Google “Aldous STAT 134” to find course web page.
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Style of course

- Blackboard and chalk (except first 5 lectures).
- We study the basic **mathematics** of probability . . .
- . . . but it’s not just algebra/calculus; you need to constantly think what the math objects **mean**.
- Roughly follow textbook.
- 12 homeworks, 2 midterms, final.
- No formal lab but strongly encouraged attend “supplemental section”.

What you might have learned in High School ("Common core" standards).

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (or, and, not).
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6.. Find the conditional probability of A given B as the fraction of Bs outcomes that also belong to A, and interpret the answer in terms of the model.

7. Apply the Addition Rule, $\text{P(A or B)} = \text{P(A)} + \text{P(B)} - \text{P(A and B)}$, and interpret the answer in terms of the model.

8. Apply the general Multiplication Rule in a uniform probability model, $\text{P(A and B)} = \text{P(A)P(B}|\text{A)} = \text{P(B)P(A}|\text{B)}$, and interpret the answer in terms of the model.

9. Use permutations and combinations to compute probabilities of compound events and solve problems.

Events and random variables (in words)

An **event** is something that may or may not happen . . . . . .
. . . . . . and so has some probability of happening.

We typically denote events as A, B, C and write P(A) for the probability that A happens.

A **random variable** is a quantity with a range of possible values, the actual value will be influenced by chance in some way.

In interesting real-world contexts of future uncertainty there are many events and RVs one might consider:

(November elections)
(Event): Obama re-elected President
(RV): number Republicans elected Congress

(Earthquakes in Bay Area)
(Event) Some $>$ 7.0 magnitude quake in next 30 years
(RV) Total cost quake damage over next 30 years.

(Tech stocks)
(RV): Facebook share price on 5/18/13
(Event): Facebook share price above IPO price on 5/18/13

(SF 49ers season)
(RV): number wins in regular season
(Event): win Superbowl

Any calculation involving real-world objects requires starting with a "probability model" that translates the question into math.

The examples above are complicated – one needs a lot of empirical data to feed into any model. We will not tackle such questions in this course.

Examples and exercises in textbooks often use "made-up" little stories with very unrealistic probability models, for example ...

a. A student must choose exactly two out of three electives: art, French, and mathematics. He chooses art with probability $5/8$, French with probability $5/8$, and art and French together with probability $1/4$. What is the probability that he chooses mathematics? What is the probability that he chooses either art or French?

b. A restaurant offers apple and blueberry pies and stocks an equal number of each kind of pie. Each day ten customers request pie. They choose, with equal probabilities, one of the two kinds of pie. How many pieces of each kind of pie should the owner provide so that the probability is about .95 that each customer gets the pie of his or her own choice?

c. Take a stick of unit length and break it into two pieces, choosing the break point at random. Now break the longer of the two pieces at a random point. What is the probability that the three pieces can be used to form a triangle?

Those examples are plainly silly. So why are they used?

(i) Tradition.

(ii) Engaging realistic issues is just difficult – takes a lot of time and fussy details. Tradition says it's more useful to spend a semester doing "the basic **mathematics** of probability" and leave applications for later.

(Every 3 years I teach a "Probability and the Real World" course which uses only real examples – some lecture write-ups are on my web site).

The history of mathematical probability began with games of chance/gambling, which involve what I pedantically call "artifacts with physical symmetry"

—- dice, playing cards, roulette wheels, "wheel of fortune"

—- bingo/lotto machines, tossed coin

In this context we assume that the different possible outcomes

1 roll of a die (6 possible outcomes)
1 pick of a card (52 possible outcomes)
1 spin of roulette wheel (38 possible outcomes)

are equally likely - a **uniform distribution**. This is our "probability model". And for these examples we're confident it's a good model so we don't distinguish between the model and the real-world objects.

These artifacts

—- dice, playing cards, roulette wheels, "wheel of fortune"

—- bingo/lotto machines, (tossed coin)

are specifically designed to have symmetry, which is what justifies the "equally likely" assumtion in the model.

Warning; do not assume "equally likely" outcomes without some good reason; not true for

— dropped thumbtack

— thrown dart

In words, we said

An **event** is something that may or may not happen . . . . . .

A **random variable** is a quantity with a range of possible values, the actual value will be influenced by chance in some way.

I will briefly mention the formal math setup, designed so that Probability can be fitted into the rest of Mathematics.

But it is more helpful to keep the verbal definitions in mind.

Events and random variables (formal math setup)

Start with an **outcome space** or **sample space** which has all the different possibile "outcomes" that may be relevant.

Example: a well-shuffled deck of cards. An "outcome" is the precise ordering of the 52 cards. So there are 52! possible outcomes.

An event is a subset of the outcome space – those outcomes that make the event happen.

"no Aces in the top 5 cards"

A RV is a function from the outcome space to numbers (usually)

"number of Aces in the top 5 cards"

A **probability model** now assigns a probability to each outcome in the outcome space. Our model of a "well-shuffled" deck is that all 52! orderings are equally likely – each ordering has probability $\frac{1}{52!}$.

Then the probability of an event is the sum of the probabilities of the outcomes that make the event happen.

(Will talk about RVs later).

The mathematical examples in this course can be fitted into this formal setup. But it's not directly useful for estimating the probability that Obama is re-elected.

Another way of thinking about probabilities for events of interest to many people is via gambling odds.

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