Earthquake Risk and Insurance

David R. Brillinger[1]

ABSTRACT

Our concern is with protection against natural catastrophes through financial insurance. Development of appropriate premium rates requires consideration of a wide variety of variables entering into the occurrence of the disaster, and a broad range of scientific and statistical investigations results. Topics discussed include: premium computation, distribution of large earthquakes in time, ground motion at sites, attenuation of energy with distance, damage description and actual practice in various countries. Statistical considerations that arise include: description, stochastic modelling, conditioning, spatial processes, (marked) point processes, uncertainty estimation and robust/resistant procedures. Study of the insurance problem is scientifically enlightening because it requires one to focus on the whole context of the problem; geology, seismology, earthquake engineering, and damage.

1. INTRODUCTION

Statistics has a long involvement with problems of risk and insurance. This occurs because of variabilities, because of uncertainties, because of estimation problems and because of choice of loss functions. Many of the techniques of contemporary statistics appear useful in problems of insurance. In this paper there are considerations of: description, stochastic modelling, conditioning, (marked) point processes, spatial processes, robust/resistant

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[1] Department of Statistics, University of California, Berkeley, CA, U.S.A. 94720.

procedures and uncertainty estimation. The basic approach is via conceptual modelling and data analysis, in contrast to a "black box" approach.

Preliminary to the problem of determining an earthquake insurance premium is that of seismic risk assessment. Seismic risk assessment may be defined as the process of estimating the probability that certain performance variates at a site of interest exceed relevant critical levels, within a specified time period, as a result of nearby seismic events. The seminal paper on the topic is Cornell (1968). Other basic works are: Vere-Jones (1973), Lomnitz (1974), McGuire (1974), Walley (1976), and Blume and Kiremidjian (1979). Addressing the insurance issue forces consideration of more than a risk problem; one needs to consider the whole sweep of geology, seismology, earthquake engineering and damage.

Generally speaking the techniques employed are applicable to other environmental risks; that is to other small probability events with substantial negative consequences.

Much of the paper is review of existing material, but some new scientific results are included. In particular an automatic method of constructing isoseismal maps, employing commercially available software, is presented and new expressions relating modified Mercalli intensity to maximum acceleration are derived. It is clear that a variety of interesting statistical and actuarial problems arise.

The layout of the paper is: Section 2 "Insurance", Section 3 "Strategy", Section 4 "Temporal Aspect", Section 5 "Spatial Aspect", Section 6 "Attenuation", Section 7 "Damage", Section 8 "Particular Practice", Section 9 "Other Aspects" and Section 10 "Discussion". There is an Appendix wherein some elementary expressions for premiums are developed.

A general reference for the background seismology is Chapter 17 of Bullen and Bolt (1985).

2. INSURANCE

Financial insurance is one means our society has devised for alleviation of disasters. There are two distinct formal procedures for setting premiums: firstly via specific formulae based on a conceptual model (Richard 1944; Beard, Pentikainen, and Pesonen 1969; Freifelder 1976; Goovaerts, De Vylder, and Haezendonck 1984; Sundt 1984; Heilmann 1988; Straub 1988) and secondly via the black box (control theory, time series) approach (Bohman 1979; Norberg 1990; Aase 1992). A concern in the black box approach is that the nonstationarity of the basic quantities may make it difficult to determine parameters of relationships. This paper will concentrate on the conceptual approach.

A variety of formulas has been proposed for the determination of premiums, assuming that a random loss may have to be compensated for. Basi-

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cally a company wants income to approximately equal outgo. The problem is sensibly focussed on two crucial components, (Smolka and Berz 1991):

- calculation of a premium commensurate with the risk,

- estimation of the size of the probable maximum loss resulting from a potential catastrophe.

To be specific, consider a time period of one year and suppose that the yearly possible loss is a random amount U. [Interest considerations will be ignored.] The *pure risk premium* for a year's insurance is given by

$$P = E\{U\} = \mu_U \ . \tag{1}$$

Because of expenses, the pure premium will have to be "loaded"; for example, the premium may be taken to be

$$P = (1 + \alpha)\mu_U \ . \tag{2}$$

The multiplier $(1 + \alpha)$ has the effect, above handling expenses, of providing some protection against random fluctuations in loss beyond the average $E\{U\}$.

Other premium formulas that have been suggested, and that take note of random fluctuations, are

$$P = \mu_U + \beta\sigma_U \quad \text{and} \quad P = \mu_U + \gamma\sigma_U^2 \tag{3}$$

with $\beta, \gamma > 0$. The latter has the property of being additive for independent risks; see Straub (1988).

A further procedure for determining premiums is to select some acceptable probability of ruin ϵ and, supposing that a reserve of R is available, to determine the premiums such that

$$\Pr\{S_U > R + S_P\} \leq \epsilon \ , \tag{4}$$

where S_U and S_P denote the sums of claims paid out and premiums paid in respectively during the year. References to the computation of ruin probabilities are: Beard *et al.* (1969), Friefelder (1976), Heilmann (1988), and Grandell (1991). In the Appendix, the form of (1)–(4) for the case of rare events is considered.

Insurance, for the particular case of earthquakes, is discussed in Straub (1973), Vere-Jones (1973), Lockett (1980), Grases (1986), Porro (1989), and Munich Re (1991).

3. STRATEGY

The usual approach to seismic risk assessment, defined in Section 1, is to break the problem down into basic components that may be investigated individually. This multistage analysis requires critical investigation of four pieces: a) sources of events, b) intermediate transmission of energy from the sources, c) the local site and d) the particular facility of concern. The earthquakes may be thought of as originating at points, on lines or within zones (the geometry). They will have different sizes and occurrence times. The intermediate transmission of the seismic signal involves attenuation of energy with distance and depends on the media traversed. Aspects of the local site include geology and ground type. In some studies the dynamic response and resistivity of the structure of interest are modelled. The fields of geology, geophysics, seismology and engineering are all involved.

