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An Empirical Investigation of the Chandler Wobble and Two Proposed Excitation Processes

By

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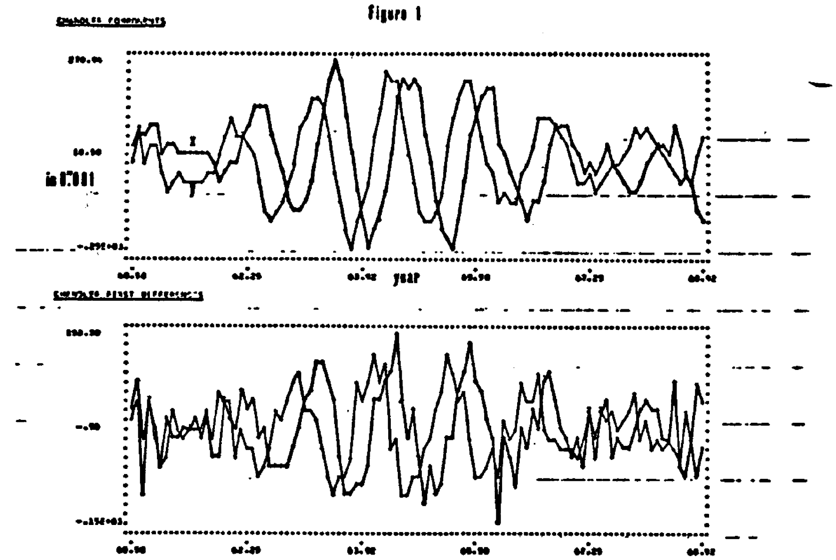
1. Introduction

The axis of instantaneous rotation of the Earth does not remain fixed relative to the body of the Earth, rather, its points of interception with the surface wander about within a region approximately the size of a tennis-court. This wandering was predicted by Euler in 1765 and confirmed by observation in 1891. The top graph of Figure 1 provides the x and y coordinates of the deviation of the North pole from its mean position for the period 1960–1969. (In units of $0''.001 = .101$ ft.) The motion of the pole produces a variation in the latitude which may be used to deduce the time path of the pole. We mention briefly how this is done.

The zenith is the direction opposite to local gravity. The altitude of a star is the complement of its zenith distance. The fundamental method of determining the latitude of an observatory is to take the average of the altitudes of a circumpolar star when it crosses the meridian above and below the pole. Since 1899 the International Latitude Service has measured the variation of

Figure 1



latitude at five stations apread along 39°08′ north latitude. A conventional pole of rotation (the C.I.O.) has been adopted. Suppose $X(t)$ denotes the displacement of the instantaneous north pole at time t from the C.I.O. towards Greenwich and $Y(t)$ the displacement towards 90° west of Greenwich. Let $\Delta\varphi_j(t)$ denote the increment in latitude at observatory j, from its mean latitude. Then estimates $x(t)$, $y(t)$ of $X(t)$, $Y(t)$ are determined by the least squares fit of the regression equation

$$\Delta\varphi_j(t) = Z(t) + X(t)\cos\lambda_j + Y(t)\sin\lambda_j + \varepsilon_j(t) \qquad (1.1)$$

$j = 1, \ldots, 5$ where λ_j denotes the longitude of the j-th observatory. For t at monthly intervals, these values are given in Vicente and Yumi (1969, 1970), which is the source of the data used in the computations of this paper. The values of $x(t)$ and $y(t)$ fall in the intervals $-0''.37$, $0''.47$ and $-0''.28$, $0''.50$ respectively. The probable errors given in Table 12 of Yumi, Ishii and Sato (1968) may be used to deduce the standard errors of $x(t)$, $y(t)$ from the above linear fit. These are $0''.057$ and $0''.048$ respectively.

Chandler (1891) suggested that the polar motion was made up of two principal components with periods one year and 428 days $\doteq$ 14 months respectively. Figure 3 below gives the logarithm of the periodogram of the data. Two peaks, at frequencies near these periods are apparent. In the next section we shall set down a differential equation that describes the motion of the pole when the Earth is subjected to arbitrary excitations. Scientific workers seem to be agreed that the component of the motion with period one year results from the excitation function possessing a strong seasonal component. 428 days corresponds to the Euler frequency of vibration of the Earth; however the source of the energy that stimulates the natural vibration is not agreed upon. We shall consider earthquakes and shifts of the mass of the atmosphere as possible sources of the energy. This 428 day component is called the Chandler component. The associated motion of the Earth is called the Chandler wobble.

In the next section we present a variety of harmonic analyses of the polar variation including; power spectrum estimation, maximum likelihood fit of a model of the spectrum, bispectrum estimation and complex demodulation. In Section 3 we carry out cross-spectrum analysis of the polar motion series with two earthquake series as well as complex demodulation of the latter. In Section 4 we repeat this analysis with an atmospheric series.

Munk and MacDonald (1960) is an excellent source of basic material concerning the rotation of the Earth. The proceedings of two symposia on the topic have appeared. These are Mansinha, Smylie and Beck (1970) and Melchior and Yumi (1972). These works show that the problem of understanding the rotation of the Earth is exceedingly rich in geophysical terms. It is also rich in statistical aspects. We mention the papers; Walker and Young (1955, 1957), Arato, Kolmogorov and Sinai (1962), Mandlebroit and McCamy (1970).

2. Analyses of the Polar Motion

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2. Analyses of the Polar Motion

The position of the pole of rotation at time t is conveniently described by the complex number

$$Z(t) = X(t) + i\,Y(t) \tag{2.1}$$

where $X(t)$, $Y(t)$ are the displacements from the C.I.O. towards Greenwich and towards 90° west of Greenwich respectively. Munk and MacDonald have investigated the dynamics of the spinning Earth. Let $\Phi(t)$ denote an excitation function whose increments, $d\Phi(t)$, describe the change in the Earth's inertia tensor in the time interval $(t, t + dt) \cdot d\Phi(t)$ is complex-valued with $\mathrm{Re}\, d\Phi(t)$ giving the change towards Greenwich and $\mathrm{Im}\, d\Phi(t)$ the change towards 90° west of Greenwich. [In the next section we shall make use of a formula for $d\Phi(t)$ when the change results from a shift of mass in an earthquake.] From classical mechanics, Munk and MacDonald deduce the equation of motion

$$dZ(t) = a\,Z(t)\,dt + d\Phi(t) \tag{2.2}$$

with $a = -\beta + i\gamma$ complex-valued and $\beta > 0$. If $\Phi(t) = 0$, then a solution of (2.2) is provided by

$$Z(t) = e^{at} = e^{-\beta t}(\cos \gamma t + i \sin \gamma t) \tag{2.3}$$

This motion is one of a damped oscillation of requency γ. The greater β, the greater will be the damping.

Suppose now that $\Phi(t)$, $-\infty < t < \infty$, is a random process with stationary increments and power spectrum $f_{\Phi\Phi}(\lambda)$. [See Brillinger (1970) for a discussion of the spectral analysis of processes with stationary increments. The definitions given there must be modified trivially to apply to complex-valued processes.] Then (2.2) will have a solution with stationary increments and power spectrum

$$f_{ZZ}(\lambda) = |\,i\lambda - a\,|^{-2} f_{\Phi\Phi}(\lambda) = [\beta^2 + (\lambda - \gamma)^2]^{-1} f_{\Phi\Phi}(\lambda) \tag{2.4}$$

This expression shows that $f_{ZZ}(\lambda)$ may be expected to inherit the peaks of $f_{\Phi\Phi}(\lambda)$ and to possess a new peak, of spread β, at $\lambda = \gamma$.

Were $Z(t)$ available for an interval $0 < t \leq T$, we would be led to base a spectral analysis of it on the Fourier-Stieltjes transform

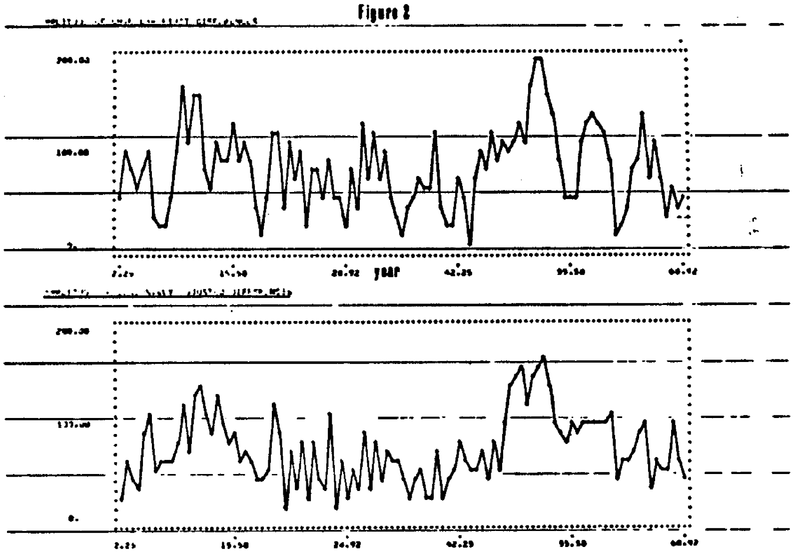
$$\int_0^T \exp\{-i\lambda t\}\, dZ(t) \tag{2.5}$$

The polar motion values we use are given at monthly intervals, and so we are led to take as basic statistic the finite Fourier transform of the first differences of $Z(t)$, namely

$$d_Z^T(\lambda) = \sum_{t=0}^{T-1} \exp\{-i\lambda t\}\, [(Zt + 1) - Z(t)] \tag{2.6}$$

$-\infty < \lambda < \infty$. The second graph of Figure 1 is a plot of the series $Z(t+1) - Z(t)$ for the time period 1960–1969. The first graph of Figure 2 is a plot of $|\,Z(t+1) - Z(t)\,|$ for the period 1902–1969.

