Election Auditing and Nonparametric Confidence Bounds

Department of Mathematics
Reed College
Portland, OR
20 November 2008

Philip B. Stark
Department of Statistics
University of California, Berkeley
statistics.berkeley.edu/∼stark

[Election Leak] [Voting Machine Wins]
[Homer Votes—sort of]

Abstract: Vote counts are subject to machine error, programming error, processing error, voter error or even deliberate fraud. Is the outcome still right?

Post-election audits hand-count ballots in a random sample of batches. Nineteen states require or allow post election audits; Oregon just passed an audit law (with serious gaps). Generally, the state-mandated audits do not control the risk of certifying an incorrect election outcome. They do not guarantee any minimum chance that there will be a full manual count when the apparent outcome is wrong.

I present a method that does. The method was tested "live" on a February 2008 ballot measure in Marin County, CA, a November 2008 bond measure in Yolo County California, with audits underway in Humboldt, Marin and Santa Cruz counties.

The method couches the problem as a sequential statistical hypothesis test. Data are collected. If the data do not allow one to reject the null hypothesis is that the apparent outcome is not the outcome a full recount would show, more data are collected. Eventually, either the hypothesis has been rejected or there has been a full manual count. Multiplicity is accounted for by adjusting the conditional levels of the sequential test. The approach amounts to a sequential nonparametric upper confidence bound for a population total.

Outline

- Voting systems: punchcard, optically scanned, DRE (VVPAT)
- Sample of 2008 sorrows: NC, CA, FL, DC, NJ, . . .
- Laws: California, New Jersey
- Random selection
- Hypothesis testing framework: the math
- The realities
- Examples: 2008 Yolo Measure W; 2008 Marin Measure A
- Complications & potential improvements
- References

Voting Systems

Punchcard & lever systems. Discouraged by *Help America Vote Act* of 2002. NY still uses—but not for much longer.

Optically scanned ballots: "bubble in" like a Scantron form. Produces auditable paper trail. Voter intent vs. machine scan.

Direct-recording Electronic (DRE): touchscreens, etc. VVPATs. Felten group, TTBR. De-certified in CA, CO, OH.

Polk County NC, 2008

Owens victory in Polk is in doubt, by Times-News staff

Ted Owens went to sleep Tuesday night thinking he had earned another term . . . A recount Wednesday showed he may not have. . . .

Computer software initially displayed figures that were different than those shown by the voting machines . . .

The software installed in the stand-alone computer that ballot results are fed into was the problem . . . [Elections Director Dale Edwards] said there was no explanation as to why the computer counted the wrong numbers, and no one is at fault.

BlueridgeNow.com Times-News, 6 November 2008 http://www.blueridgenow.com/article/20081106/NEWS/811050255

Santa Clara County, CA, 2008

Few problems reported in area despite record turnout, by by Karen de Sá and Lisa Fernandez

Record-high voting in the Bay Area on Tuesday mostly defied predictions of unwieldy waits and overwhelmed polls. But in Santa Clara County, concerns about touch-screen voting machines will likely increase following significant malfunctions.

Fifty-seven of the county's Sequoia Voting Systems machines failed on Election Day, resulting in hourslong delays before replacements arrived.

Mercury News, 4 November 2008 http://www.mercurynews.com/elections/ci_10901166?nclick_check=1

Leon County, FL, 2008

Ballots not being recorded at two Leon County polling places by Angeline J. Taylor

Leon County Supervisor of Elections Ion Sancho has reported that ballots . . . are not being read properly. The problem, he said, rests with a new machine that has been purchased for polling sites throughout the state. . . .

"Certain ballots are being rejected across the state," he said. . . . If the machine reads the ballot card as too long, the [] machine will simply not read the card.

Tallahassee Democrat, 20 October 2008 http://www.tallahassee.com/article/20081020/BREAKINGNEWS/81020024

Palm Beach County, FL, 2008

Florida Primary Recount Surfaces Grave Voting Problems One Month Before Presidential Election, by Kim Zetter

At issue is an August 26 primary election in which officials discovered, during a recount of a close judicial race, that more than 3,400 ballots had mysteriously disappeared after they were initially counted on election day. The recount a week later, minus the missing ballots, flipped the results of the race to a different winner.

. . . officials found an additional 227 ballots that were never counted on election day . . . in boxes in the county's tabulation center.