In a naive assessment one might postulate: i) that there is a single source with the point process of events Poisson of rate μ and with earthquake magnitudes distributed exponentially, [Pr{magnitude $> M$} $= \exp\{-\beta M\}$], ii) that intensity of motion falls off with magnitude and distance in accordance with the following relation:

$$I = \beta_0 + \beta_1 M + \beta_2 \log d + \text{noise} \ , \tag{5}$$

d being the source-to-site distance and "noise" being a normal variate with mean 0 and variance σ^2, and iii) that the behavior of the structure is effectively described by I. The risk may be evaluated explicitly in this case as follows:

$$\text{Pr}\{\text{intensity } i \text{ exceeded within time period of } u\} = 1 - \exp\{-u\mu G(i)\}$$

where

$$G(i) = \exp\left\{-(i - \beta_0 - \beta_2 \log d)\beta/\beta_1 + \frac{1}{2}\sigma^2\beta^2/\beta_1^2\right\} \ .$$

It needs to be mentioned that each of these assumptions is debatable and that variants have been investigated.

In the insurance case one needs to continue and to model the loss that could be experienced. This may be done through the percent of damage likely to be experienced for a given building type; see discussion in Section 7.

There is a need for models, for parameter estimates and for the recognition of statistical regularities in the work.

4. TEMPORAL ASPECT

As indicated in the previous section, some means of describing the temporal rates of occurrence of damaging earthquakes is required. The basic

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series of times involved is often modelled by a stochastic point process. A variety of specific point process models have been suggested; see Vere-Jones (1970). When for example total damages are associated with the earthquake times, one has a marked point process.

The following is an explicit example of the development and fitting of a point process model. Pallett Creek is an area in Southern California lying by the San Andreas Fault. In Brillinger (1982) and Sieh, Stuiver, and Brillinger (1989) the times between large seismic events there are modelled as independent Weibull variates. A Weibull variate may be defined as follows: if x denotes the time elapsed since the preceding event, then the hazard function

$$h(x) \approx \Pr\{\text{event in } (x, x + \Delta) \,|\, \text{last at time } 0\}/\Delta$$

for small $\Delta > 0$, has the form

$$\frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \qquad x > 0 \ .$$

For example, if $\beta = 1$ it is constant and if $\beta > 1$ it increases steadily with x. The reasonableness of this assumption may be assessed by a cumulative hazard plot; see Nelson (1972). Figure 1 provides such a plot based on the Pallett Creek data. One graphs the times between events and checks to see if they fall near a straight line. The Weibull assumption does not appear invalidated in this case.

A difficulty that arose for the Pallett Creek data was that in one case, it could be inferred that an earthquake had taken place between two others. The dates of the bounding two could be estimated directly, but the date of the event in between could not. This led to one observation that was the sum of two Weibulls. Also in forming the likelihood, the censorship involved in the open interval starting at 1857 and measurement error had to be taken account of. Details are provided in Sieh, et al. (1989). The maximum likelihood estimates determined were

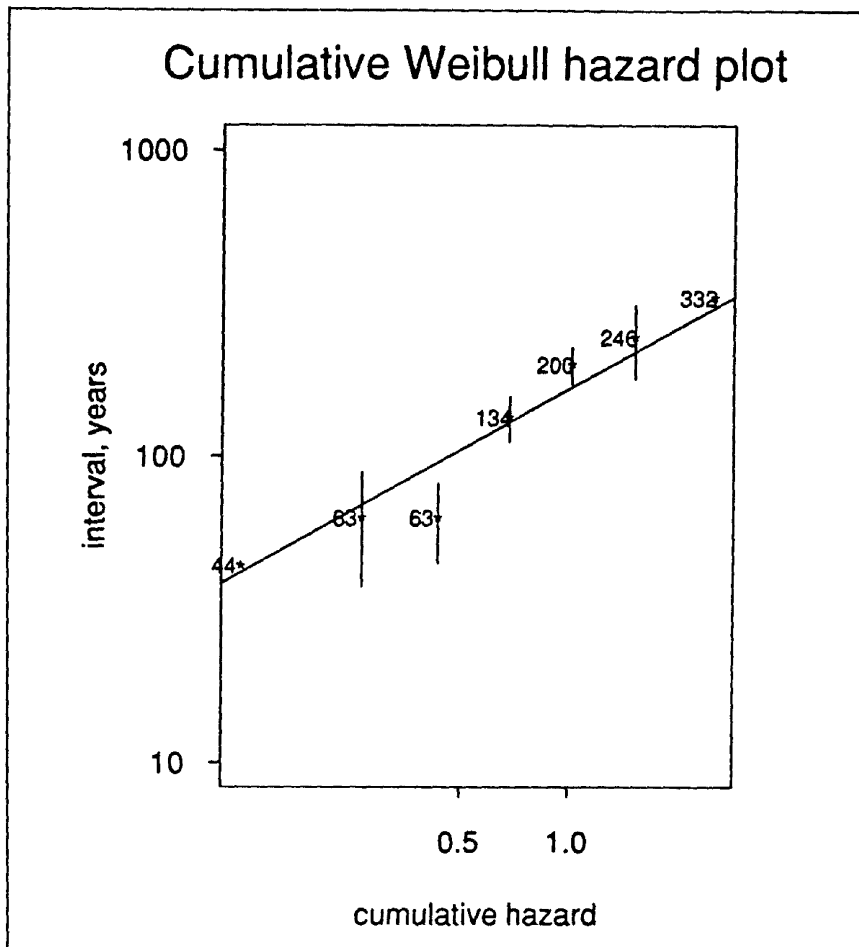$$\hat{\alpha} = 166.1 \pm 44.5 \qquad \hat{\beta} = 1.50 \pm .80 \ .$$

The following risk estimate was then determined

$$\Pr\{\text{event in next year} \,|\, \text{last in } 1857\} \doteq .008$$

with a corresponding approximate 95% confidence interval of $(.004, .026)$.