Figure 2

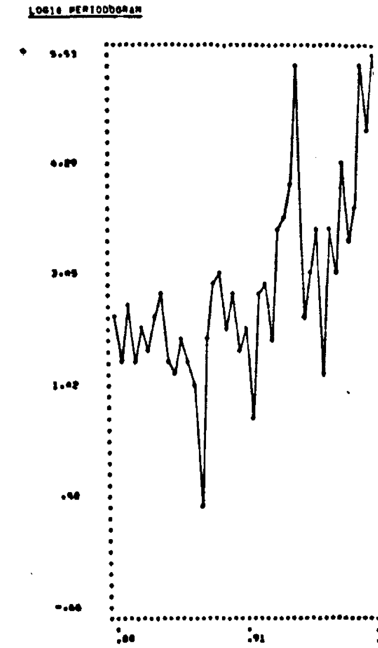


In Figure 3 we have plotted $\log_{10}$ of the periodogram $I_{ZZ}^{(T)}(\lambda) = (2\pi T)^{-1} | d_Z^{(T)}(\lambda) |^2$ for $.88 \leq \lambda/2\pi \leq 1.00$. $I_{ZZ}^{(T)}(\lambda)$ may be considered to be a highly unstable estimate of $f_{ZZ}(\lambda)$. In the case that the process $\Phi(t)$, $-\infty < t < \infty$, is mixing, the periodogram will be asymptotically exponential with mean $f_{ZZ}(\lambda)$. The standard deviation of the curve in Figure 3 will be approximately .43. Peaks are present in this graph at frequencies $\lambda/2\pi = .917$, .929 corresponding to rotations in a negative direction with periods $\doteq$ 12 months, 14.1 months respectively. It has long been understood that the process $\Phi(t)$, $-\infty < t < \infty$, would contain a strong component of period 12 months because of the seasonal variation of the loading of the Earth through, shifts of the atmosphere, melting of snow, tides and the like. [See Jefferys (1959).] This would account for the peak in Figure 3 corresponding to a period of 12 months. Before smoothing the periodogram in order to obtain a more stable estimate of the power spectrum, we therefore removed the seasonal variation from the series of first differences by subtracting monthly means. The values subtracted are given in Table 1. They correspond to a figure of ellipsoidal shape. Figure 4 is $\log_{10}$ of the spectral estimate obtained by

Table 1

(units of 0″.001)

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
x	−41	−17	−2	22	33	43	49	31	−2	−34	−40	−42
y	11	28	43	34	22	7	−12	−35	−40	−45	−19	5



smoothing 8 adjacent periodogram
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asymptotic standard error is .15.

The smooth curve in Figure 4 corr
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a plot of $| Z'(t+1) - Z'(t) |$, th
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(2.2) retains the form

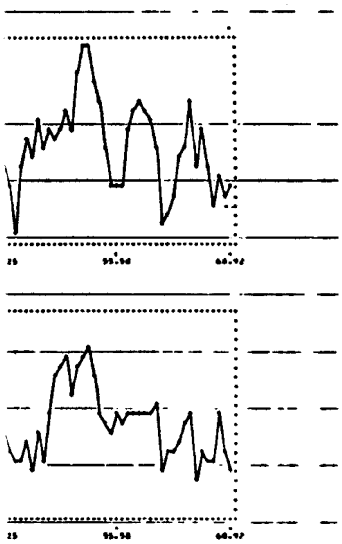
$$dZ'(t) = a$$

We may solve the equation (2.7) and

$$Z'(t) = $$

Noting the assumed removal of s
assume that $\Phi'(t)$ is a noise proce
and var $\{d\Phi'(t)\} = \sigma^2$. [Were we t
would be the model of Arato et al.

$$\Delta Z'(t) = Z$$



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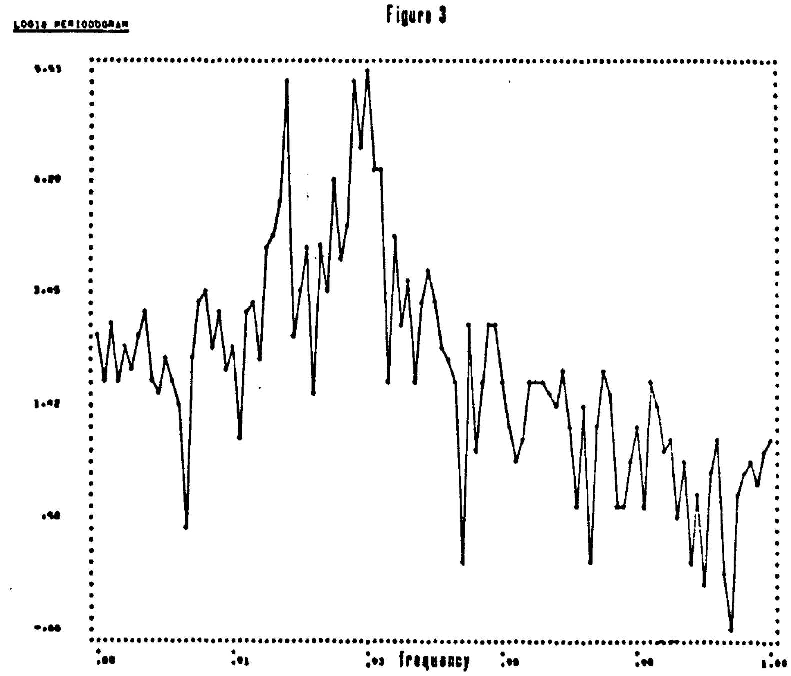
y	Aug.	Sept.	Oct.	Nov.	Dec.
49	31	−2	−34	−40	−42
12	−35	−40	−45	−19	5

Figure 3



smoothing 8 adjacent periodogram ordinates based on the seasonally corrected values. The bandwidth of this estimate is .01 cycles/month. Its asymptotic standard error is .15.

The smooth curve in Figure 4 corresponds to a fitted model whose construction we now describe. Suppose that we denote the seasonally corrected version of $Z(t)$, $\Phi(t)$ by $Z'(t)$, $\Phi'(t)$ respectively. The second graph of Figure 2 is a plot of $|Z'(t+1) - Z'(t)|$, the amplitude of the seasonally adjusted first differences. It is seen to peak around the years 1910 and 1950. The model (2.2) retains the form

$$dZ'(t) = a\,Z'(t) + d\Phi'(t) \qquad (2.7)$$

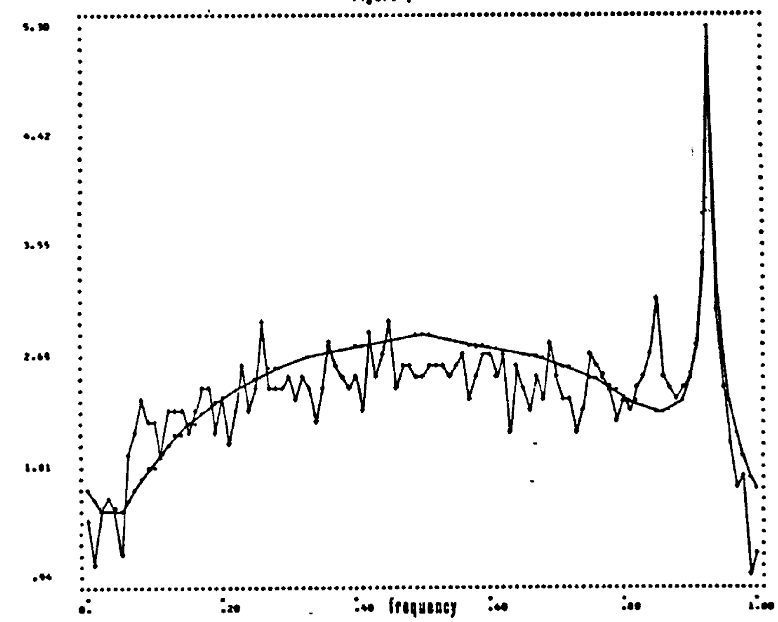
We may solve the equation (2.7) and obtain

$$Z'(t) = \int_{-\infty}^{t} e^{a(t-u)}\, d\Phi'(u) \qquad (2.8)$$

Noting the assumed removal of seasonal components from $\Phi(t)$, we now assume that $\Phi'(t)$ is a noise process with stationary orthogonal increments and var $\{d\Phi'(t)\} = \sigma^2$. [Were we to assume it Gaussian as well, then (2.7) would be the model of Arato et al. (1962).] Consider the series of increments

$$\Delta Z'(t) = Z'(t+1) - Z'(t) \qquad (2.9)$$

Figure 4



$t = 0, \pm 1, \ldots$. We see from the representation (2.8) that this series has autocovariance function

$$c_{\Delta Z', \Delta Z'}(u) = \sigma^2 \exp\{-\beta \mid u \mid\} \exp\{i \gamma u\}/2\beta \qquad (2.10)$$

$u = 0, \pm 1, \ldots$ and hence power spectrum

$$f_{\Delta Z', \Delta Z'}(\lambda) =$$

$$= \frac{\sigma^2}{2\pi} \frac{1 - \exp\{-2\beta\}}{2\beta} \frac{1}{1 - 2\exp\{-\beta\}\cos(\lambda - \gamma) + \exp\{-2\beta\}}$$

for $-\infty < \lambda < \infty$.

