Joint Model for Exchange Rate Dynamics and Influence of Presetting Correlation between Stock Price and Exchange Market

Jian Wang

University of California, Berkeley

wj_wangjian1992@berkeley.edu

I. INTRODUCTION:

American Depository Receipts (ADRs) are securities issued by non-US companies to cross list their stocks on both domestic and foreign stock exchanges. In addition to stocks, there are also options traded in these markets denominated in their respective currencies. As such, ADR options are exposed to foreign exchange rate risk. In the summer research in Columbia University we studied the implied volatilities of options issued by a non-US company and their US traded ADR counterparts. Under the stochastic assumption of exchange rate we applied the joint model for implied volatility and exchange rate to explain the options price discrepancies. For each company by mean square minimization we extracted the exchange rate implied volatility, which turned to be pretty effective and could catch the clue of currency depreciation up to three months in advance. In the dynamic model, the correlation between stock price and exchange rate was a critical factor determining the exchange rate implied volatility and we maximized the mean square error over the exchange rate volatility and correlation. Yet the implied correlation was quite unstable and if we constrained the correlation between -1 and 1 we actually got lots of extreme implied correlation, which was not realistic. In the independent research this semester, we can examine the sensitivity of implied exchange rate volatility respect to correlation. By reasonable preset of the correlation, the expression of implied volatility becomes explicit for direct analysis. There are three different measurements of correlation, dynamic correlation over a short period, correlation as regression line over long period and industrialized correlation to exchange rate. Though different measurement of correlation between foreign stock market and exchange rate caught the similar trend, they gave significant different levels of implied exchange rate volatility. By examining the results of three measurements of correlation, we can compare the advantages and limitations for these methods. The study improves the model we built in the summer to extract foreign exchange rate implied volatility and provides a procedure to better understanding the perceived exchange rate risks for different stocks across exchanges.

II. JOINT MODEL FOR IMPLIED VOLATILITY AND EXCHANGE RATE

Theoretical background[1]:

In the whole research, for an ADR option, we deal with the foreign stock struck in domestic currency (here USD)[2]. The dynamic model of the entire economy, under objective measure P, is as follows

$$dX = X\alpha_X dt + X\sigma_X d\overline{W}$$

$$dS_d = S_d \alpha_d dt + S_d \sigma_d d\overline{W}$$

$$dS_f = S_f \alpha_f dt + S_f \sigma_f d\overline{W}$$

$$dB_d = r_d B_d dt$$

$$dB_f = r_f B_f dt$$

where

$$\overline{W} = \begin{matrix} \overline{W}_1 \\ \overline{W}_2 \\ \overline{W}_3 \end{matrix}$$

Is a three-dimensional Winner process (with independent components)

For the foreign call struck in domestic currency, the claim, expressed in domestic term, is given by

$$Z^d = \max[X(T) * S_f(T) - K, 0]$$

Use the Black-Schoels formula we can obtain the price function and the implied volatility is given by

$$\|\sigma_f + \sigma_X\|$$

where σ_f is the foreign stock volatility and σ_X is the foreign exchange rate volatility

by no-arbitrage argument, the implied volatility of foreign stock struck in domestic currency should equal to the implied volatility of domestic options σ_d

$$\|\sigma_f + \sigma_X\| = \sigma_d$$

[1] *Detailed proof refers to Bjoerk book Arbitrage Theory in Continuous Time Chapter 12*
[2] *See Appendix 1 for all the ADRs used in this project*

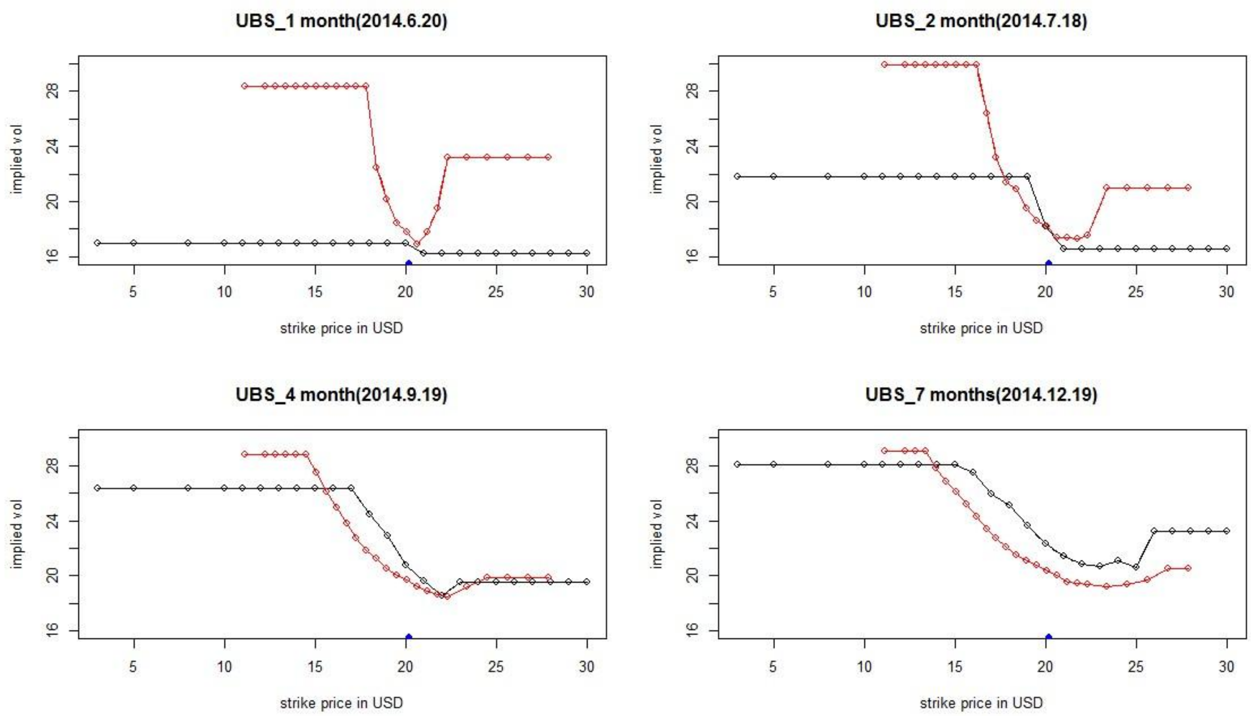
if we denote ρ as the correlation between σ_f and σ_X, the equation is

$$\sqrt{\sigma_f^2 + \sigma_X^2 + 2\sigma_f\sigma_X\rho}=\sigma_d$$

And we can get the implied exchange rate volatility $\widehat{\sigma_X} = \sqrt{\sigma_f^2\rho^2 - (\sigma_f^2 - \sigma_d^2)} - \sigma_f\rho$

Mean Square minimization[3]:

In practice, given a stock for a specific day, there are many option choices with different strike price and expiration date. Notice the implied volatilities have a smile curve. After converting the strike price of foreign options into domestic price, we can compare the implied volatilities of foreign options and domestic options for the same stock. Here we use UBS as illustration[4].



The data is from Bloomberg on May 28th .The implied volatility from emerging market is shown in red and the implied volatility from the ADR market is shown in black. The blue dot points the current spot price of ADR.

We see the implied volatility differs a lot in some extreme strike price, which is actually not a problem since in reality those options are not traded in the market. We only need concern about the implied volatility of options with the strike around the spot price.

[3] *Corroborative work with Professor Tim Leung and Connie Lee in Columbia University in summer 2014*

[4] *See Appendix 2 for more examples and illustrations of implied volatility curves*

As expected, the curve of implied volatility in foreign market is higher than that in domestic market (U.S. market), since the ADRs in U.S. market also carry the currency risk that increase the options price.