5. SPATIAL ASPECT

After a sizeable earthquake many measurements of consequences are made in the surrounding regions. For example, strong motion seismometers

Cumulative Weibull hazard plot

Figure 1. A hazard probability plot to assess the reasonableness of the Weibull distribution for the intervals between earthquakes at Pallett Creek. The points plotted correspond to estimates of the intervals between events. The vertical bars indicate plus and minus twice their standard errors. If the distribution is reasonable the points should fall near a straight line. For reference a fitted line has been added.

will be examined to see if they were triggered. In the case that they were, the maximum acceleration recorded will be noted. In addition reports are received from selected observers on a verbally described scale, the scale of modified Mercalli (MM) intensities. This scale provides 12 discrete levels of increasing severity. For example the description of MM_{VIII} reads,

Damage slight in specially designed structures; considerable in ordinary substantial buildings, with partial collapse; great in poorly built structures. Panel walls thrown out of frame structures. Fall of chimneys, factory stacks, columns,

hazard plot

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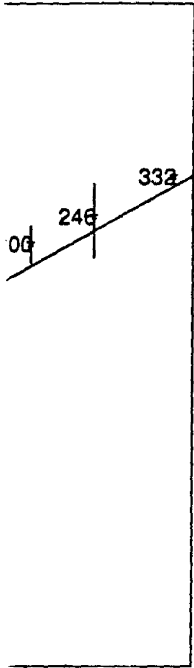
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monuments, walls. Heavy furniture overturned. Sand and mud ejected in small
amounts. Change in well water. Disturbs persons driving motor cars.

There are certainly basic difficulties with the MM scale. An important one
is that there are not susceptible structures at every location so that possible
damage there could not be recorded. Various aspects of the MM scale are
discussed in Reiter (1990) and critical remarks may be found in Steinbrugge
and Algermissen (1990). Most workers seem to agree however that for dam-
age studies it is the best thing generally available. An improved scale is
suggested in Brazee (1979).

When such acceleration or intensity data are examined, there is found
to be a general fall-off in severity of effect with distance from the earth-
quake source; however substantial variability and irregularity are invariably
present. This may be seen in Figure 4.

In the case of intensity data, isoseismal maps are prepared. The purpose
of such maps is to show the pattern of ground-shaking and associated dam-
age. The isoseismals are meant to be contours of equal intensity, to bound
areas within which the predominant intensity is the same. The drawer seeks,
for example, to draw a curve encircling all the MM$_{VIII}$ values, but scattered
VIII's will be ignored. The drawing of the contours is highly personal, eg.
Reiter (1990), p. 37 states "... drawing isoseismals can be a subjective pro-
cess that may lead to different outcomes for different analysts." Bruce Bolt
has emphasized to this writer a critical aspect of existing isoseismal maps,
namely they are conservative in two senses. First the indicated intensity level
at a location is the highest one noted. Second the isoseismals themselves are
drawn as far out from the source as reasonable to include all locations with
given intensity.

One intention of this paper is to indicate that it is in fact possible to
employ formal algorithms to generate isoseismals. Figure 2 presents some
preliminary results for the Loma Prieta CA event of 17 October 1989. This,
"World Series", disaster took place near Santa Cruz, California. It had
magnitude 6.9, duration 10 seconds, and led to 63 deaths, 1300 buildings
destroyed and 5.9 billion dollars damage. The largest MM intensity was IX.
[Further details of the event may be found in the October 1991 number of
Bull. Seismol. Soc. America.] In Figure 2 the small triangles indicate the
positions of the measurements. The source of the earthquake is marked by
a hexagon.