We have mentioned previously that the series $Z(t)$ is not observed directly, but is measured subject to error. Let

$$z'(t) = Z'(t) + \varepsilon(t) \qquad (2.11)$$

denote the observed series, corrected for seasonal effects, where we assume that $\varepsilon(t)$, $t = 0, \pm 1, \ldots$ is a stationary noise series with var $\varepsilon(t) = \psi^2$. It follows that the power spectrum of the first differences of $z'(t)$ will be given by

$$f_{\Delta z', \Delta z'}(\lambda) = f_{\Delta Z', \Delta Z'}(\lambda) + \frac{\psi^2}{2\pi} \mid 1 - e^{-i\lambda} \mid^2 \qquad (2.12)$$

This last constitutes our propose seen to involve four unknown p wobble, β the damping constant. corrected excitation function and errors.

We fit the model by the metho

$$\hat{f}_s = I^{(T)}_{\Delta z', \Delta z'}\Big(\frac{2}{}$$

for $s = 0, \ldots, T - 1$. Under a $s = 0, \ldots, T - 1$ are approxima likelihood function of the data th

$$L = I$$

Let θ, θ' denote any two of the p

$$\frac{\partial \log L}{\delta \theta} =$$

$$E\left[\frac{\partial \log L}{\partial \theta} \frac{\partial l}{}\right.$$

The maximum likelihood equat to 0 for the various parameters. of scoring [see Rao (1965), p. 3 producing estimates of the asymp the recursion with estimates de procedure stabilised after two rou

parameter	$\gamma/2\pi$
estimate	.9294
s. e.	.0026

The indicated results for γ lead t Chandler period to be from 13.2 fidence interval for β is from .000 determined accurately at all. The the source of the excitation of the fluctuations of the excitation ha monthly values. It is interesting t

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fferences of $z'(t)$ will be given by

$$\frac{\psi^2}{2\pi} \,|\, 1 - e^{-i\lambda} |^2 \qquad (2.12)$$

This last constitutes our proposed model for the spectrum of Figure 4. It is seen to involve four unknown parameters; γ the frequency of the Chandler wobble, β the damping constant, σ the standard deviation of the seasonally corrected excitation function and ψ the standard deviation of the measurement errors.

We fit the model by the method of maximum likelihood. Set

$$\hat{f}_s = I^{(T)}_{\Delta z', \Delta z'}\left(\frac{2\pi s}{T}\right), f_s = f_{\Delta z', \Delta z'}\left(\frac{2\pi s}{T}\right)$$

for $s = 0, \ldots, T-1$. Under a variety of conditions, the variates $\hat{f}_s/f_s$, $s = 0, \ldots, T-1$ are approximately independent standard exponentials. The likelihood function of the data therefore has the approximate form

$$L = \prod_s f_s^{-1} \exp\{-\hat{f}_s/f_s\}$$

Let θ, θ' denote any two of the parameters, then

$$\frac{\partial \log L}{\partial \theta} = - \sum_s \frac{(f_s - \hat{f}_s)}{f_s^2} \frac{\partial f_s}{\partial \theta} \qquad (2.13)$$

$$E\left[\frac{\partial \log L}{\partial \theta} \frac{\partial \log L}{\partial \theta'}\right] = \Sigma f_s^{-2} \frac{\partial f_s}{\partial \theta} \frac{\partial f_s}{\partial \theta'}$$

The maximum likelihood equations are obtained by setting (2.13) equal to 0 for the various parameters. We solve these equations by the method of scoring [see Rao (1965), p. 302]. This procedure has the advantage of producing estimates of the asymptotic standard errors incidentally. We began the recursion with estimates determined by the method of moments. The procedure stabilised after two rounds. We obtained the following results.

Table 2

parameter	$\gamma/2\pi$	β	σ	ψ
estimate	.9294	.0050	7.1	31.9
s. e.	.0026	.0023	.33	.62

The indicated results for γ lead to a 95 per cent confidence interval for the Chandler period to be from 13.2 months to 15.3 months. A 95 per cent confidence interval for β is from .0005 month^{-1} to .0095 month^{-1}. It has not been determined accurately at all. The estimate of σ is important in searching for the source of the excitation of the whobble. It suggests that the non-seasonal fluctuations of the excitation have standard deviation of order 0''.007 for monthly values. It is interesting to compare the magnitude of the standard deviation of the observational errors as estimated here with the values 0″.057, 0″.048 mentioned in the introduction. In the present notation they correspond to $\psi = 0''.075$ a value larger than the 0″.032 found here. In either case the observational errors are large compared to the magnitude of the phenomenon under study.

In Figure 4 we have plotted expression (2.12) using the parameter values of Table 2. The fit seems consistent with the standard error .15 of the estimate except for the peak just to the left of the Chandler peak. Surprisingly, this peak is centered at a frequency, (.846) (2π) that is near the sum of the seasonal and Chandler frequencies. This occurrence led us to suspect the presence of a non-linear phenomenon. We therefore estimated the modulus of the bicoherency

$$\frac{|f_{\Delta z\,\Delta z,\Delta z}(\lambda_1, \lambda_2)|}{\sqrt{f_{\Delta z,\Delta z}(\lambda_1) f_{\Delta z,\Delta z}(\lambda_2) f_{\Delta z,\Delta z}(\lambda_1 + \lambda_2)}} \tag{2.14}$$

(See the Appendix for the definition of the third-order spectrum appearing here.) Table 3 below presents an estimate for frequencies in the immediate neighborhood of the seasonal, the Chandler and the seasonal plus the Chandler. The bandwidth of the estimate is .01. In the null case the square of the estimate is distributed asymptotically as an exponential with mean $T/2\pi N$, if N denotes the number of third-order periodograms averaged in forming the estimate. [The sampling properties of such estimates are discussed in Brillinger and Rosenblatt (1967) and Huber et al. (1970).] The 99 per cent point for the values of Table 3 is 2.52, corresponding to $N = 95$. There is a clear suggestion that the values of Table 3 are larger than would be expected in the null case. No dramatic peaks are present in the table however. Our conclusion is that the excitation process or the measurement error process, is not quite normal. Table 4 presents an estimate of (2.14) covering the whole frequency domain. Here the bandwidth adopted was .05 and $N = 1950$. The 99 per cent point of the null distribution is now .56. Again there are no dramatic peaks in the function, rather the whole collection of values is larger than would be expected in the null case. It appears that the data are somewhat non-normal.

Table 3

frequency/2π	.87	.88	.89	.90	.91	.92	.93	.94	.95	.96
.87	2.5	.7	.2	.7	.9	1.2	1.6	1.6	.9	1.5
.88		1.1	1.1	.3	2.1	.4	.9	1.3	.9	.2
.89			2.3	.7	.6	1.3	1.8	.7	.9	.5
.90				1.1	.8	.4	2.6	1.1	2.2	1.2
.91					.5	1.4	1.3	.9	2.2	.5
.92						2.9	2.5	2.7	.1	1.9
.93							.8	.2	1.5	.3
.94								1.3	.6	.5
.95									2.5	2.3
.96										.7

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$$\frac{\ldots)|}{{}_{4x}(\lambda_1+\lambda_2)} \tag{2.14}$$

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92	.93	.94	.95	.96
ı.2	1.6	1.6	.9	1.5
.4	.9	1.3	.9	.2
1.3	1.8	.7	.9	.5
.4	2.6	1.1	2.2	1.2
1.4	1.3	.9	2.2	.5
2.9	2.5	2.7	.1	1.9
	.8	.2	1.5	.3
		1.3	.6	.5
			2.5	2.3
				.7

Table 4

frequency/2π

	.00	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90	.95
.00	1.9	.7	.8	.3	.4	.5	.3	.6	.5	.5	.9	.8	.7	.7	.6	.5	.4	.3	.4	.5
.05		.2	.2	.3	.3	.3	.3	.5	.1	.2	.2	.4	.4	.2	.2	.4	.3	.5	.1	.1
.10			.5	.2	.3	.0	.4	.3	.2	.2	.1	.2	.3	.2	.2	.6	.2	.5	.2	.3
.15				.3	.4	.1	.3	.3	.1	.2	.6	.3	.3	.6	.3	.4	.1	.2	.7	.3
.20					.6	.3	.3	.1	.2	.1	.2	.1	.4	.3	.1	.2	.4	.3	.1	.4
.25						.3	.3	.3	.2	.2	.2	.5	.1	.6	.3	.1	.1	.3	.1	.3
.30							.3	.2	.4	.3	.2	.2	.2	.3	.1	.3	.3	.2	.2	.2
.35								.6	.2	.3	.1	.3	.2	.6	.4	.1	.2	.4	.4	.2
.40									.8	.6	.5	.0	.2	.5	.4	.1	.1	.4	.3	.1
.45										.1	.1	.3	.4	.3	.4	.6	.3	.2	.2	.4
.50											.8	.2	.2	.3	.3	.0	.4	.3	.7	.3
.55												.3	.4	.5	.3	.2	.5	.2	.5	.3
.60													.7	.5	.1	.2	.1	.2	.2	.3
.65														.4	.2	.2	.1	.1	.1	.2
.70															.1	.5	.4	.5	.3	.2
.75																.4	.1	.3	.3	.5
.80																	.2	.0	.4	.2
.85																		.3	.4	.4
.90																			1.1	.8
.95																				.0