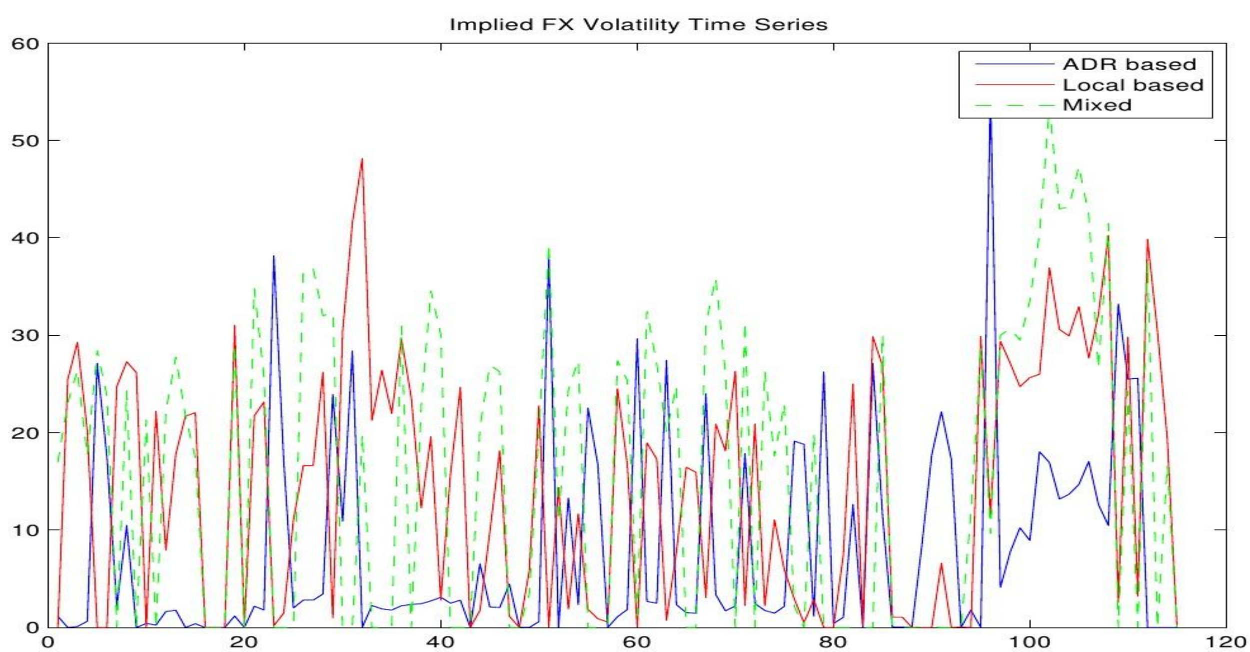
We aim to find $\hat{\rho}$ and $\widehat{\sigma_X}$ that minimize the mean square error term between two curves

$$(\hat{\rho}, \widehat{\sigma_X}) = \operatorname{argmin} \sum_{i=1}^{n} ({\sigma_f}^2 + {\sigma_X}^2 + 2\sigma_f\sigma_X\rho - {\sigma_d}^2)_i$$

subject to $-1 \leq \rho \leq 1;\ \sigma_X \geq 0$

where n is total number of points of foreign option strike price struck in USD, σ_{fi} is foreign implied volatility for ith strike price expressed in USD and σ_{di} is the ADR implied volatility of the same strike price computed by interpolation.

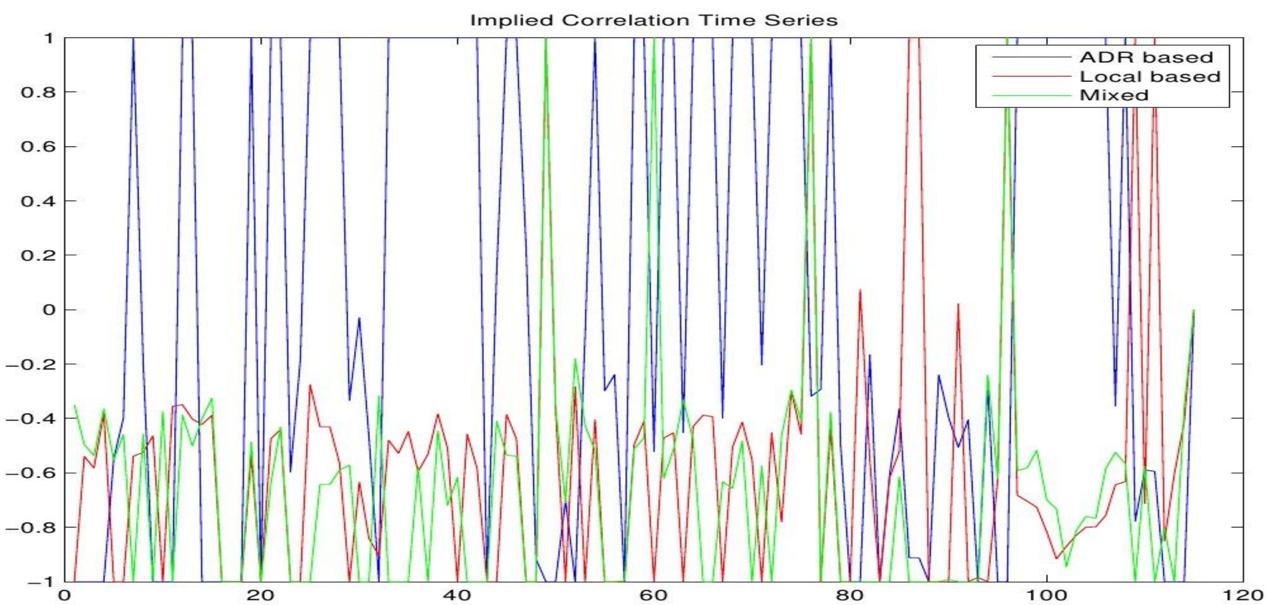
In practice since the strike prices are not exactly the same and we need interpolate either σ_{fi}



(ADR based methods) or σ_{di} (local based methods) or mix them weighted by the number of points. Here we examine the implied exchange rate volatility from the implied volatility disparity (data from 2014/01/02 to 2014/05/29 with expiration date 2014/12/15) of Deutsche Bank (DB in ADR market and DBK in Germen market). The time series of σ_X gave us a general impression of exchange rate dynamics.

Due to the constraint of σ_X very rarely the optimal σ_X is 0 and we can just interpret it as the very low exchange rate volatility. In general the analysis gave us a good impression about the implied exchange rate volatility, from which we can know people's expectation about the exchange rate in advance from the options implied volatility disparity.

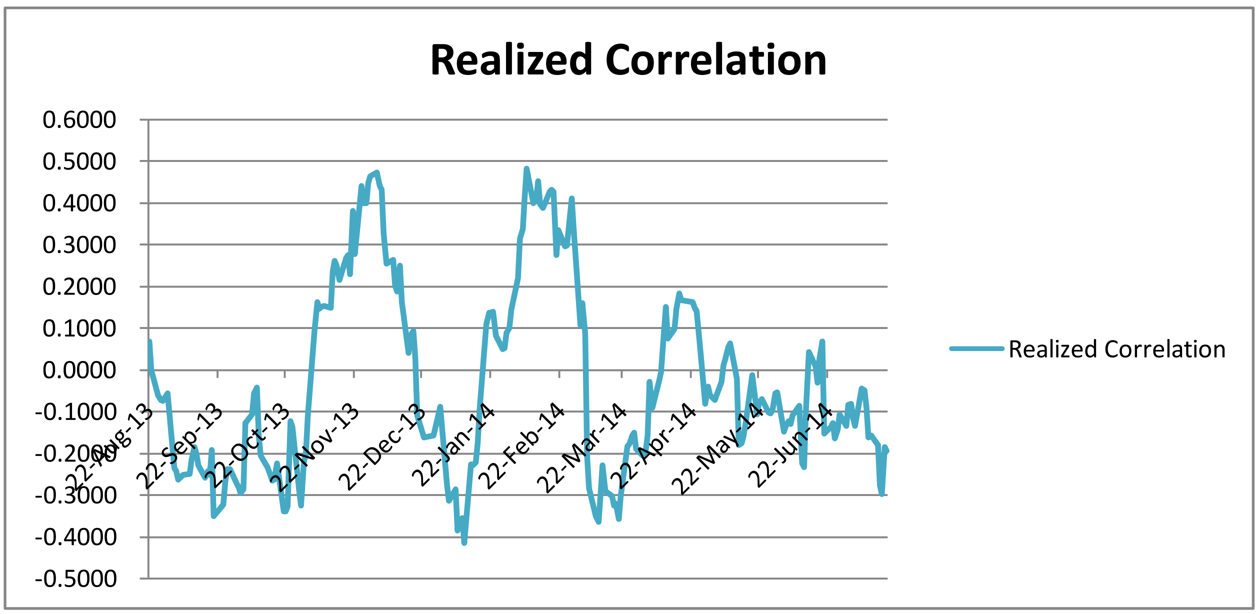
The similar analysis of implied correlation between foreign stock prices and exchange rate is comparatively less significant.



