STATISTICS 134 Practice Final

There are 9 questions, worth a total of 49 points. Calculations should be worked through to an explicit numerical answer. Show your work!

1. **[5 points]** Let U be a continuous r.v. with uniform distribution on $(0, 1)$. Let $X = \log \frac{U}{1-U}$. Find a formula for the density function $f(x)$ of X.

2. **[5 points]** A box contains n tags numbered $1, 2, \ldots, n$. Two tags are drawn without replacement, giving two numbers: write X for the smaller and Y for the larger number. Calculate $P(Y = X + 1)$.

3. **[5 points]** Consider Poisson random scatter with intensity λ on the plane. Let (X, Y) be the coordinates of the random point of the scatter which is closest to the origin. Find the joint density function $f(x, y)$ of (X, Y).

4. **[5 points]** A roulette wheel has 38 slots, of which 18 are red and 18 are black. In 100 spins of the wheel, let R be the number of "reds" and let B be the number of "blacks". Calculate the correlation $\text{cor}(R, B)$.

5. **[5 points]** A statistics class has 23 students. As part of an assignment, each student tosses a coin 200 times and records the number of heads. What is the chance than no student gets exactly 100 heads?

6. **[5 points]** Let X and Y be independent r.v.'s with $EX = EY = \mu$ and $\text{var } X = \text{var } Y = \sigma^2$. Write $Z = XY$. Calculate $\text{var } Z$ in terms of μ and σ.

7. **[8 points]** Let X and Y be continuous r.v.'s with joint density

$$\begin{aligned} f(x, y) = 0.5 + 2xy \qquad & \text{if } 0 < x < 1 \text{ and } 0 < y < 1 \\ = 0 \qquad & \text{if not .} \end{aligned}$$

(a) Find the marginal density of X.
(b) Do X and Y have the same marginal density? Explain.
(c) Are X and Y independent? Explain.
(d) Calculate $P(X + Y < 1)$.

8. **[6 points]** Let X_1 and X_2 be independent continuous r.v.'s with distribution function

$$F(x) = \exp(-e^{-x}),\ -\infty < x < \infty$$

(a) What is the distribution function of $X_1 + c$, for constant c ?
(b) What is the distribution function of $M = \max(X_1, X_2)$?
(c) True or false (and explain): for a certain constant c, the random variable $X_1 + c$ has the same distribution as the random variable M.

9. **[5 points]** Let X and Y be independent r.v.'s with standard Normal$(0, 1)$ distribution. Find the conditional distribution of X given $X = Y$.