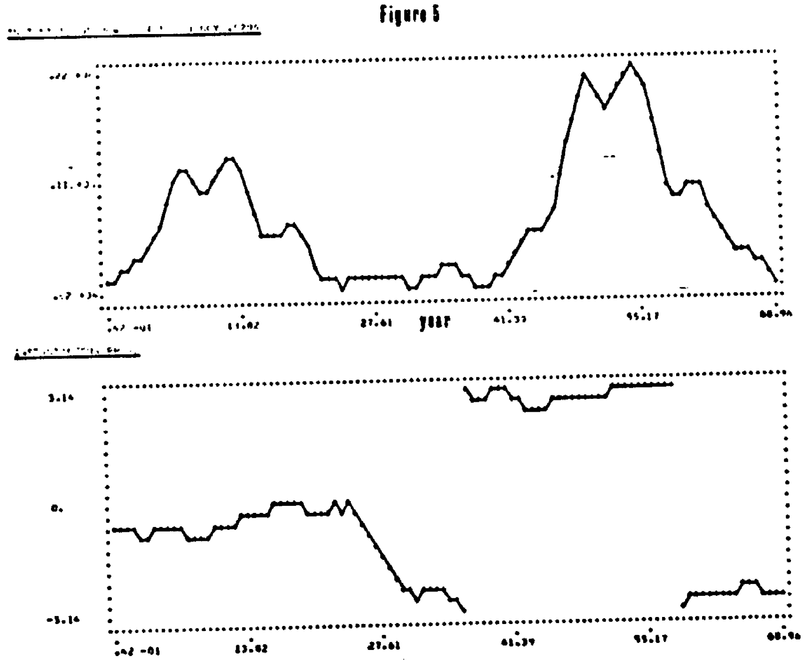
In order to be able to better understand the behavior of the polar motion and in order to get an idea of the character of the excitation process, $\Phi(t)$, we carried out a complex demodulation of the series $\Delta z(t)$ at several frequencies. [This procedure is described for real-valued series in Tukey (1961).] Specifically we formed the series

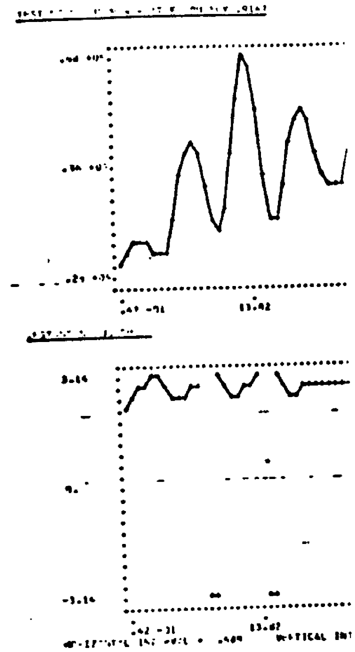
$$\zeta_{\Delta z}(t, \lambda) = \frac{1}{\sqrt{2\pi(2L+1)}} \sum_{|t-u| \leq L} \Delta z(u) \exp\{-iu\lambda\} \qquad (2.15)$$

$t = 0, 1, \ldots, T-1$ for a variety of λ and $L = 48$. We note that $|\zeta_{\Delta z}(t, \lambda)|^2$ $t = 0, \ldots, T-1$, is a running periodogram for the data at frequency λ. Its average across the whole time domain would provide an alternate estimate of $f_{\Delta z, \Delta z}(\lambda)$. Variations in it are indicative of temporal variations in the power at frequency λ. The time path of $\arg \zeta_{\Delta z}(t, \lambda)$ gives information concerning the value of the dominant frequency component in the neighborhood of λ. If its path is a straight line with slope ν for some time period, then the component with frequency $\lambda + \nu$ is dominant in that time period.

Figure 5 is a plot of $|\zeta_{\Delta z'}(t, \lambda)|^2$, $\arg \zeta_{\Delta z'}(t, \lambda)$, $t = 0, \ldots, T-1$ for $\lambda/2\pi = .9294$, the Chandler frequency. The Chandler component is seen to be strong in the period 1910—1914 and very strong in the period 1948—1955.

Figure 5





The second graph suggests that
period 1925—1940 when the pow
and $\arg \zeta_{\Delta z}(t, \lambda)$ for $\lambda/2\pi = .91$
the seasonal component is seen t
period. Figure 7 is especially intere
for $\lambda/2\pi = .8460$, corresponding
The component at this frequency
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We now turn to the problem
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ı of the series $\Delta z(t)$ at several fre-
r real-valued series in Tukey (1961).]

$$\sum_{-|u| \leq L} \Delta z(u) \exp\{-iu\lambda\} \qquad (2.15)$$

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of temporal variations in the power
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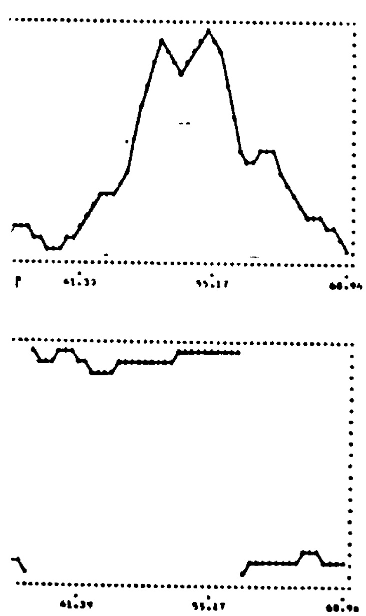
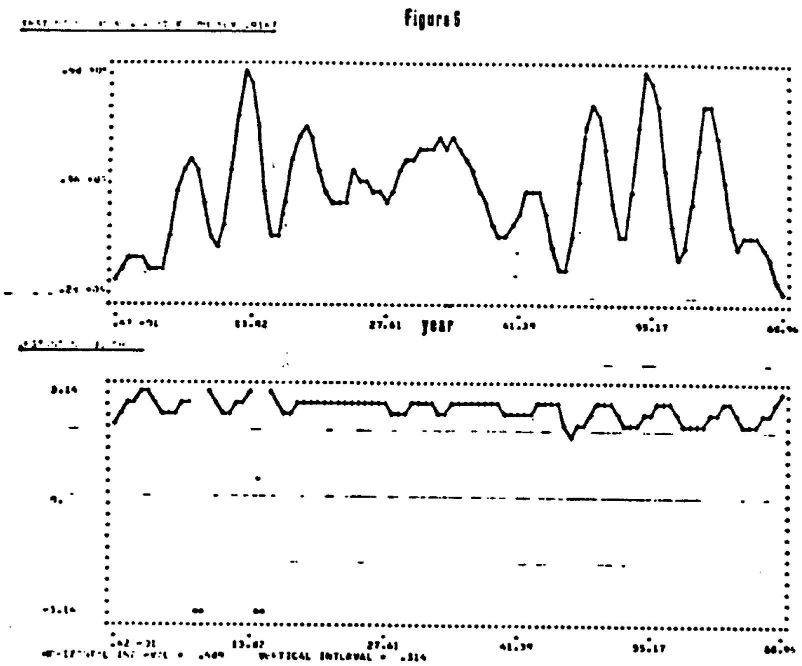
5



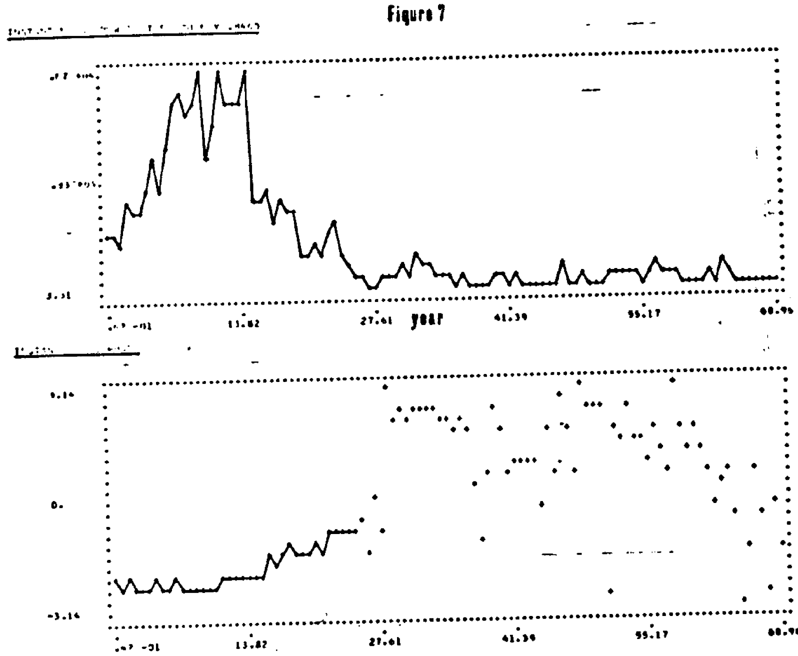


The second graph suggests that the Chandler frequency decreased in the period 1925–1940 when the power was low. Figure 6 is a plot of $|\zeta_{\Delta z}(t, \lambda)|^2$ and $\arg \zeta_{\Delta z}(t, \lambda)$ for $\lambda/2\pi = .9167$, the annual component. The power of the seasonal component is seen to be reasonably stationary across the whole period. Figure 7 is especially interesting. It is a plot of $|\zeta_{\Delta z'}(t, \lambda)|^2$, $\arg \zeta_{\Delta z'}(t, \lambda)$ for $\lambda/2\pi = .8460$, corresponding to the Chandler frequency plus the seasonal. The component at this frequency is seen to be present effectively only for the period 1905–1914. I have no explanation for this behavior. Perhaps it is due to a fault in the data processing for the early years. A perturbation analysis of the equations of motion leading to (2.2) suggests that non-linearities would lead to the appearance of harmonics of the seasonal, but not the appearance of a seasonal plus Chandler frequency component.