Due to the constraint of correlation in the minimization problem there is lots of extreme value as optimal correlation especially for the ADR based methods. First the extreme value lost its statistical significance since the optimal value of σ_X here may be far from correct implied value without the constraint of correlation. In addition we should look at the nature of implied correlation and implied exchange rate volatility. People's expectation could be quite fluctuated in respond to the information in exchange market, which corresponding the time series of σ_X. Yet the other factor, the implied correlation between the foreign stock and exchange rate represented people view about the correlation, which is hard to change dramatically during a short time period. The relative stable implied correlation and fluctuate exchange rate volatility is more reasonable. Thus to understand the implied exchange rate dynamics we need further exploration about the other factor, the implied correlation between foreign stock price and exchange rate.

III. SENTIVITY OF IMPLIED EXCHANGE RATE VOLATILITY TO CORRELATION

The historical correlation between stock price and exchange rate actually is not stable. Here we take DBK and EUR as illustration



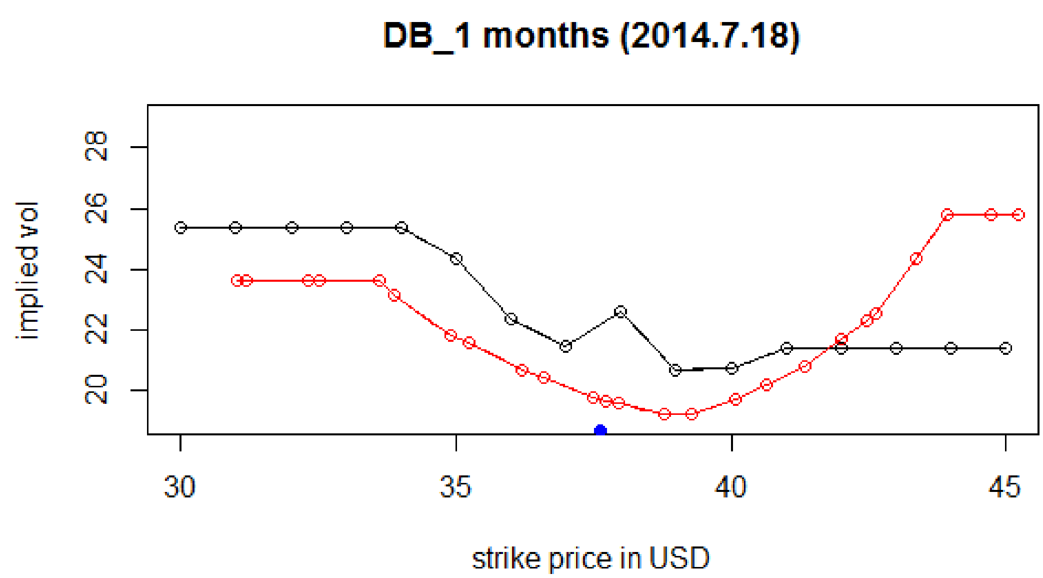
The historical correlation fluctuate from -0.5 to 0.6, which is a very broad range of correlation. Thus we need to examine how sensitive of ρ to correlation.

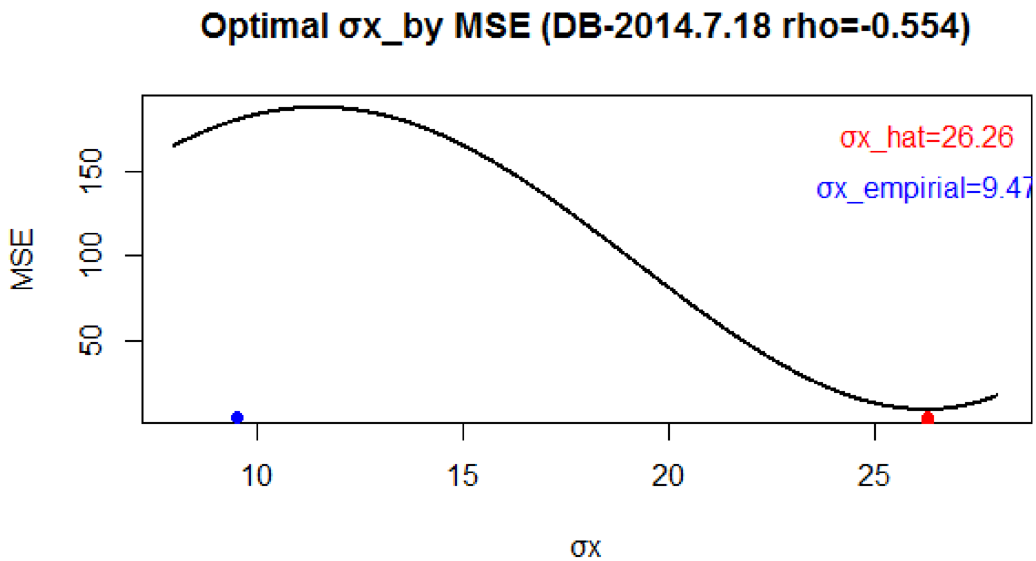
To see how the σ_X is affected by rho I apply the same methods of MSE to DBK and find the implied volatility of exchange rate but for different rhos obtained from different time scale.

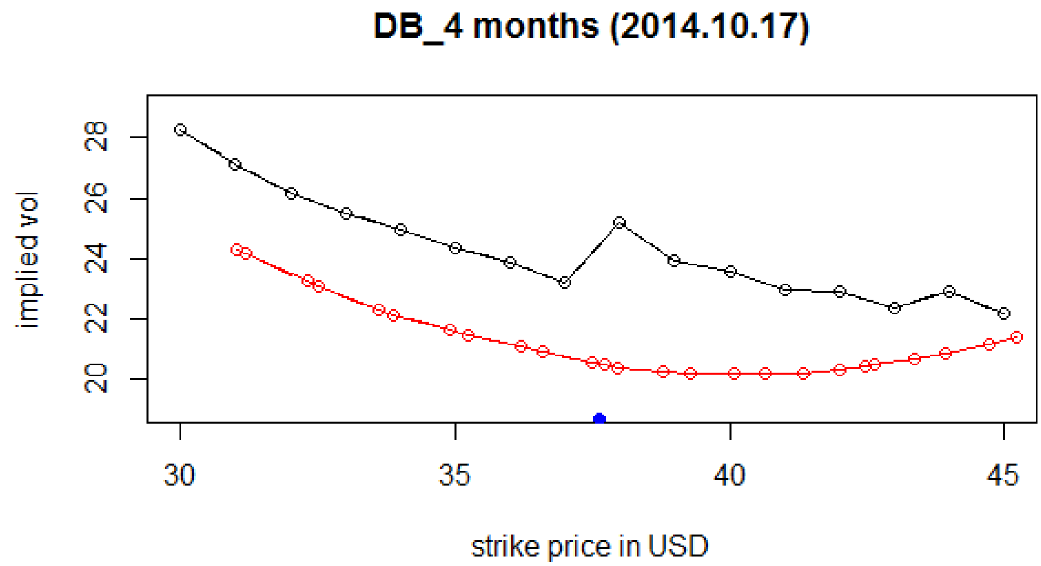
	Rho=-0.18 (1w)	-0.62 (2w)	-0.52 (1m)	-0.55 (2m)	-0.32 (3m)	-0.14 (4m)	-0.06 (6m)	-0.005 (1y)	0.30 (2y)
2014.07.18	13.11	28.01	24.32	25.29	17.56	12.14	10.16	8.91	4.61
2014.10.17	15.74	30.62	26.91	27.88	20.16	14.77	12.77	11.48	6.64
2015.01.16	16.56	31.41	27.71	28.67	20.97	15.59	13.58	12.28	7.31
2016.01.15	18.50	33.88	30.04	31.04	23.07	17.49	15.40	14.03	8.66