The MM intensity data analyzed are those employed in Stover, Reagor,
Baldwin, and Brewer (1990). There were 921 observations. The isoseis-
mals appearing in Figure 2a were prepared via the procedure "loess", de-
scribed in Cleveland and Devlin (1988), or Chapter 8 of Chambers and
Hastie (1992). This is a local regression procedure that smooths the data in
a robust/resistant fashion. In simplest form, the "smoothed value" at the
position with latitude and longitude (x, y) is $\hat{g}(x, y) = g((x, y) | \hat{\theta}_{x,y})$ where
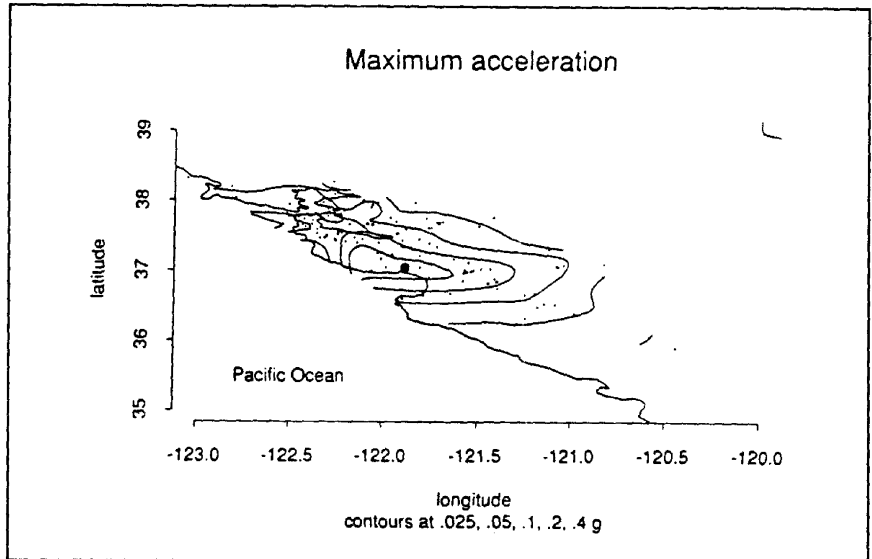
Figure 2a. Smoothed values of the maximum acceleration at logarithmically spaced levels for the Loma Prieta event. The triangles give the locations of the measurements.
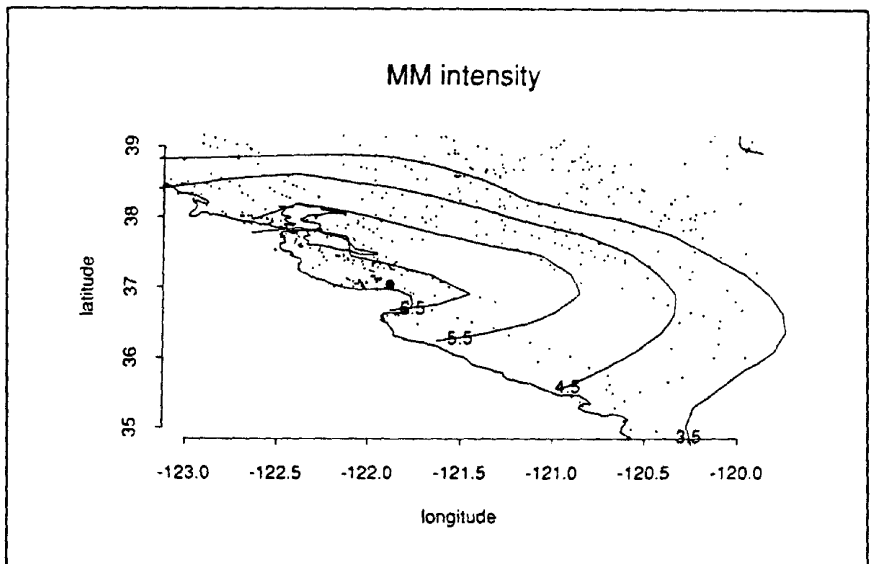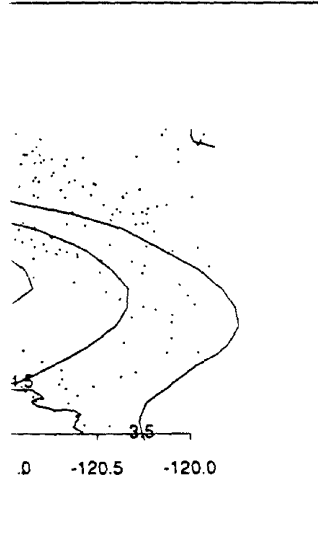

Figure 2b. Isoseismals obtained by employing the procedure loess of Chambers and Hastie (1992). For example the region between the contours 4.5 and 5.5 is meant to correspond to an MM intensity of V. The triangles give the locations of measurements.

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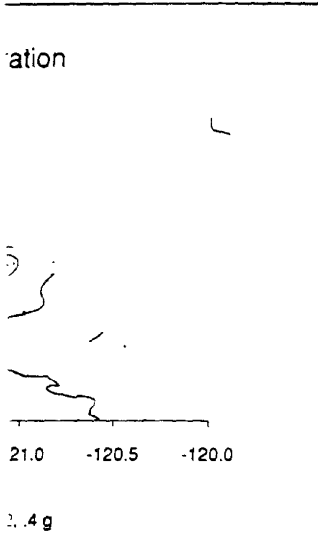
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$\hat{\theta}_{x,y}$ is the value minimizing

$$\sum_i w_i(x,y)\{I_i - g[(x_i,y_i)\,|\,\theta]\}^2 \qquad (6)$$

and for example $g(x,y)$ is a function assumed linear in θ and x,y. The data are intensities I_i recorded at location (x_i,y_i), for $i = 1,\ldots,n$. In the loess procedure,

$$w_i(x,y) = W[d_i(x,y)/d_{(q)}(x,y)]$$

where

$$W(u) = \begin{cases} (1 - |u|^3)^3 & \text{for } |u| < 1 \\ 0 & \text{otherwise,} \end{cases}$$

with $d_i(x,y)$ the distance from (x,y) to (x_i,y_i) and $d_{(q)}(x,y)$ the qth smallest of these distances. One includes in (6) a prespecified fraction q/n of the points, here 0.1. There is also a robust/resistant variant that down-weights outliers. [Details may be found in Chambers and Hastie (1992).] The smoothed values, at points on a grid, are then contoured for plotting. The resultant Figure 2a is notably similar to the United States Geological Survey's officially produced map, Stover, et al. (1990). The spread of the MM intensities about the smoothed values has $\hat{\sigma} = 1.09$. [The values are very scattered.] One can also obtain standard errors for the smoothed function values.

Figure 2b presents the results of the same computations for the maximum accelerations. In this case the data are taken from Boore, Seekins, and Joyner (1989). There are 266 observations. The general fall-off of strength of motion with distance from the source of the earthquake is again apparent.

The similarity of Figures 2a and 2b is noteworthy for seismologists and seismic engineers. These professionals have been concerned with relating Mercalli intensity and maximum acceleration. One reason is that intensity estimates are available for historical earthquakes for important regions, while maximum accelerations have only been recorded routinely in the last thirty years. Contemporary seismic risk analyses are often based on acceleration values. When acceleration data are unavailable, there is impetus to include estimates for old events based on MM intensities. A serious difficulty in constructing a conversion relationship however is that, even when recorded for the same event, the intensities and accelerations are usually measured at different places. The solution employed here is to obtain smoothed values of both quantities at common grid locations and then fit a relationship. The smoothing has the effect of reducing "measurement error".

