including wider-band spectra of chaotic, severe seas as well as narrow-band spectra of near-regular swell.

4. The extreme wave crest elevation during time T with risk α is independent of spectral shape for the Gaussian case; however, the extreme wave height is not. It is therefore concluded that it is preferable to formulate engineering design problems in terms of wave amplitudes vice wave heights.

What does the information presented in this paper mean from a practical standpoint? Clearly, an offshore platform must be designed so that waves will not slap the underside of the structure's vulnerable deck. This paper has presented formulation necessary to determine the minimum elevation of the deck above mean water level so that sufficient space is available for the maximum crest to pass underneath. Further, the maximum forces and moments on submerged structural members are dominated by the wave-induced velocities and accelerations of the water particles. During the design phase of a structure, these loads are calculated based on a design maximum wave. As discussed in this paper, due to the uncertainties introduced by the spectral bandwidth of the seaway, the maximum wave height is not necessarily twice the value of the maximum wave crest; thus, if design forces and moments are calculated without taking this fact into consideration, difficulties could occur (e.g., overestimating the maximum horizontal drag force by 10-30% during static analysis).

The statistics of ocean waves are still not completely understood, expecially for non-Gaussian conditions, but this paper, an interim report of an ongoing development effort, has introduced certain new techniques from the field of extreme value theory to the field of wave statistics. These techniques are providing the statistical tools necessary to approach the problem of extreme ocean waves and, coupled with the continuing effort, have the potential of providing further insight into the statistical properties governing severe seas.

NOMENCLATURE

a_T = Normalizing constant
b_T = Normalizing constant
$C(r)$ = Covariance with lag r
f = Wave frequency
$F(u_T)$ = Distribution function of u_T
$G(x)$ = Statistical distribution of maxima
H_{max} = Expected value of maximum wave height
H_s = Significant wave height
H_T = Extreme wave height during interval (0,T)
$H_{T,\alpha}$ = Extreme wave height during the time T with risk of exceedance α
M_T = Extreme wave crest elevation during the interval (0,T)
$M_{T,\alpha}$ = Extreme wave crest elevation during the time T with risk of exceedance α
M_T^X = Maximum value of non-Gaussian X(t) during interval (0,T)
M_T^Y = Maximum value of Gaussian Y(t) during interval (0,T)
N = Number of wave cycles
r = Time lag
S(f) = Spectral wave density
t = Time
T = Interval length
T_o = Dimensionless parameter
u = Arbitrary level
$u_T = x/a_T + b_T$
U(T) = Number of upcrossings at the level $u > o$
x = Variable
X(t) = Random ocean surface displacement
X'(t) = First derivative of X(t)
Y(t) = Gaussian transformation of non-Gaussian X(t)
z = Variable
α = Risk of exceedance; exponent of statistical distributions of Gnedenko
β = Constant
ζ = Dimensionless risk parameter
λ_2 = Second moment
μ = Expected number of u-upcrossings per unit time

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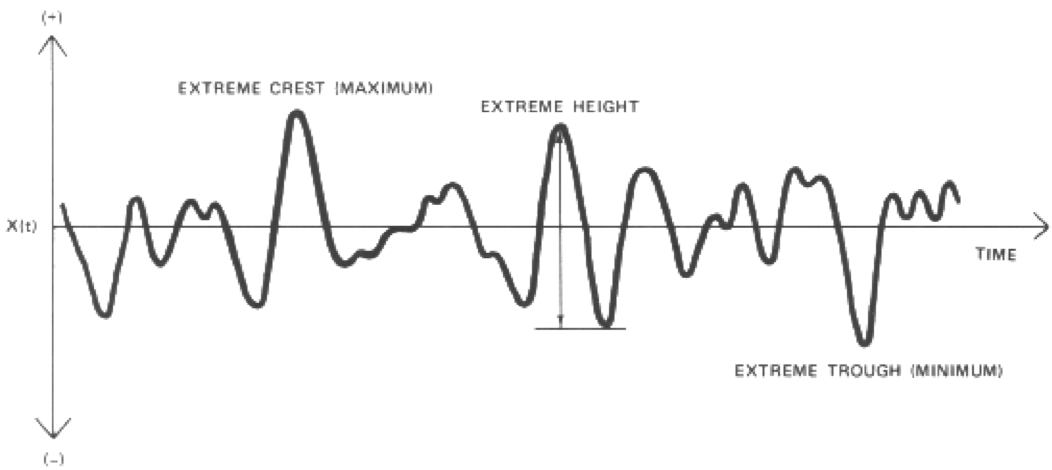

Fig. 1 - Sample wave profile.