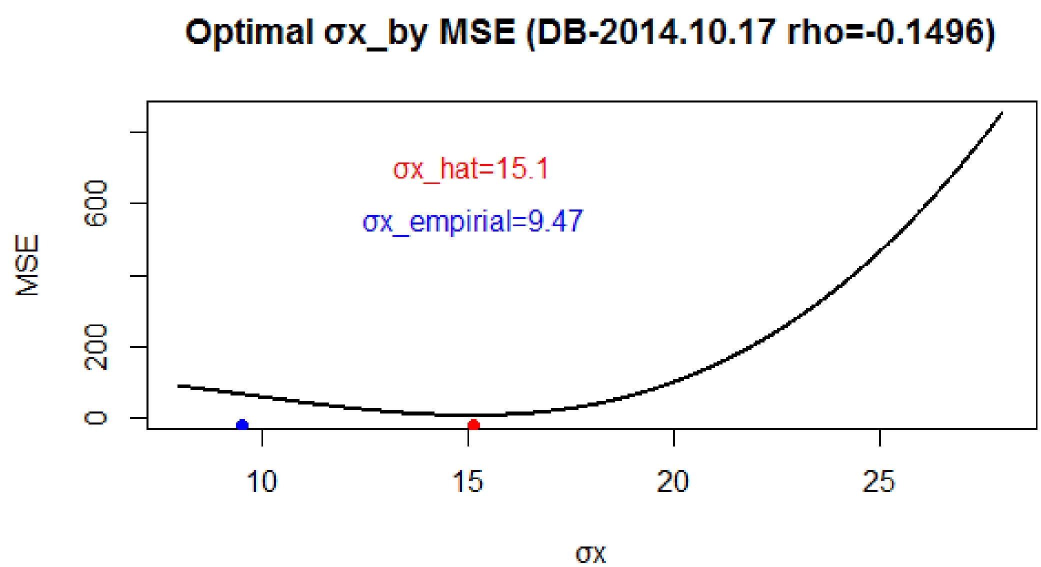
σ_X from empirical data for last two years:
9.47

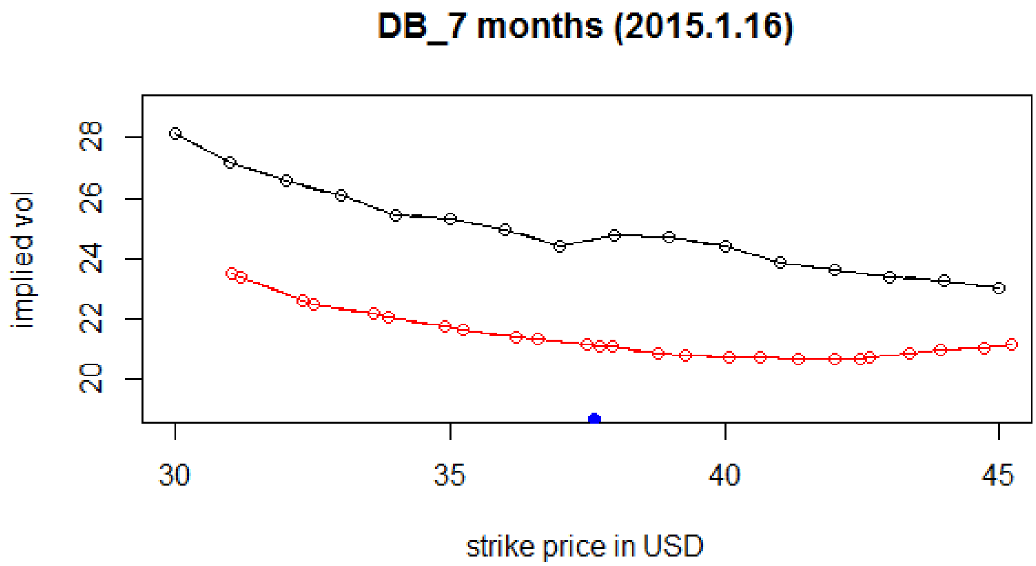
The σ_X is actually quite sensitive to rho so we need choose the proper rho carefully. One possible solution is to use the rho from same length of maturity (i.e. if we calculate σ_X of options with three month maturity, we use the rho from past three months)

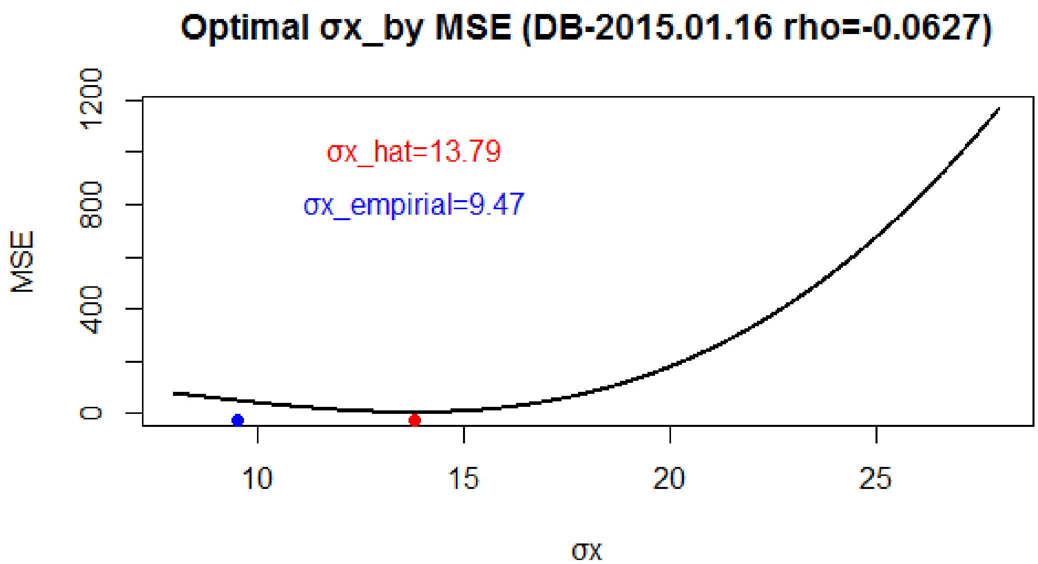


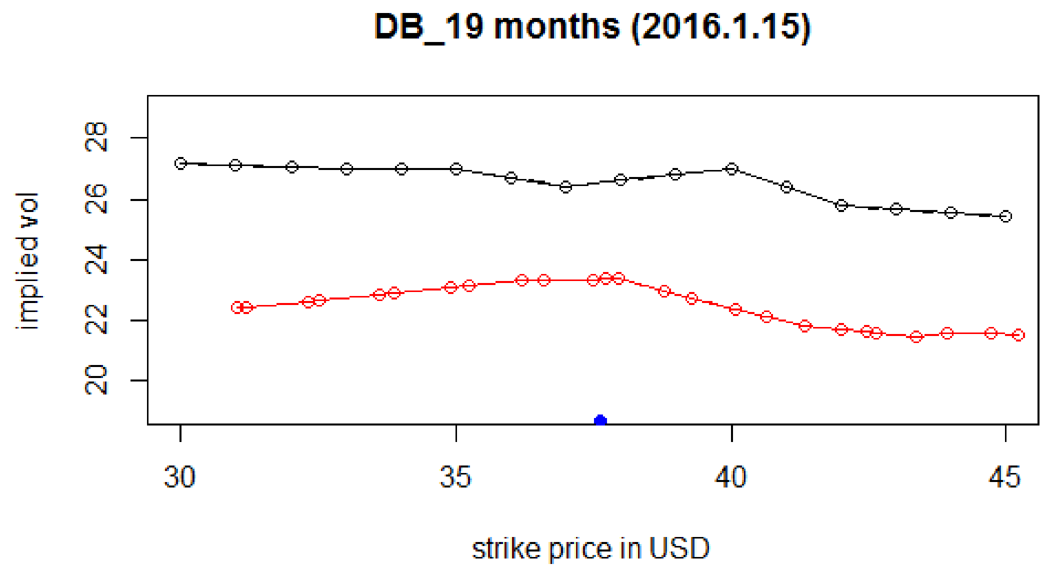

Optimal σx_by MSE (DB-2014.7.18 rho=-0.554)
σx_hat=26.26
σx_empirial=9.47
MSE
50
100
150
10
15
20
25
σx