Figure 3 is a scatter diagram of the smoothed acceleration values of the grid of Figure 2b against the corresponding smoothed MM values of 2a. There is a suggestion of approximate linearity with considerable scatter.
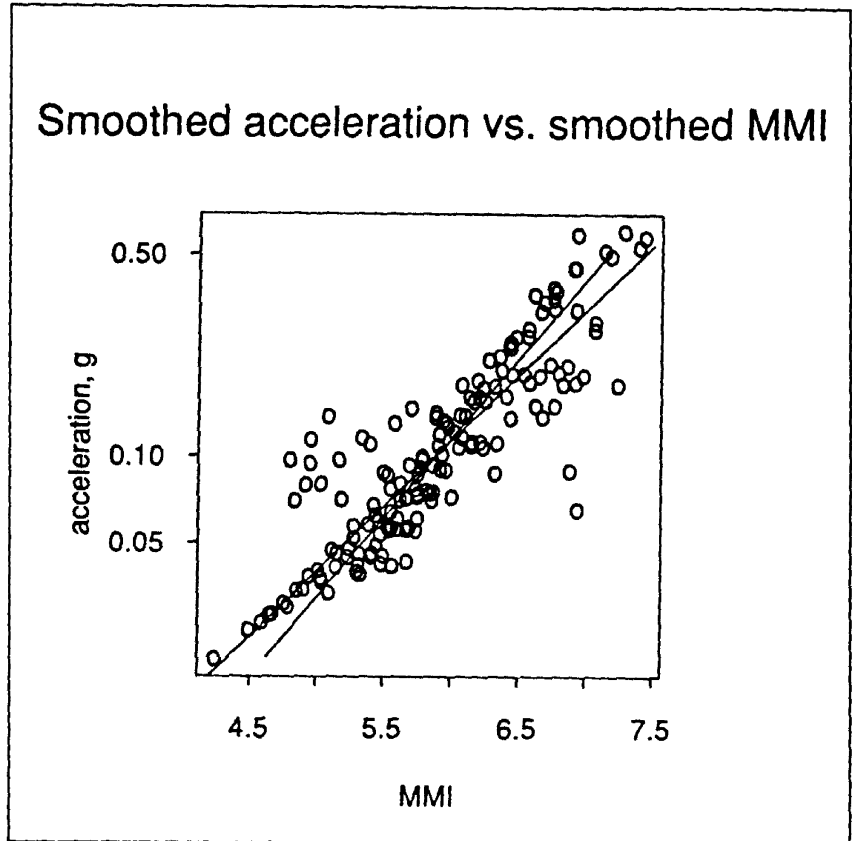
Figure 3. Corresponding smoothed values of accelerations and MM intensity on which Figures 2a and 2b were based are plotted against each other. The lines are robust/resistant regression lines of A on I_{MM} and vice versa.

Robust/resistant prediction lines of each variate on the other have been added. The prediction relations determined are:

$$\ln \hat{A} = -8.44 + 1.04 I_{MM} \qquad IV \leq I_{MM} \leq VIII \qquad (7)$$

$$\hat{I}_{MM} = 7.71 + .79 \ln A \qquad .02 \leq A \leq .50 \ . \qquad (8)$$

Here acceleration is measured in units of $g = 980$ cm/sec^2. There follows a table of predictor and corresponding predicted values (Table 1). No standard errors have been provided for these fits because in their computation, note would be taken that the values at the different grid points are statistically dependent. [Standard errors will be developed in later work.] For comparative purposes one can note that Trifunac and Brady (1975) determined the
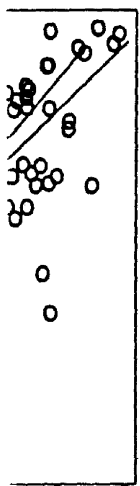
following relationship for horizontal accelerations,

$$\ln \hat{A} = -6.558 + .691 I_{MM} \quad \text{for} \quad IV \leq I_{MM} \leq X \ ,$$

while Bolt (1978) found,

$$\ln \hat{A} = -7.671 + .721 I_{MM} \ ,$$

and Dowrick (1989) determined, for New Zealand,

$$\ln \hat{A} = -7.772 + .721 I_{MM} \quad \text{for} \quad I_{MM} \leq IX \ .$$

Standard errors are needed to assess whether these various relationships are essentially different. It is important to have the two relationships, (7) and (8), because on some occasions one wishes to replace A and on some occasions I_{MM}.

I_{MM}	$\hat{A}, g$	A, g	$\hat{I}_{MM}$
8	.89	1.0	7.7
7	.31	.5	7.2
6	.11	.25	6.6
5	.04	.125	6.1
4	.01	.063	5.5

Table 1. Left-hand side — fitted acceleration as a function of intensity. Right-hand side — fitted intensity as a function of acceleration.

An interesting question is how to indicate the uncertainty of maps like those of Figure 2. Musmeci (1984) proposes the use of a bootstrap procedure. Bootstrap procedures are discussed in Diaconis and Efron (1983), for example.

Rubeis, et al. (1992) have also proposed an objective procedure for constructing isoseismal maps and discuss the importance of having such. Their procedure involves least squares fitting of polynomials within circles. However, it does not handle outliers, does not downweight points smoothly with distance, nor does it provide standard errors.

6. ATTENUATION

For assessing seismic risk and determining insurance premiums at a particular site, an attenuation law like (5) above is needed.
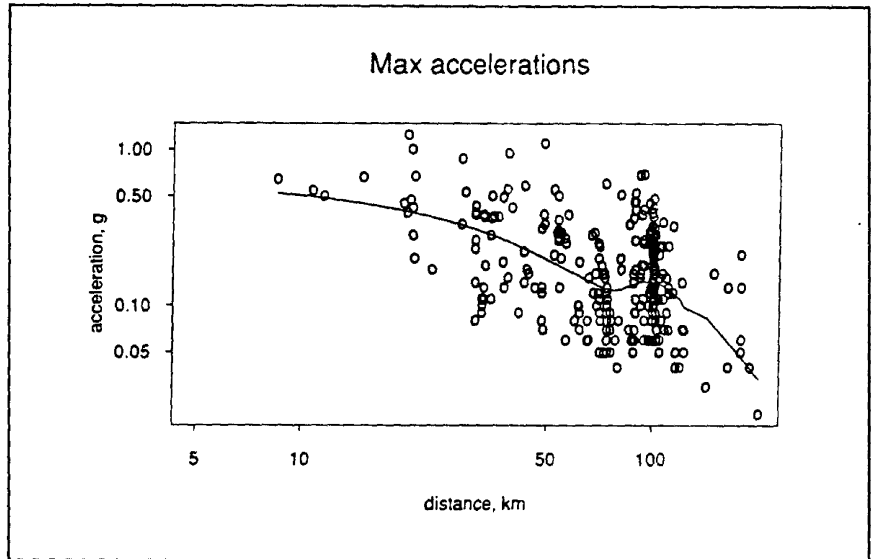
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Max accelerations