We now turn to the problem of discerning, if possible, the character of the excitation process $\Phi(t)$. An examination of expression (2.12), (with the parameter values of Table 2), as plotted in Figure 4 suggests that the true process $Z'(t)$ dominates only for frequencies in the immediate neighborhood of the Chandler frequency. It follows that we will have

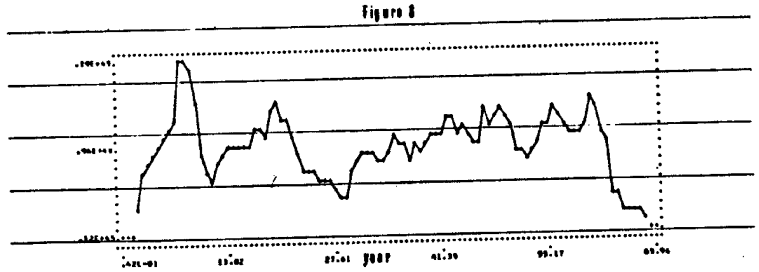
$$\zeta_{\Delta z'}(t, \lambda) \doteq \zeta_{\Delta z'}(t, \lambda) \doteq (i\lambda - a)^{-1} \zeta_{\Delta \Phi'}(t, \lambda) \qquad (2.16)$$

Figure 7



for, and only for, $\lambda/2\pi = .9294$. Figure 5 is therefore especially important in the search for the process, $\Phi(t)$, exciting the Chandler wobble. The instantaneous power of this process, at frequency .9294, must have behaved in the manner of the top graph of Figure 5, namely been high for the period 1910–1914 and very high for the period 1948–1955. I do not know of a process that has behaved in this manner. I would appreciate suggestions that anyone has. In the next sections I examine two processes that have been proposed.

Figure 8



3. Excitation by Earthquakes

Earthquakes were proposed at a
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The series is one of earthquake en
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We notice that the energy released

Figure 9 gives $|\zeta(t,\lambda)|^2$, arg ζ (
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Clearly the effect any earthquak
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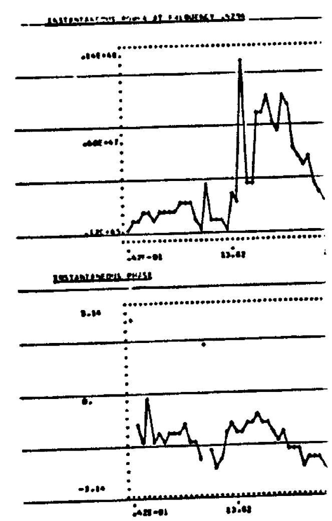
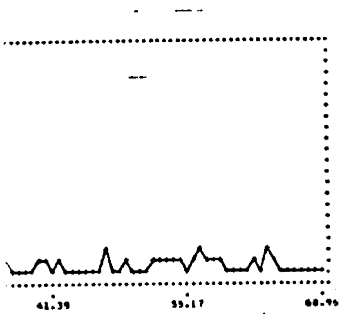
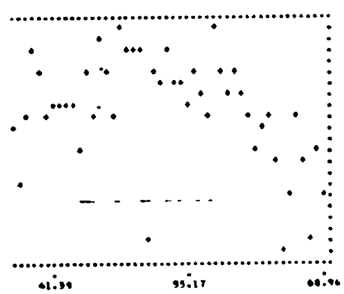
its movement. The above analysis
(1971, 1972) has developed expres







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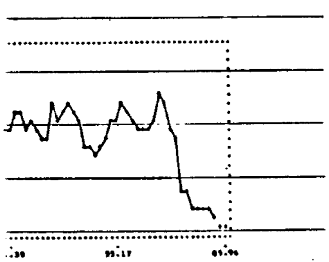
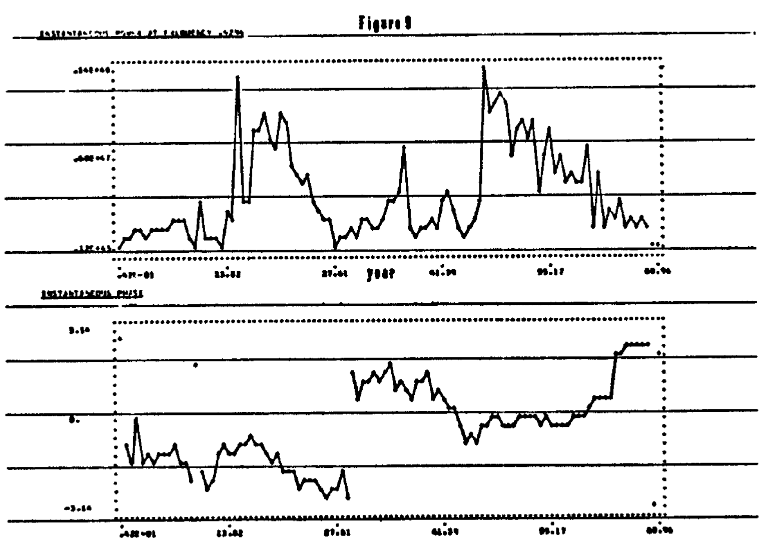


3. Excitation by Earthquakes

Earthquakes were proposed at an early time as a cause of the Chandler wobble [see Cecchini (1928)]. A number of serious investigations of this possibility have been carried out recently [see Mansinha et al. (1970), Dahlen (1971, 1972) for example]. We begin this section with an examination of a series of monthly earthquake energy. We computed such a series from values given in Dubourdieu (1972), which were based on the data in Duda (1965). The series is one of earthquake energy (in ergs) released per month by earthquakes of magnitude $\geqslant 7.0$ throughout the world in the period 1904—1965. Figure 8 is a plot of the square of an 8 year running average of this series. We notice that the energy released was greatest in the period 1904—1910.

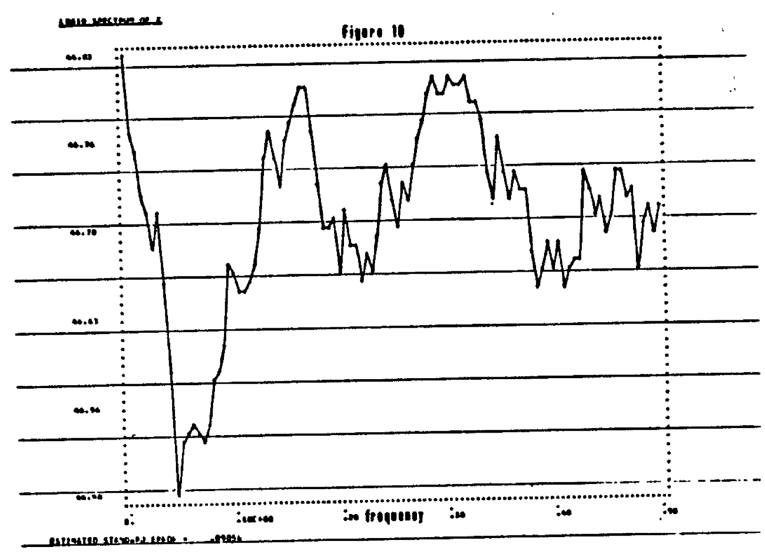
Figure 9 gives $|\zeta(t, \lambda)|^2$, $\arg \zeta(t, \lambda)$, $t = 0, \ldots, T-1$ for this series with λ corresponding to the Chandler frequency. The instantaneous power at this frequency is seen to be greatest for the periods 1914—1925, 1946—1954. A comparison of this plot with the top graph of Figure 5 suggests that the two do not match too well with respect to either shape or timing. Figure 10 is a graph of $\log_{10}$ of the estimated power spectrum of this series. The approximate standard error of this estimate is .09, suggesting that the population spectrum is not far from constant. The bandwidth is .05.

Clearly the effect any earthquake has on the motion of the pole will depend on the location of the earthquake within the Earth and on the direction of its movement. The above analysis takes no note of this dependence. Dahlen (1971, 1972) has developed expressions for the change in the Earth's inertia



tensor as a function of an earthquake's latitude, longitude, depth, strike, dip, slip and magnitude. Let θ_j denote all of these parameters for the j-th earthquake. Let τ_j denote the time of occurrence of the j-th earthquake. Dahlen (1972) has derived an expression for $C(\theta)$, the change in the Earth's inertia tensor for an earthquake with parameter θ. The excitation function $\Phi'(t)$ may now be written

$$\Phi'(t) - \Phi'(s) = \sum_{s < \tau_j \leq t} C(\theta_j) \qquad (3.1)$$



The model (2.11) therefore takes the form

$$z'(t) = Z'(t) + \varepsilon(t) \qquad (3.2)$$

with

$$dZ'(t) = a\,Z'(t) + d\Phi'(t) \qquad (3.3)$$

and $d\Phi'(t)$ given. This is a model of a linear causal relationship involving two observed processes, $z'(t)$, $\Phi'(t)$. If we assume that these processes have stationary increments, then we can carry out a frequency domain analysis of the processes in the manner of Section 4 of Brillinger (1970).