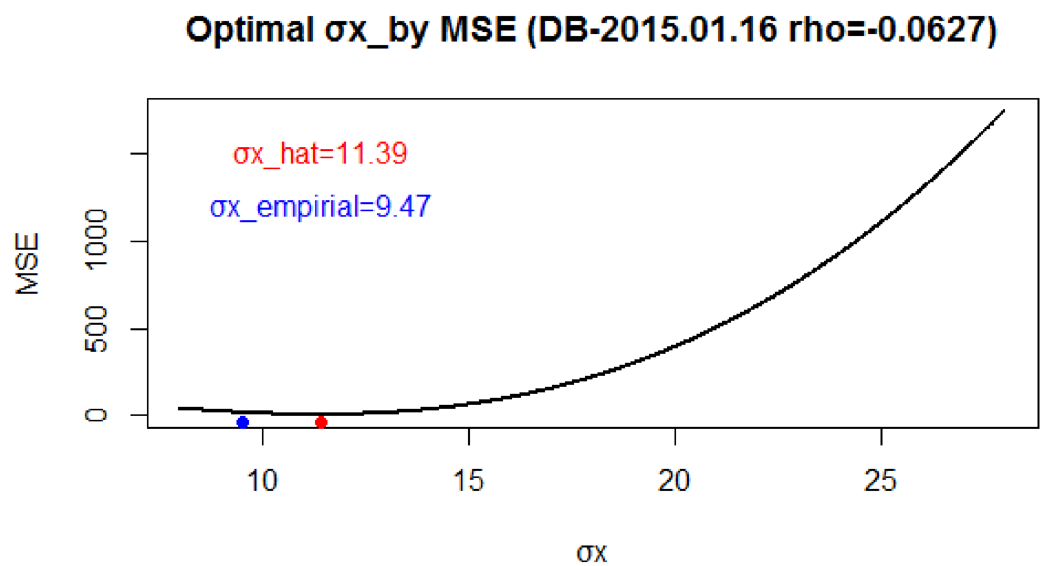

DB_4 months (2014.10.17)
implied vol
20
22
24
26
28
30
35
40
45
strike price in USD


Optimal σx_by MSE (DB-2014.10.17 rho=-0.1496)
σx_hat=15.1
σx_empirial=9.47
MSE
0
200
600
10
15
20
25
σx


DB_7 months (2015.1.16)
implied vol
20
22
24
26
28
30
35
40
45
strike price in USD


Optimal σx_by MSE (DB-2015.01.16 rho=-0.0627)
σx_hat=13.79
σx_empirial=9.47
MSE
0
400
800
1200
10
15
20
25
σx





IV. DIFFERENT MEASUREMENTS OF CORRELATION

Instead of minimizing the mean square error over σ_X and ρ we can preset the correlation and calculate the implied exchange rate volatility. In the original methods, the implied correlation is extremely unstable and as the analysis above it could significantly influence the optimal exchange rate volatility. By presetting the correlation, the minimization problem is simplified to the quadratic minimization

$$\widehat{\sigma_X} = \text{argmin} \sum_{i=1}^{n} (\sigma_f^{\,2} + \sigma_X^{\,2} + 2\sigma_f \sigma_X \rho - \sigma_d^{\,2})_i$$

We can give the explicit formula for $\widehat{\sigma_X}=\frac{\sum_{i=1}^{n}\sigma_f\rho}{n}$

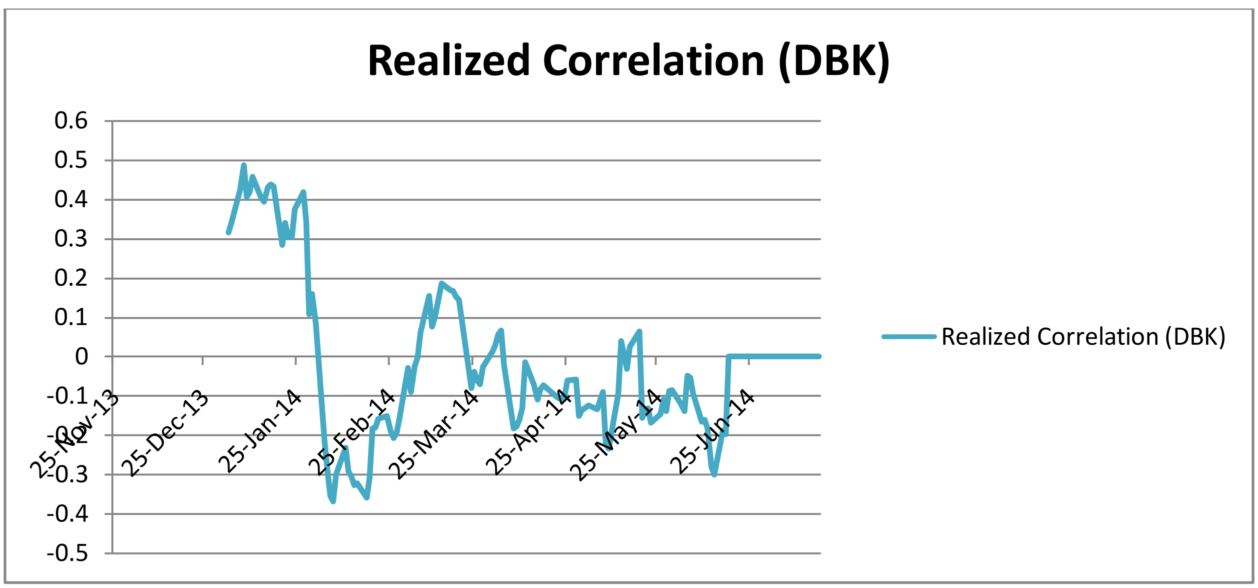
From the explicit formula the implied exchange rate volatility is determined by the foreign implied volatility and correlation between foreign stock and exchange market. The fluctuated foreign stock and s strong positive correlation between the stock and exchange market correspond the larger implied exchange rate volatility. By presetting the correlation, the pattern of implied exchange rate volatility becomes more clear and simple.

The other important parity also came from the explicit formula. The implied exchange rate volatility represents people's expectation about the volatility of exchange rate and only depends on the information on the macroeconomic factors. In other words, the ADRs implication is the approach to get the implied exchange volatility and theoretically using different ADR stock we should get the same implied volatility. Back to the formula, which says $\widehat{\sigma_X}$ is equal to the average of product of stock price implied volatility and its correlation with exchange rate for different stocks. In this project the parity is used to test the result of $\widehat{\sigma_X}$. One potential application is to compare the implied correlation for different stocks if we know the implied exchange rate volatility, which may be further studied in the future.