Figure 4a. A scatter diagram of observed maximum acceleration values versus distance for the Loma Prieta event. The curve added is a robust/resistant smooth. The data is taken from Boore, *et al.* (1989).
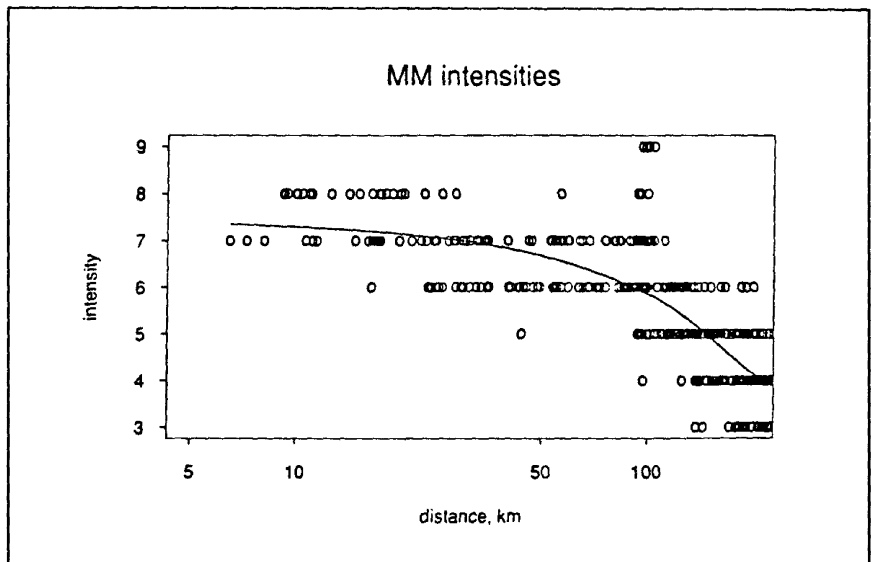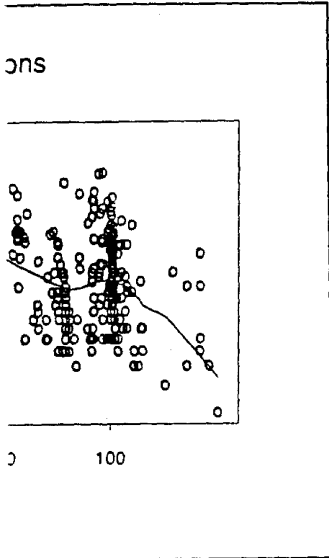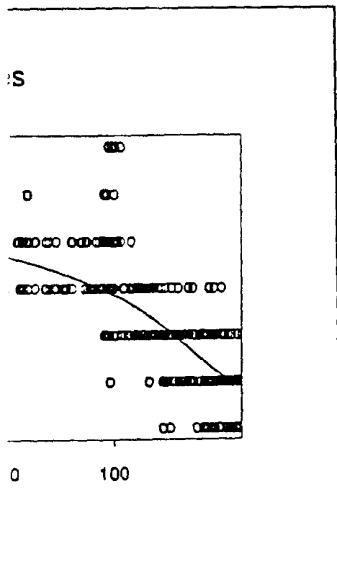
MM intensities

Figure 4b. A scatter diagram of observed MM intensities versus distance for the Loma Prieta event. The curve added is a robust/resistant smooth. The data is taken from Boore, *et al.* (1989).

Loma Prieta Data

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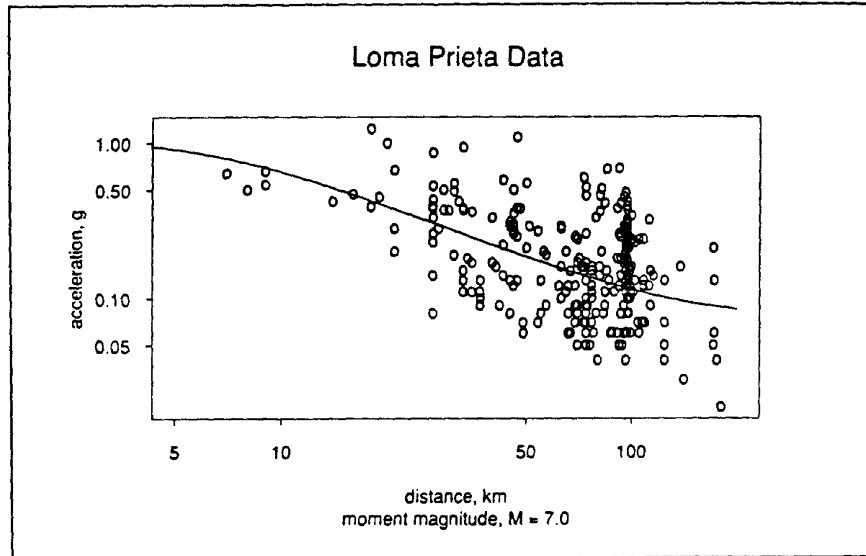/resistant smooth. The data is

Figure 5. The data of Figure 4a with a fitted line derived under the model (9) added.

Figure 4 is a graph of the accelerations and MM intensities of the Loma Prieta event plotted versus distance from the source. [In the case of the intensities, distances are taken from Boore, *et al.* (1989).] A substantial amount of scatter is present, and there is a falloff of severity with distance. The smooth curves on the figures correspond to robust/resistant smoothing of the data. The cluster of high values at a distance of about 100 km corresponds to the extreme motions recorded in the San Francisco/Oakland region, perhaps due to local geology, as discussed by Lomax and Bolt (1992).