A difficulty presents itself at the beginning of the analysis. Not all of the components of θ are available for most of the earthquakes. After reading Dahlen (1971) and discussions with Professor B. A. Bolt, Professor T. V. McEvilly and W. Peppin, I have approached this difficulty as follows. Earthquakes tend to occur in belts along the edges of great plates on the Earth's surface [see Calder (1972)]. I constructed 11 strips at plate to plate boundaries

within which the majority of the e
map of Chase (1972). (I took as
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continental plates at the oceanic
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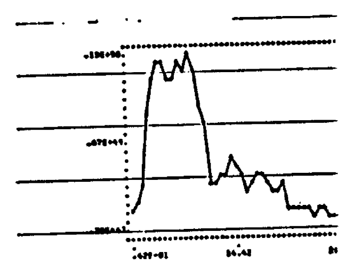

Figure 11 is a plot of the squar
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The horizontal line is an estima
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Figure 13 is a plot of the com
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compute instead

$$\xi_{\Phi'}(t, \lambda) = \frac{1}{\sqrt{2\pi(2L}}$$

Figure 13 has no semblance with
exciting the pole. Figure 14 is an
The bandwidth is .05 and appro
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(1970). Notice the low level of th

latitude, longitude, depth, strike,
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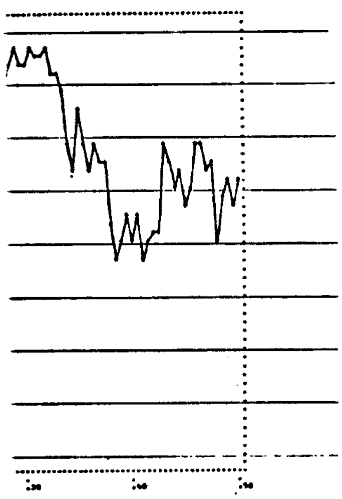
$$\sum_{<\tau_j<t} C(\theta_j) \qquad (3.1)$$



$$+\ \varepsilon(t) \qquad (3.2)$$

$$+\ d\Phi'(t) \qquad (3.3)$$

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strips at plate to plate boundaries

within which the majority of the events occurred (166 out of 187) using the map of Chase (1972). (I took as basic data the earthquakes of magnitude $\geqslant 7.9$ occurring in the period 1900–1971.) For the unknown components of θ, I then took the parameters suggested by the direction and overall motion of the plates. I read average strike angles from the map of Chase (1972). I used the dip angles of Davies and Brune (1971). I assigned slip angles by assuming that the oceanic plates were plunging at a 45° angle under the continental plates at the oceanic transform faults. The remaining 19 events had unknown parameters assigned at random.

Figure 11

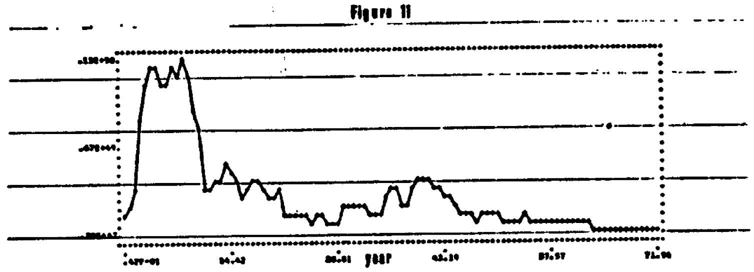


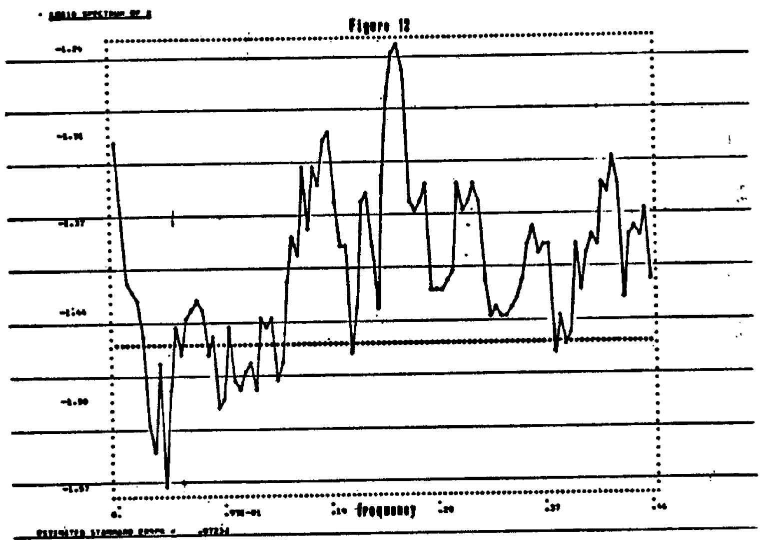
Figure 11 is a plot of the square of an 8 year running average of the energy released by earthquakes of magnitude $\geqslant 7.9$ for the period 1900–1971. Notice the large value of this function for the period 1904–1911. Figure 12 is an estimate of the $\log_{10}$ spectrum of the series of times of these events. The horizontal line is an estimate of the asymptote of the curve. The approximate standard error of the estimate is .072. The graph is suggestive that the corresponding population curve is near constant. (It would be constant for a stationary Poisson process.)

Figure 13 is a plot of the complex demodulate at the Chandler frequency of the process $\Phi'(t)$. Because of the point process with ancillary variate character of $\Phi'(t)$ we compute the demodulate differently from (2.15). We compute instead

$$\zeta_{\Phi'}(t, \lambda) = \frac{1}{\sqrt{2\pi(2L+1)}} \sum_{|t-\tau_j|<L} C(\theta_j) \exp\{-i\lambda\tau_j\} \qquad (3.4)$$

Figure 13 has no semblance with Figure 5, as it should were the earthquakes exciting the pole. Figure 14 is an estimate of the spectrum of the process $\Phi'(t)$. The bandwidth is .05 and approximate standard error .086. The Figure suggests that the population spectrum is near constant. The average level suggests a monthly excitation standard error of .74, far below the value 7.1 found from the polar coordinate data in Section 2. Figure 15 gives an estimate of the coherence between $z'(t)$ and $\Phi'(t)$ computed in the manner of Brillinger (1970). Notice the low level of the curve. The 95 per cent point of the approx-

Figure 12



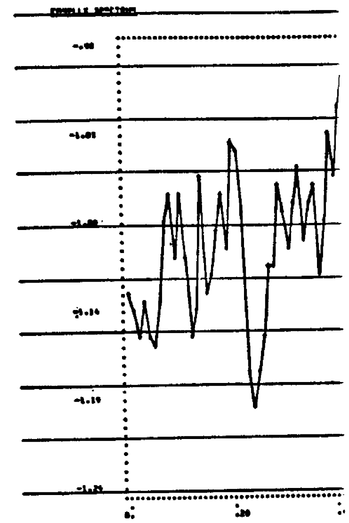
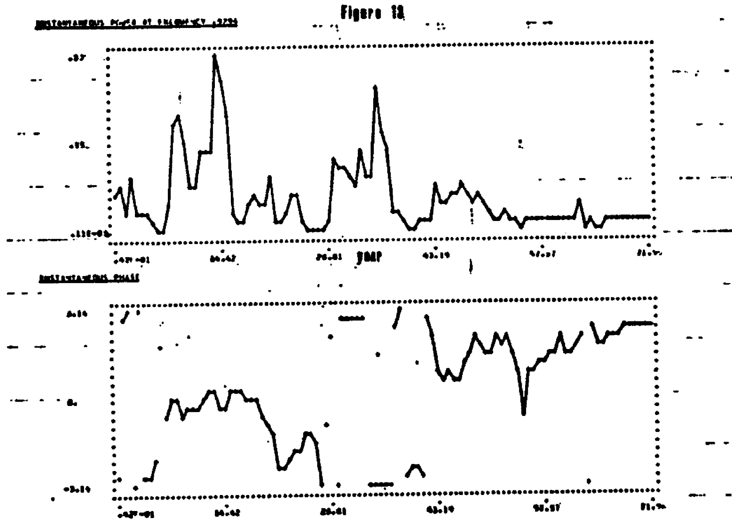
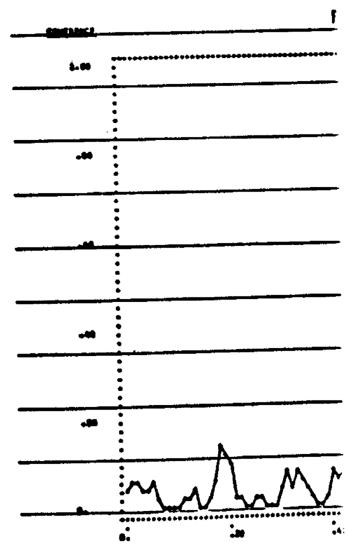