Finally we can use three different measurements of correlation

Correlation as time series over a short period

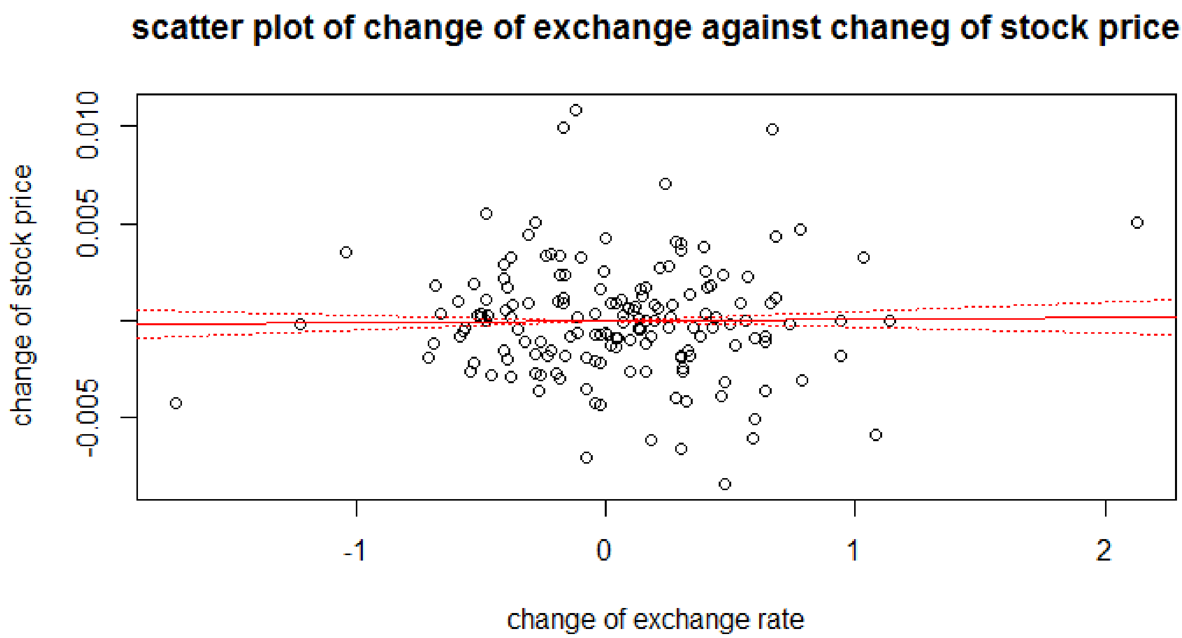
As discussed previously, we can use the realized correlation as the measurement of implied correlation since people's view of correlation is relatively stable.



The realized correlation from 2014.01.02 to 2014.07.18 is within the range between -0.4 and 0.5. Compared to the implied correlation we got from minimizing both σ_X and ρ, the change of realized correlation changes steadily.

Correlation as regression over long period

Another measurement of correlation is the average correlation over long period and we can regard the correlation as coefficients of regression of change of exchange rate against stock price. Although compared to the first measurement it's not dynamic, instead we can get the confidence interval of the correlation, which will gave us the range of the implied exchange rate volatility.

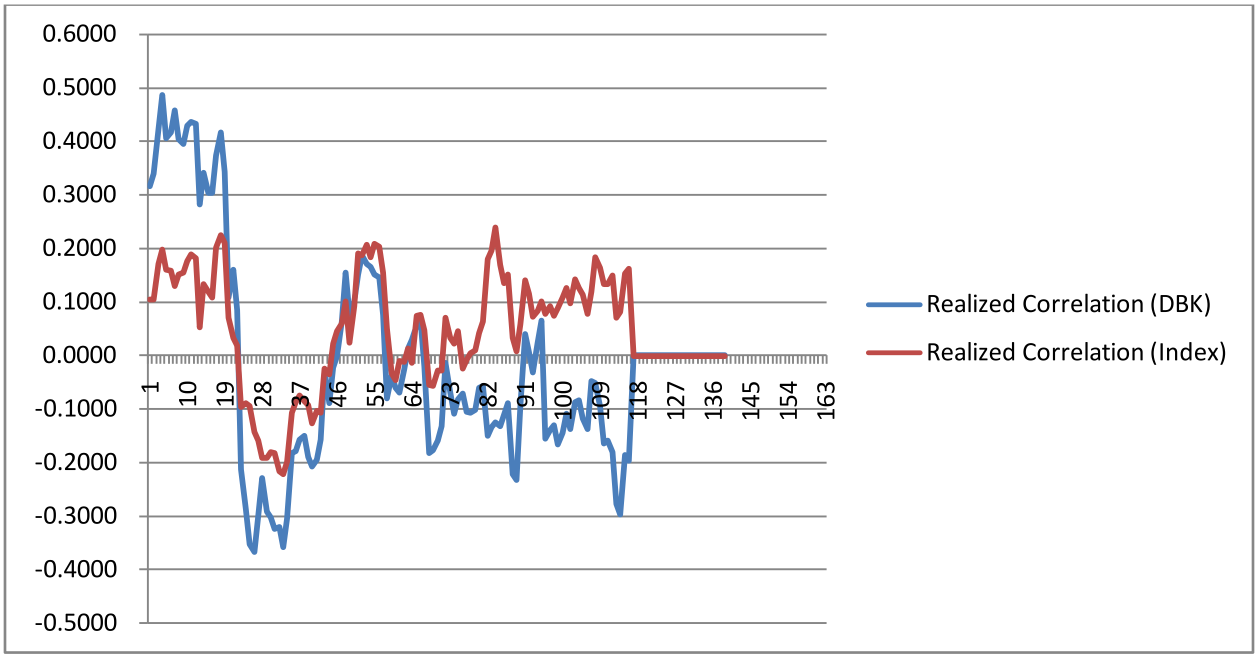



```
lm(formula = diff(DBK[, 2]) ~ diff(DBK[, 1]))

Residuals:
      Min        1Q     Median        3Q       Max
-0.0084750 -0.0018027 -0.0001058  0.0012873  0.0109077

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    -1.713e-05  2.347e-04  -0.073    0.942
diff(DBK[, 1])  8.777e-05  4.956e-04   0.177    0.860

Residual standard error: 0.002961 on 159 degrees of freedom
Multiple R-squared:  0.0001972,	Adjusted R-squared:  -0.006091
F-statistic: 0.03136 on 1 and 159 DF,  p-value: 0.8597
```

The standard deviation of the slope is pretty small and thus we have a narrow confidence interval of the correlation as shown in the dashed line. The long-run correlation between stock price and exchange rate is approximate zero with a very tiny range.

Industrialized Correlation for Different Time Scale

Since the implied exchange rate volatility represent the fluctuation of exchange rate and should be independent of our choice of specific stocks, we can also looked up for the aggregate stock market index. Here we can preset the correlation as the correlation between the bank industrial index (KBW bank index)[5] and exchange rate.



From the Comparison of realized correlation of index and realized correlation of a specific stock (DBK here), as expected, they follow the similar trends over time and the correlation of index is more stable.

V. CONCLUSION

The exchange market has various forms and is among one of the most volatile and unpredictable market in the worlds. In addition the unpredicted currency depreciation could cause serious impact on the whole financial market. For instance, the Brazilian Real lost its value up to 10% in the recent currency crisis in South America from early 2014. We can find the way to predict the potential increasing volatility of exchange rate from the implied volatility

[5] *See Appendix 3 for definition and explanation for KBW Index*

discrepancy of ADR options in U.S. market and home market. After the financial crisis in 2008, the U.S. stock market yield a very low return and investors turned to foreign market especially for emerging market like Brazil for higher return rate. Thus the portion of ADRs increased significantly and the options market in such countries has rapidly developed, which gave us the more adequate and statistically significant data. In the project, based on the result of summer research collaborated with Professor Tim Leung and Connie Lee, I do more sensitivity analysis for the implied exchange rate volatility respect to correlation between foreign stock market and exchange rate. By presetting the correlation, the expression for implied exchange rate volatility becomes simpler and explicit. There are three different measurements of correlation, short term dynamic correlation, long-term correlation as regression and correlation with industrialized index. In practice we can combine the measurements and get the more comprehensive impression about correlation and prediction of implied exchange rate volatility.

The connection between ADR options volatility discrepancy and exchange rate can be further explored. In the project we choose the bank industrial as example since the banks are more sensitive to the exchange rate. Since the implied volatility of exchange rate is industry independent we can possibly examine from other industrial with different sensitivity. All in all, the methods connected the ADR market and exchange market and provided a good insight of exchange market dynamics. It’s possible to make further and deeper exploration in the future.