It is convenient to have a particular functional form for the attenuation. One such that has been proposed by Joyner and Boore (1981) for the maximum acceleration at distance d for an event of moment magnitude M is as follows:

$$A = \frac{1}{d}e^{\beta M}e^{-\gamma d} \ ,$$

which involves parameters β and γ. In Brillinger (1989) a variant of this is fit to data (d_{ij}, M_i, A_{ij}) for $I = 23$ western USA events, the ith having J_i data values, i.e. $j = 1, \ldots, J_i$ and $i = 1, \ldots, I$. The model fitted is

$$\log A_{ij} = \alpha_i + \beta_i M_i - \log \sqrt{d_{ij}^2 + \delta_i^2} - \gamma_i \sqrt{d_{ij}^2 + \delta_i^2} + \epsilon_{ij} \ , \qquad (9)$$

where the $\alpha_i, \beta_i, \gamma_i, \delta_i$ are independent random effects for the ith earthquake with normal distributions and the ϵ_{ij} are independent normal noise values. The relationship (9) could be converted to one for I_{MM} via the prediction formula (8).

Figure 5 provides the result of fitting model (9) to the Loma Prieta acceleration data in the manner of Brillinger (1989). The curve provides an estimate of the acceleration, for the Loma Prieta event, at a given distance from the source.

In the analysis local site effects have been ignored since the emphasis is on damage and on the relationship of A and I_{MM}.

7. DAMAGE

By damage is here meant economic loss caused by an earthquake. It relates to the performance of structures. Most of the discussion to follow has in mind damage to insured properties. Damage is commonly described by a loss ratio that varies with the strength of shaking and type of structure. For a given strength of motion a variety of damage levels are seen to occur, necessitating the use of distributions. Dowrick and Rhoades (1990) for example found the distribution of damage ratios in an intensity zone to be approximately lognormal. One needs something like a motion-damage relationship or damage-probability matrix; see Panel on Earthquake Loss Estimation Methodology (1989) in order to proceed. Table 2 is an example of loss ratios for buildings, by risk category in percent.

Risk type vs MMI	VI	VII	VIII	IX	X
residential	.4%	1.7%	6%	17%	42%
commercial	.8%	3.5%	11%	27%	60%
industrial	.1%	.7%	3%	11%	30%

Table 2. Values are taken from Figure 42 in Munich Re (1991). Loss ratios (%) for buildings.

Specific formulas have been proposed on occasion. Following California studies, Steinbrugge and Algermissen (1990) suggest, in the case of pre 1940 dwellings,

$$Y = (.114M + .259)\left(8.534\, F\, e^{-.05389X}\right) \quad ,$$

where Y is loss over the deductible in percent, X is deductible in percent, M is the event's magnitude and F is an uncertainty factor, eg. 1.50. There are similar formulas for other ages and types of structures. McGuire (1986) suggests

$$D = 24.7 + 17.8\log_{10}(A) \qquad \text{for} \qquad A > .041$$

where D is damage as a percentage of value and A is in units of g. This could be rewritten in terms of I_{MM} using the relationship (7).

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of shaking and type of struc-
of damage levels are seen to
Dowrick and Rhoades (1990)
ratios in an intensity zone to
ething like a motion-damage
·e Panel on Earthquake Loss
oceed. Table 2 is an example
percent.

VIII	IX	X
6%	17%	42%
11%	27%	60%
3%	11%	30%

nich Re (1991). Loss ratios (%)

occasion. Following California
uggest, in the case of pre 1940

$e^{-.05389X}$) ,

t, X is deductible in percent,
-tainty factor, eg. 1.50. There
of structures. McGuire (1986)

or $A > .041$

and A is in units of g. This
relationship (7).

There is an extensive discussion of this topic in Panel on Earthquake Loss Estimation Methodology (1989). A difficulty in carrying out damage studies is that the data are often proprietary.

8. PARTICULAR PRACTICE

The practice and laws of earthquake insurance vary with country and even within country. A brief description of some follows.

CALIFORNIA

In California some companies use 3 "territories". For example in Territory 1 (mainly Imperial County) for a frame house with a deductible of 10% the cost of insurance is 6.50 dollars per 1000 coverage. There has been a State run plan, the California Residential Earthquake Recovery Program, providing coverage for houses of $15,000 with a deductible of 1000 dollars for a cost of 60 dollars/year.

ISRAEL

The Israeli case is detailed in Kahane (1988). The insurance is part of the homeowners policy. The country pays reinsurance premiums on the order of $15,000,000 and is said to be, "one of the largest customers of earthquake insurance in the world market" *ibid*.

JAPAN

The insurance for homes is an endorsement to fire coverage. Premiums are based on a statistical analysis; see Matsushima (1989). Historical records provided evidence of 349 damaging earthquakes during the 485 years between 1494 and 1978. Their magnitudes and hypocentres were estimated. The probable amount of damage that these would cause if they occurred in the present year was estimated. The pure premium was this amount divided by 485. Finally loadings were included. In the analysis the country was divided into 5 zones. For example, the rates per thousand yen insured for wooden buildings were 4.80 for Zone 5 (Tokyo, Kanagawa, and Shizuoka). Reinsurance is provided by the government.

NEW ZEALAND

The New Zealand approach changed recently. It is part of fire insurance. For homes there is a limit of $100,000 coverage at a cost of 0.50 per 1000. The country has a billion dollars of reinsurance cover for 1992 for $31,698,000

for claims above $1.25 billion; see EQC (1992). The New Zealand reinsurance program is thought to be the largest catastrophe coverage in the world (Steven 1992).

The following quotes indicate some of the flavor of earthquake insurance practice.

"The rate of premium has never been actuarially based."