Palm Beach County was using new optical-scan machines that it recently purchased from Sequoia Voting Systems for $5.5 million.

[In a re-scan of ballots the machines had rejected] Officials expected the machines would reject the same ballots again. But that didn't happen. During a first test of 160 ballots, the machines accepted three of them. In a second test of 102 ballots, the machines accepted 13 of them . . . When the same ballots were run through the machines again, 90 of the ballots were accepted.

[T]he county then re-scanned two batches of 51 ballots each that had initially been rejected for having no vote cast in the judicial race, but that were found in a manual examination to contain legitimate votes for one candidate or the other. The first batch of 51 ballots were found to have legitimate votes

for Abramson. The second batch of 51 ballots were found to have legitimate votes for Wennet.

In the first batch of 51 ballots . . . 11 of the ballots that had previously been rejected as undervotes were now accepted . . . the remaining 40 ballots were rejected as having no votes. In the second batch of 51 ballots . . . the same machine accepted 2 ballots and rejected 49.

The same two batches of ballots were then run through the second . . . machine. [I]n the first batch . . . the machine accepted 41 . . . and rejected 10 others. In the second batch . . . the machine accepted 49 of the ballots and rejected 2 – the exact opposite of the results from the first machine.

Wired News, 7 October 2008, http://blog.wired.com/27bstroke6/2008/10/florida-countys.html

Washington, DC, 2008

Report Blames Speed In Primary Vote Error; Exact Cause of Defect Not Pinpointed by Nikita Stewart

Speed might have contributed to the Sept. 9 primary debacle involving thousands of phantom votes, according to a D.C. Board of Elections and Ethics report issued yesterday. . . . [T]he report does not offer a definitive explanation. . .

The infamous Precinct 141 cartridge "had inexplicably added randomly generated numbers to the totals that had been reported," according to the report written by the elections board's internal investigative team.

. . . 4,759 votes were reflected instead of the actual 326 cast there.

Washington Post, 2 October 2008; Page B02

see also hearings at http://www.octt.dc.gov/services/on_demand_video/channel13/October2008/10_03_08_PUBSVRC_2.asx

New Jersey 2008

County finds vote errors: Discrepancies discovered in 5% of machines by *Robert Stern*

Five percent of the 600 electronic voting machines used in Mercer County during the Feb. 5 presidential primary recorded inaccurate voter turnout totals, county officials said yesterday . . .

23 February 2008, New Jersey Times

New Jersey 2008–contd.

Judge Suppresses Report on Voting Machine Security
by Andrew Appel

A judge of the New Jersey Superior Court has prohibited the scheduled release of a report on the security and accuracy of the Sequoia AVC Advantage voting machine. . . . [NJ] mostly uses Sequoia AVC [DRE] models. None of those DREs can be audited: they do not produce a voter verified paper ballot.

2 October 2008, Freedom to Tinker

Ohio 2004

Machine Error Gives Bush Thousands of Extra Ohio Votes by *John McCarthy*

COLUMBUS, Ohio – An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said. Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

5 November 2004, Associated Press

Florida 2004

Broward Machines Count Backward by *Eliot Kleinberg*

. . . Early Thursday, as Broward County elections officials wrapped up after a long day of canvassing votes, something unusual caught their eye. Tallies should go up as more votes are counted. Thats simple math. But in some races, the numbers had gone . . . down.

Officials found the software used in Broward can handle only 32,000 votes per precinct. After that, the system starts counting backward.

. . . The problem cropped up in the 2002 election. . . . Broward elections officials said they had thought the problem was fixed.

5 November 2004, The Palm Beach Post

Machine (Voting System) Counting

- Want to count votes by machine: saves time and money (or so we are told).

- Machine counts are subject to various kinds of error. (So are hand counts, but they're the gold standard. Progress on accuracy, too.)

- Counting errors $\Rightarrow$ risk that machines name the wrong winner.

Statistical Audits

Can *limit* and *quantify* that risk.

Could guarantee that,

> If the outcome is wrong, there's a at least a 99% chance of a full manual count

even if an evil adversary built the hardware and wrote the software.

(Of course, could just manually count 99% of *all* contests at random, but that's a lot of counting: avoidable by statistics.)

Essential that voters create an audit trail.

Essential to select batches at random.

California Elections Code §15360

. . . the official conducting the election shall conduct a public manual tally of the ballots tabulated by those devices, including absent voters' ballots, cast in 1 percent of the precincts chosen at random by the elections official . . .