REFERENCE:

[Tomas Bjork, 1998] Arbitrage Theory in Continuous Time chapter 12 Currency Derivatives p167-181

[Stefan Eichler, Dominik Maltritz, 2008] Currency Crisis Prediction Using ADR Market Data-An Option-Based Approach, *International Journal of Forecasting 26*

[Stefan Eichler, Alexander Karmann, Dominik Maltritz, 2009] The ADR Shadow Exchange Rate as an Early Warning Indicator for Currency Crisis, *Journal of Banking and Finance 33*

[Dilip B. Madan, 2012] Joint Modeling the Prices of American Depository Receipts, the Local Stock and the U.S. Dolalr, *Journal of Investment Strategy Volume1/Number 4*

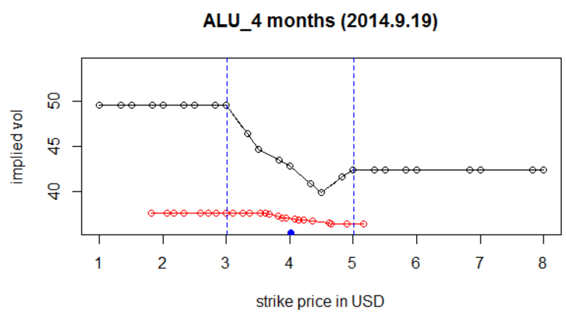
[Louis Ederington, Wei Guan, 2005] The Information Frown in Option Prices, *Journal of Banking and Finance 29*

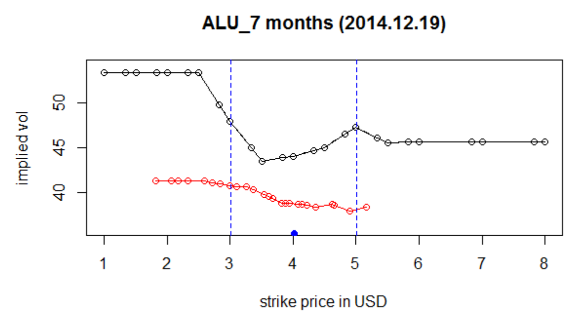
Appendix 1

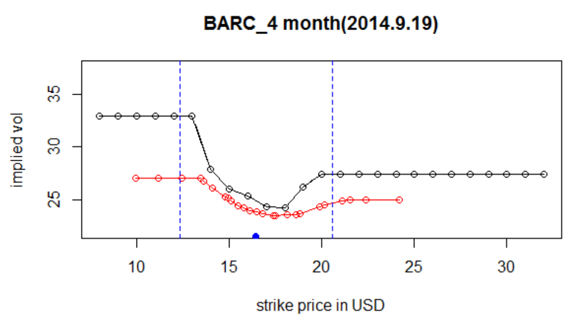
Name	Country	Stock symbol in US	Stock symbol in original country	Ratio DR:ORD	Foreign currency	Industry
UBS	Switzerland	UBS	UBS	1:1	CHF	Bank
Barclay	British	BCS	BARC	1:4	GBX	Bank
Santander	Spain	SAN	SAN	1:1	EUR	Bank
GlaxoSmithKline	British	GSK	GSK	1:2	GBX	healthcare
Novartis	Switzerland	NVS	NOVN	1:1	CHF	healthcare
TEVA	Israel	TEVA	TEVA	1:1	NIS	healthcare
British Petroleum	British	BP	BP	1:6	GBX	Oil and gas
Royal Dutch Shell	British	RDS	RDS	1:2	GBX	Oil and gas
Alcatel-Lucent	France	ALU	ALU	1:1	EUR	telecommunication
America Movil	Mexico	AML	AML	1:20	MXN	telecommunication
China Mobile	China	CHL	941	1:5	HKD	telecommunication
Cemex	Mexico	CM	CEMEX	1:10	MXN	Building matrials
BHP Biliton	Australia	BHP	BHP	1:2	AUD	Metal and mining
Nokia	Finland	NOK	NOK1V	1:1	EUR	Tech equipment

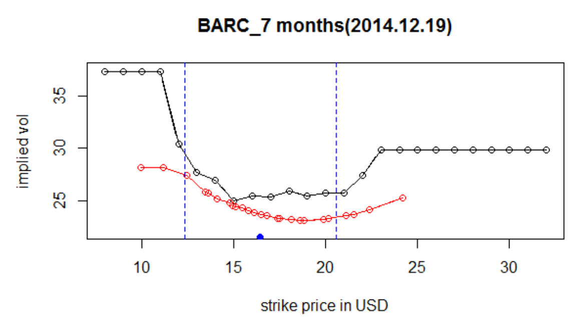
Appendix 2

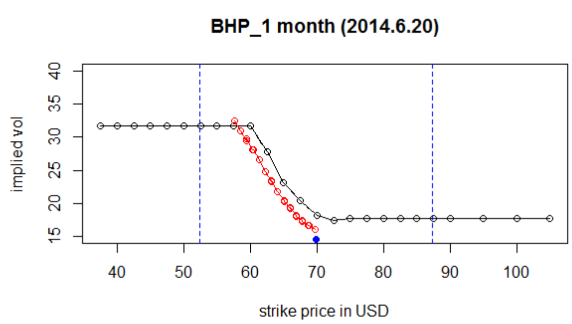
In the plots, the ADR volatility are shown in black and the original market volatility are shown in red. All the strike price is converted into USD using the spot exchange rate assuming the exchange rate doesn't change until expiry. The blue doc points out the spot price and the blue dashed lines are 75% and 125% of spot rate. Basically we only need compare the implied volatility between the dashed lines.