(Hellberg 1984)

"We are a California company so cannot offer earthquake insurance because of the risk."

(California Casualty 1989)

"earthquake loss estimation is presently more an art than a science"

(Rojahn and Sharpe 1985)

"There are surprises after every earthquake"

(Steinbrugge 1989)

"The calculation and enforcement of 'correct' premium rates is, of course, important in the long term. Nevertheless, there are further economic and social factors that also influence the rates charged eventually."

(Smolka and Berz 1991)

Some of the practice of reinsurance companies in determining rates is described in Porro (1989) and Munich Re (1991). The latter may be described as follows: one has Table 2 of damage percents. For the site of interest one determines a table of annual probabilities such as Table 3. For residential properties this leads to a net premium of

$$.4 \times .04 + 1.7 \times .014 + 6 \times .005 + 17 \times .003 + 42 \times .001 = .1628\%$$

or 1.63 per $1000 coverage. Loadings would be added to this figure.

I_{MM}	VI	VII	VIII	IX	X
Prob.	.04	.014	.005	.003	.001

Table 3. Hypothetical probabilities as a function of intensity for a site of interest.

9. OTHER ASPECTS

This paper has so far focussed on only part of the loss estimation story. Attention has been directed to the problems of event timings, attenuation of

. The New Zealand reinsur-
trophe coverage in the world

avor of earthquake insurance

ased."

(Hellberg 1984)

hquake insurance because of

(California Casualty 1989)

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(Rojahn and Sharpe 1985)

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$$3 + 42 \times .001 = .1628\%$$

added to this figure.

II	IX	X
)5	.003	.001

if intensity for a site of interest.

TS

t of the loss estimation story.
event timings, attenuation of

energy and damage laws. However in a study for a given region the particular locations of faults are needed. As an example for the case of California, see Wesnousky (1986). Magnitudes or intensities are also needed for each fault. There are functional forms based on the lengths of faults. Attenuation laws, like (5) and (9), are next applied followed in turn by damage laws.

Nonscientific issues arise too. Regulatory agencies are concerned with the solvency of insurance companies. In California the concept of probable maximum loss (PML) is employed in assessing this; see California Department of Insurance (1990). The PML is defined as the average probable maximum monetary loss which will be experienced by 9 out of 10 buildings in a given earthquake building class in the specified earthquake PML zone. There are 8 such zones for California. An event of Richter magnitude 8.25 is assumed. A yardstick sometimes employed to assess solvency is that the potential loss on one risk should not exceed 10% of the surplus, *ibid*.

Insurers protect themselves by reinsurance. This is the sharing of insurance risks with other insurers. It is used extensively for earthquakes and catastrophes. As an example of just how extensive this is, note that for California Zone B (Los Angeles and Orange Counties), only 26% of the PML is retained by the insurer of record, according to the California Department of Insurance (1990). However, wanting reinsurance is one thing, getting it is another, as Japan and New Zealand have found on occasion.

10. DISCUSSION

General needs of the insurance industry are discussed in Holden and Real (1990) and Workshop Report (1990) and the practice is evolving. Both New Zealand (New Zealand Government 1988), and the USA are involved with major changes and controversies. In the U.S. case the insurance industry sponsored the Earthquake Project, a proposal to the Federal government for a joint program in the event of a major earthquake. The Federal Emergency Management Agency (FEMA) was not impressed and recommended against the program. In California too the government program was viewed negatively and the California Earthquake Recovery Program ended.

Another aspect of earthquake insurance is how it is treated under tax laws. One point of dispute relates to the taxation of reserves. Kahane (1988) remarks on the "need for tax rules allowing for larger reserves for unexpired risks".

The focus of this paper has been on earthquake insurance, but the basic principles apply to other catastrophes as well. We may mention: floods, hail, firestorms, landslides, tsunamis, volcanic eruptions, windstorms.

It is clear that many technically interesting problems remain for geologists, seismologists, engineers, statisticians and actuaries.

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APPENDIX

Attention will focus on a single year and a damaging event that can happen at most once in that year. Let π denote the probability of the event's occurrence, $\Pr\{U \neq 0\}$, and if the event does occur, suppose the random loss is L.

Define an indicator variable I that equals 1 if the event occurs and 0 if it does not. Then the loss, U, of Section 2 may be written as $U = IL$. From this one sees that

$$\mu_U = \pi \mu_L$$

and

$$\sigma_U^2 = \pi(1 - \pi)\mu_L^2 + \pi\sigma_L^2 \approx \pi E\{L^2\}$$

respectively, with the last relationship assuming that π is small.

In the case that many units are involved, L will be the sum of many individual losses and the distribution may be approximated by a normal. If $\Phi(\cdot)$ denotes the normal cumulative, then the probability of (4) is given by

$$\Pr\{IL > R + P\} = \pi \Pr\{L > R + p\} \approx \pi\{1 - \Phi[(R + P - \mu_L)/\sigma_L]\} \quad .$$

For the premium rule (1), $P = \pi\mu_L$, this becomes

$$\pi\{1 - \Phi[(R - (1 - \pi)\mu_L)/\sigma_L]\} \quad ,$$

which may be used to compute the probability of ruin given the reserve, R, and the values of μ_L and σ_L. Alternately, for given ϵ one can solve for the premium as follows:

$$P = \mu_L + \sigma_L \Phi^{-1}\left(1 - \frac{\epsilon}{\pi}\right) - R \quad .$$

The effects of several years operation can be studied as in Benjamin (1986).

Suppose that $R = 10\mu_L$, the yardstick proposed in Section 9, the ruin probability becomes

$$\pi\{1 - \Phi[(9 + \pi)\mu_L/\sigma_L]\} \quad .$$

This is seen to increase with the coefficient of variation σ_L/μ_L. The expression can be used to study ruin probabilities.

Other premium rules can be investigated in a similar fashion. Note that the effects of employing estimates for parameters, rather than actual values, remains to be addressed generally.