

Statistics 240 Lecture Notes

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Revised 11 February 2008

1 Assignment 2

1. Write a brief (1–2 page) research proposal for your term project. The proposal should include the scientific question you intend to address and the source of the data. If you intend to re-analyze data from a published study, please cite the original paper. If you need help picking a topic, see me.
2. Explain the “randomization model” and some of its advantages and limitations. State the strong null hypothesis for the randomization model.
3. For the randomization model, state some pros and cons of the Wilcoxon rank sum test versus a permutation test based on the sample mean and the two-sample t -test.
4. Consider an experiment involving 9 subjects, 5 assigned at random to treatment and 4 to control. We want to test the null hypothesis that “treatment makes no difference” at significance level 10%. For each individual, we measure a quantitative response. Think about the omnibus alternative, about the one-sided shift alternative that treatment increases the response, and about the two-sided shift alternative that treatment increases or decreases the response. Consider four tests: the Wilcoxon rank-sum test (using mid-ranks for ties), a permutation test based on the difference in the sample means for the control and the treatment groups, the Smirnov test, and a 2-sample t -test based on the difference in the sample means for the control and treatment groups.

- (a) Explain the assumptions of each test, including a precise statement of the null hypotheses.
- (b) State strong and weak versions of each null hypothesis. Note whether the nominal significance level of each test is for the strong or weak null hypothesis.
- (c) Find or estimate by simulation the power of each test against the one-sided shift alternative that treatment increases each individual's response by 1 unit. If the power depends on additional unspecified features of the treatment effect or on features of the baseline responses of the subjects, explain what those features are, and find the power for a few different values of those features.
- (d) For one-sided versions of the Wilcoxon rank-sum test, the permutation test using the sample mean, and the t -test, and for the Smirnov test, find the P -values for the following hypothetical data, and the power against a shift alternative that treatment increases the response by one unit:

treatment	1	2	3	3	4
control	0	1.5	2.5	3.5	
- (e) Which test do you think is best in this situation, absent any additional information about the nature of the experiment? Why?

5. Consider the Smirnov test for an experiment involving 5 subjects, 2 assigned at random to treatment and 3 to control.

- (a) Enumerate all possible values of the test statistic and their probabilities under the strong null hypothesis of no treatment effect, assuming no ties among the data.
- (b) Now estimate the probabilities by simulation, using 10,000 replications.
- (c) Calculate the true (theoretical) standard error of the simulated probabilities.
- (d) What is the joint distribution of the number of times the test statistic takes each of its possible values? Sketch how you would test the hypothesis that the true probabilities are equal to the values you calculated in the first part of this question using the empirical frequencies you observed in the second part.
- (e) What software package are you using to do the calculations? What is its default algorithm for calculating pseudo-random numbers? What is the period of that pseudo-random number generator? What is the largest number of objects for which that generator can give you all permutations? Is there an option in the package to use a better pseudo-random number generator? If so, which one?