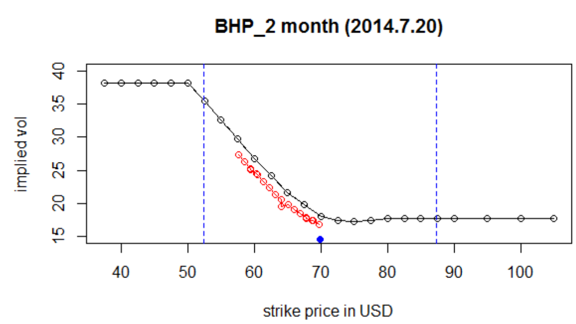


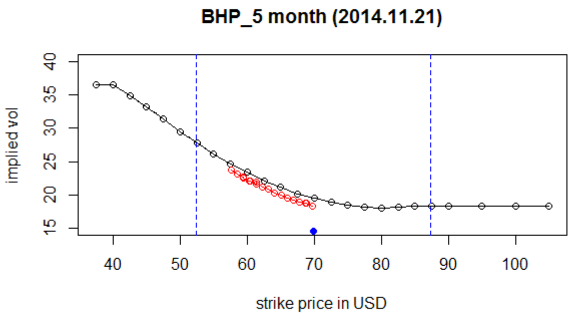


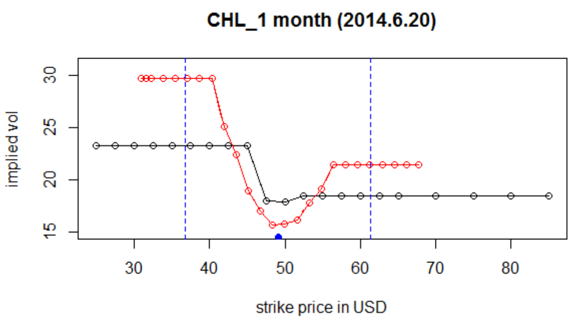


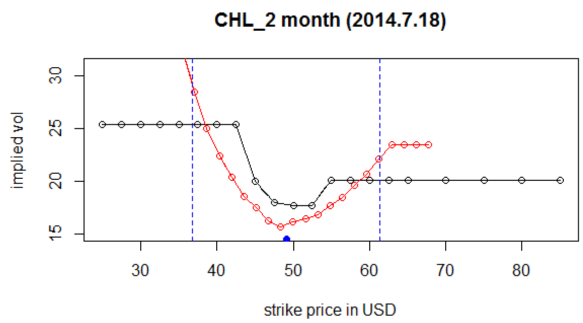


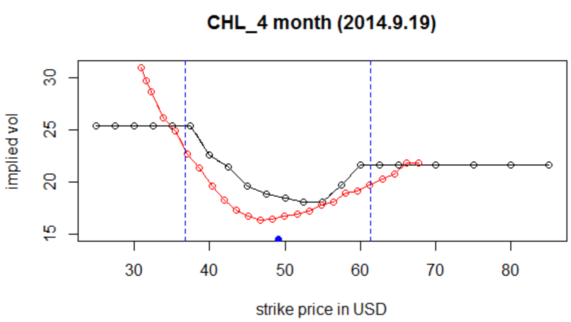


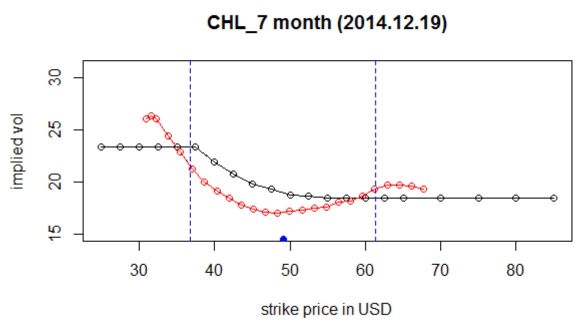


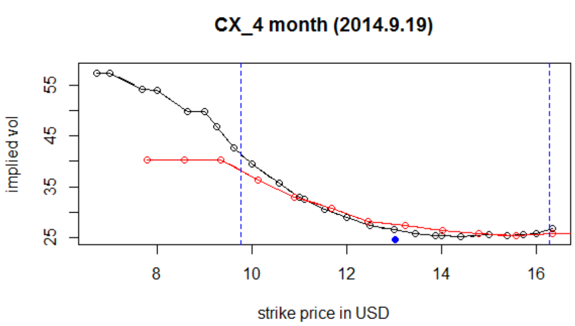

BHP_5 month (2014.11.21)
implied vol
strike price in USD

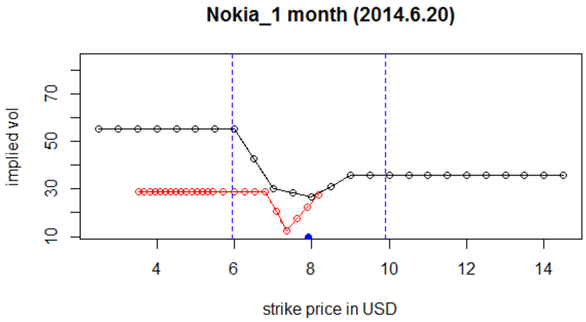

CHL_1 month (2014.6.20)
implied vol
strike price in USD

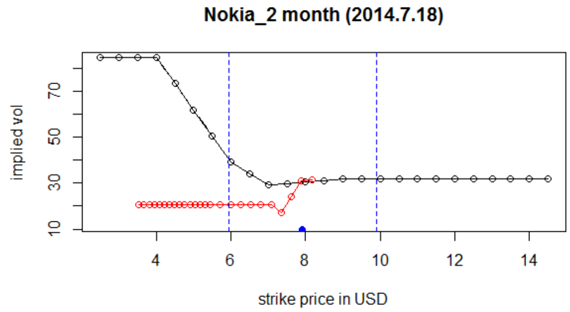

CHL_2 month (2014.7.18)
implied vol
strike price in USD

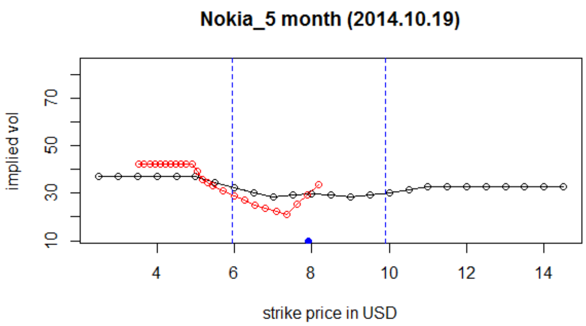

CHL_4 month (2014.9.19)
implied vol
strike price in USD

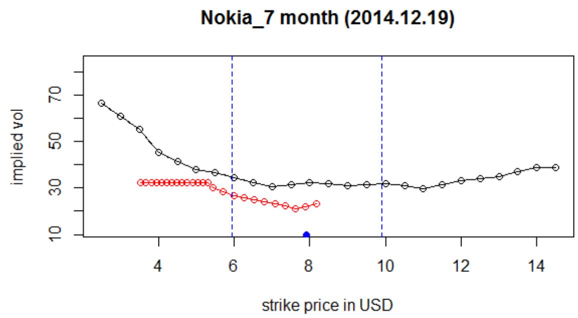

CHL_7 month (2014.12.19)
implied vol
strike price in USD

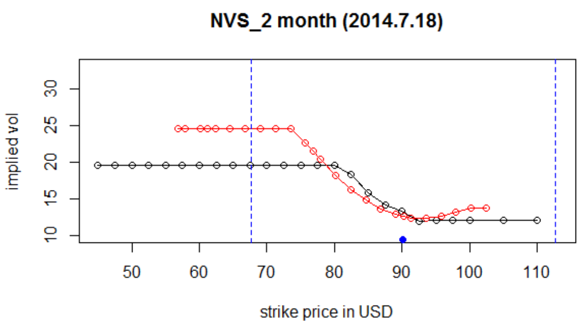

CX_4 month (2014.9.19)
implied vol
strike price in USD

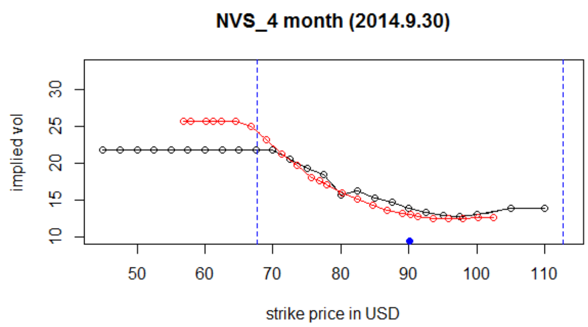

Nokia_1 month (2014.6.20)
implied vol
strike price in USD

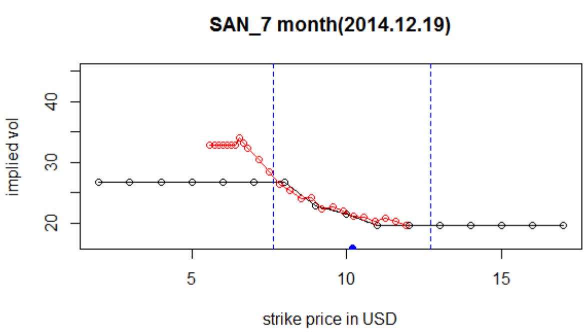

Nokia_2 month (2014.7.18)
implied vol
strike price in USD


Nokia_5 month (2014.10.19)
implied vol
strike price in USD


Nokia_7 month (2014.12.19)
implied vol
strike price in USD


NVS_2 month (2014.7.18)
implied vol
strike price in USD


NVS_4 month (2014.9.30)
implied vol
strike price in USD


SAN_7 month(2014.12.19)
implied vol
strike price in USD

Appendix 3

The KBW bank index is an economic index consisting of the stocks of 24 banking companies. This index serves as a benchmark of the banking sector. This index trades on the Philadelphia Stock Exchange, where it was created. The KBW Index is named after Keefe, Bruyette and Woods, a recognized authority in the banking industry. The KBW Index trades under ticker symbol BKX. The index is weighted according to capitalization and represents major banks and money centers from across the country. Mathematically, the index is based on a tenth of the value of the Keefe, Bruyette and Woods Index (KBWI). It began trading options in September of 1992.