Figure 13





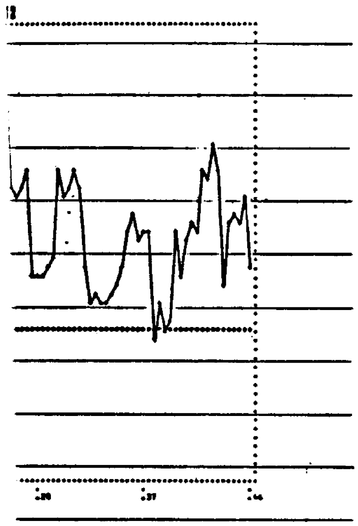


Figure 14

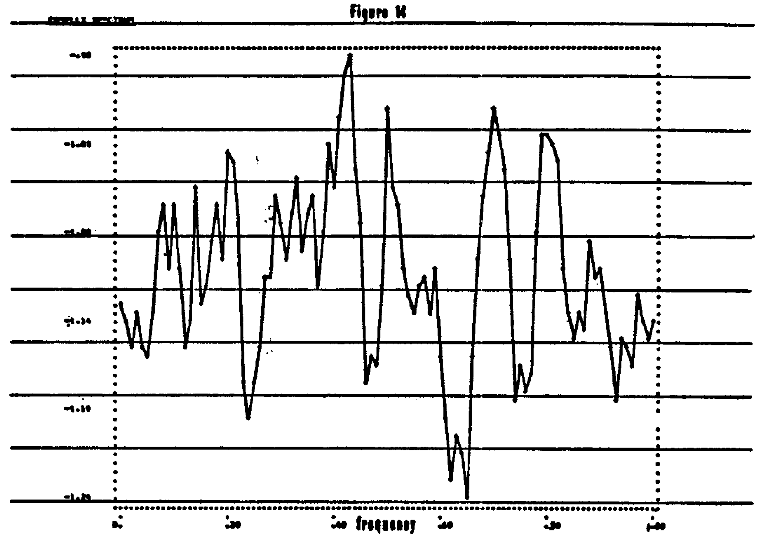
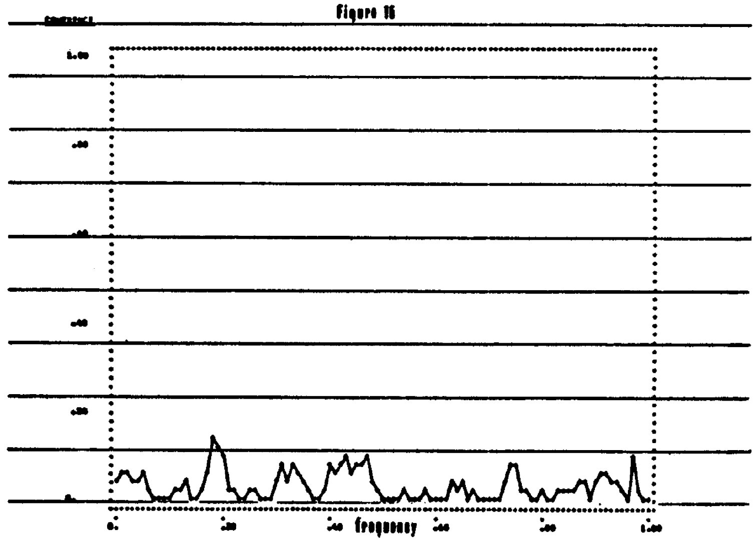


Figure 15

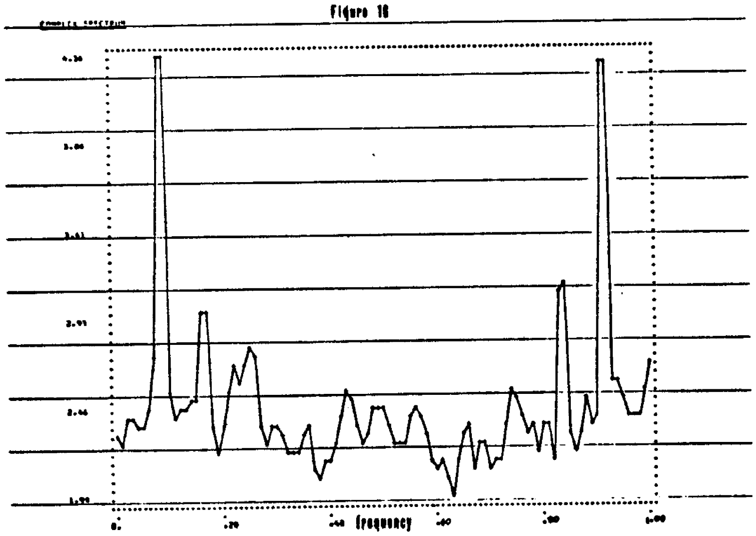


imate null distribution is .069. We have no evidence for a linear time invariant connection between the process $z'(t)$ and the computed series of earthquake effect. Perhaps the most telling thing against earthquakes being a principal cause of the Chandler wobble is an elementary comparison of Figures 11 and 2. Earthquake energy was high at the beginning of this century and has been trailing off since. The wobble amplitude has not been trailing off, in fact it reached its highest level around 1950.

4. Excitation by the Atmosphere

In 1901 Spitaler suggested that the seasonal component of polar motion was due to changes in the inertia tensor of the atmosphere. Hassan (1960) estimated the atmospheric product of inertia, $\psi(t)$, on a monthly basis for the period 1900—1950. Munk and Hassan (1961) carried out a cross-spectral analysis of this data with the polar motion. We carry out a further analysis here. Figure 16 gives $\log_{10}$ of the estimated power spectrum of this atmospheric data. The bandwidth of the estimate is .02. The approximate standard error of the curve is .13. The peaks appearing occur at the seasonal frequency and its harmonics. Figure 17 gives $|\zeta_\psi(t,\lambda)|^2$, arg $\zeta_\psi(t,\lambda)$ for λ at the seasonal frequency and $L = 48$. The instantaneous power is quite level after 1913. The phase is near constant also. The coherence between the series $\Delta z(t)$ and the series $\psi(t)$ is .88 at the seasonal frequency corresponding to polar motion in a negative direction. Figure 18 gives $|\zeta_{\psi'}(t,\lambda)|^2$, arg $\zeta_{\psi'}(t,\lambda)$ for λ the Chandler frequency. There is clearly not much power at this frequency, nor does its variation appear the same as that of the Chandler wobble. Figure 19

Figure 18



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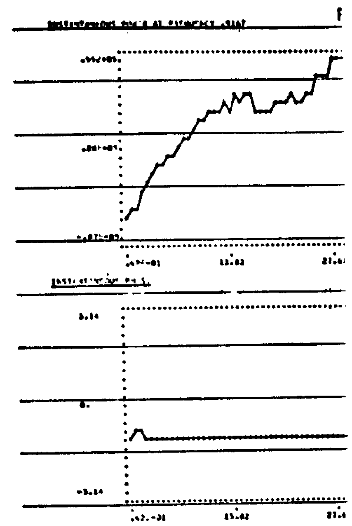

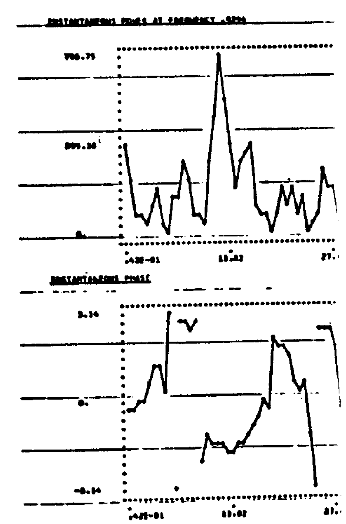

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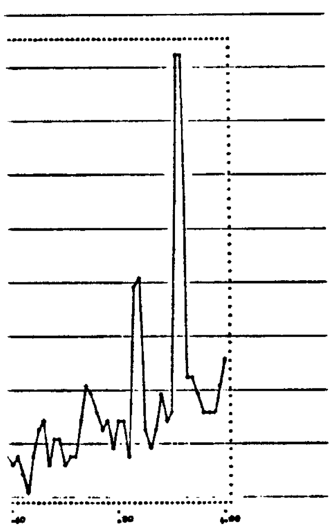


gives an estimate of the coherence between the process $\Delta z'(t)$ and the seasonally corrected process $\psi'(t)$. The bandwidth here is .05. The 95 per cent point of the null distribution is .095. We have no evidence to suggest that there exists a linear time invariant relation between the polar series and the atmospheric series at any but the seasonal frequency.

Figure 17

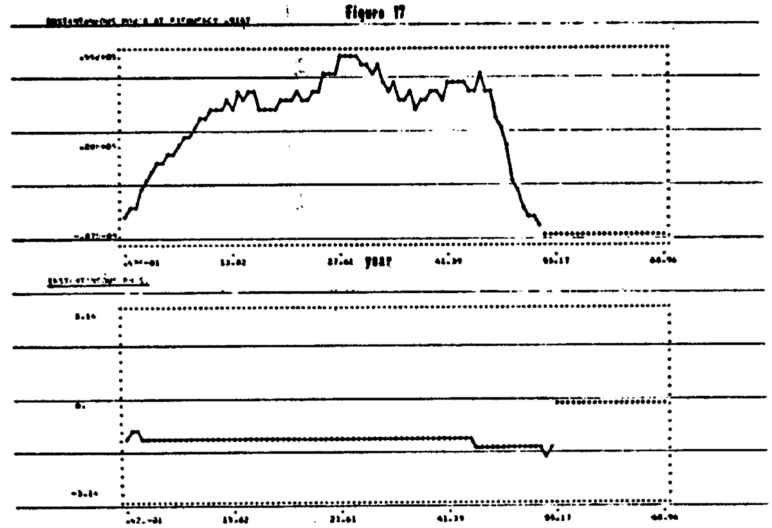


Fig. 18

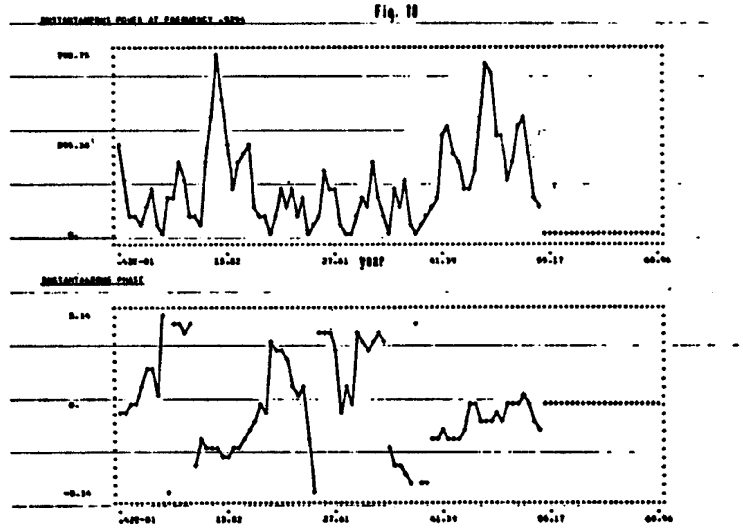
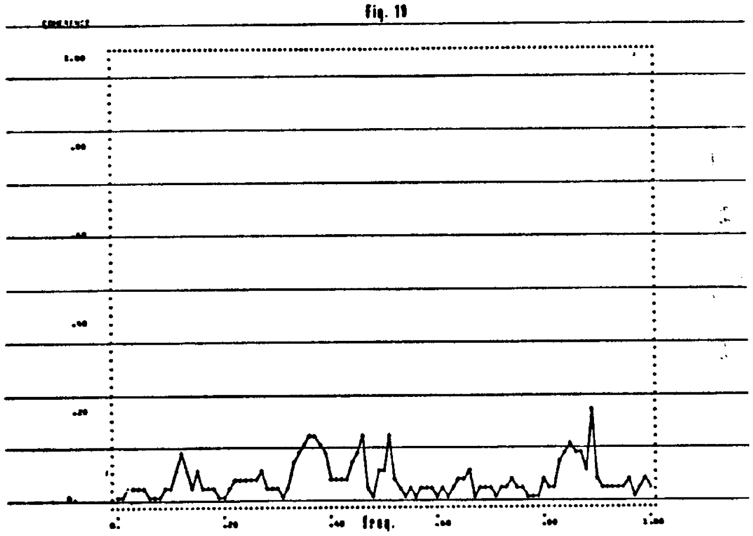


fig. 11



Acknowledgement

I would like to acknowledge helpful conversations concerning the material of Section 3 with Professors B. A. Bolt. T. V. McEvilly, D. Vere-Jones and Mr. W. Peppin. The research was carried out through the support of NSF Grant GP-31411.

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Appendix on Complex-Valued Proce

The simplest approach to the definitio process is through the spectral represent valued process with stationary increment

$W(t) = \int$

Then the spectrum $f_{W \ldots W, W \ldots W}(\lambda_1, \cdot$ comma and l after, is given by

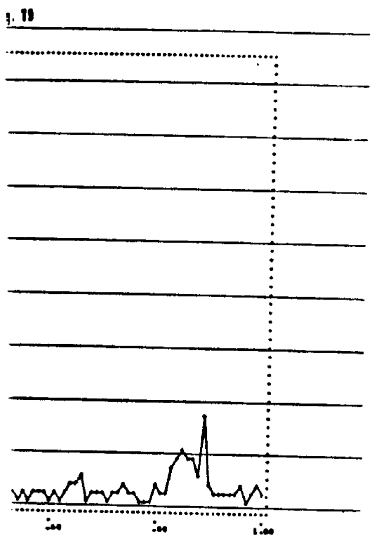
cum $\{dZ_W(\lambda_1), \ldots, dZ_W($

$\delta(\lambda_1 + \ldots + \lambda_k - \lambda_{k+1} - \ldots - \lambda_{l+k}$

In particular, the power spectrum of a ze

$E\, dZ_W(\lambda_1)\, \overline{dZ_W(\lambda_2)}$

Sinai (1963) discusses some aspects of



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Appendix on Complex-Valued Processes

The simplest approach to the definition of the spectral parameters of a complex-valued process is through the spectral representation. Let $W(t)$, $-\infty < t < \infty$, be a complex-valued process with stationary increments and spectral representation

$$W(t) = \int \frac{e^{i\lambda t} - 1}{i\lambda} dZ_W(\lambda)$$

Then the spectrum $f_{W \dots W, W \dots W}(\lambda_1, \dots, \lambda_{k+l-1})$, where there are k W's before the comma and l after, is given by

$$\text{cum } \{dZ_W(\lambda_1), \dots, dZ_W(\lambda_k), \overline{dZ_W(\lambda_{k+1})}, \dots, \overline{dZ_W(\lambda_{k+l})}\} =$$
$$\delta(\lambda_1 + \dots + \lambda_k - \lambda_{k+1} - \dots - \lambda_{l+k}) f_{W \dots W, W \dots W}(\lambda_1, \dots, \lambda_{k+l-1})\, d\lambda \dots d\lambda_{l+k}.$$

In particular, the power spectrum of a zero mean series is given by

$$E\, dZ_W(\lambda_1)\, \overline{dZ_W(\lambda_2)} = \delta(\lambda_1 - \lambda_2) f_{W, W}(\lambda_1)\, d\lambda_1\, d\lambda_2$$

Sinai (1963) discusses some aspects of the spectral theory of complex-valued processes.

Summary

The axis of rotation of the Earth does not remain fixed relative to the body of the Earth. Instead it has a motion composed of a movement with period 12 months and another movement with period 14.2 months (the Chandler wobble). The 12 month component appears to result from annual fluctuations in the loading of the Earth. The period 14.2 months corresponds to the fundamental frequency of vibration of the Earth. Scientific workers are not agreed upon the cause of the vibration however.

In this paper we use harmonic analysis to examine the possibility that either major earthquakes or annual fluctuations of the atmosphere are the cause. Our computations suggest that neither of these phenomena provides the source of the energy for the vibration.

Résumé

L'axe de rotation de la Terre ne reste pas fixe par rapport au globe terrestre. Son mouvement se décompose en deux composants: un mouvement de periode 12 mois et un autre mouvement de periode 14,2 mois (le mouvement de Chandler). La première composante provient des fluctuations annuelles dans la repatition des masses de la Terre. La deuxième composante correspond à la fréquence fondamentale de vibration de la Terre. Les chercheurs scientifiques ne sont pas d'accord sur le cause de la vibration a la fréquence fondamentale.

Dans cet article, nous utilisons l'analyse harmoniques pour examiner la possibilité que cette vibration proviendre de grands tremblements de terre ou de fluctuations annuelles de l'atmosphere. Nos calculs indiquent qu'auncune de ces séries n'est la source d'energie de la vibration.

DISCU

Discussant: P. Bloomfield, USA
Paper under discussion: Brillinger, An empiri

It is always a pleasure to see new work by I
It is especially interesting to see an example
tionary increments, and its use in detecting line
a technique which will allow the over-workec
many cases.

Two questions come to mind concerning
is about the representation of data as a comp
analysis of the data, such as the estimation
Furthermore, if these data do indeed follow a
valued-series is entirely appropriate. Howev
motion that there is probably considerable
that an analysis based on Hermitian quadrat
introduced by the measurement error. For
were correlated, this would not be detected.
effects are, in fact, present in the data.

The second point concerns the use of seco
methods are most easily understood when ap
they will be appropriate when used on a poi
tities, no one of which dominates the rest.
quakes on the Chandler wobble do, in fact
major earthquakes, such as the 1960 in Chile
information might be obtained by examinir
However, as Dahlen (1972) has pointed o
negative, or at least to show discrepancies b

Discussant: A. M. Walker, UK
Paper under discussion: Brillinger, An Emp

I much enjoyed reading this very impress
in the analysis which puzzled me, and I sho

The first relates to the model defined by e
with stationary orthogonal increments. Th
first order autoregressive continuous time
Walker and Young in their 1955 and 1957
the autocovariance function of $Z'(t)$ rather t
function of $\Delta Z'(t)$ led to the result

$$\frac{\sigma^2}{2\beta} e^{-\beta|u| + i\gamma u} \{$$

$$\frac{\sigma^2}{2\beta} \{e^{-2\beta} + (e^{-\beta}$$

Moreover, the discrete time process $\{Z'(t$
it would be natural to fit the model to va
I must somehow have misunderstood Dr. B
for some explanation.

The second point concerns the equatior
citation function $\Phi(t)$ is such that its inc

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