The elections official shall use either a random number generator or other method specified in regulations . . .

The official conducting the election shall include a report on the results of the 1 percent manual tally in the certification of the official canvass of the vote. This report shall identify any discrepancies between the machine count and the manual tally and a description of how each of these discrepancies was resolved . . .

NJ S507 [1R] (Gill)

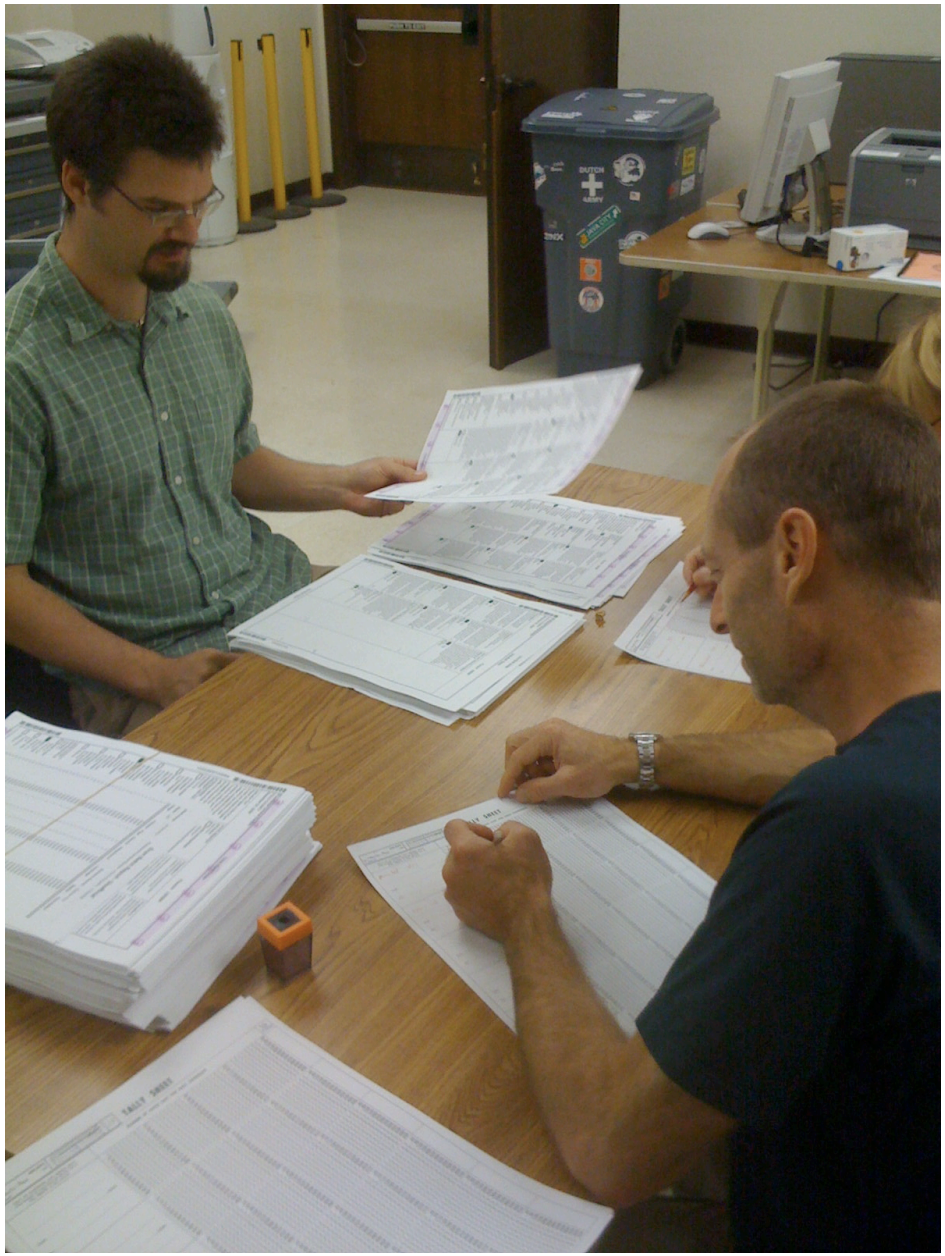
. . . shall conduct random hand counts of the voter-verified paper records in at least two percent of the election districts where elections are held for federal or State office . . .

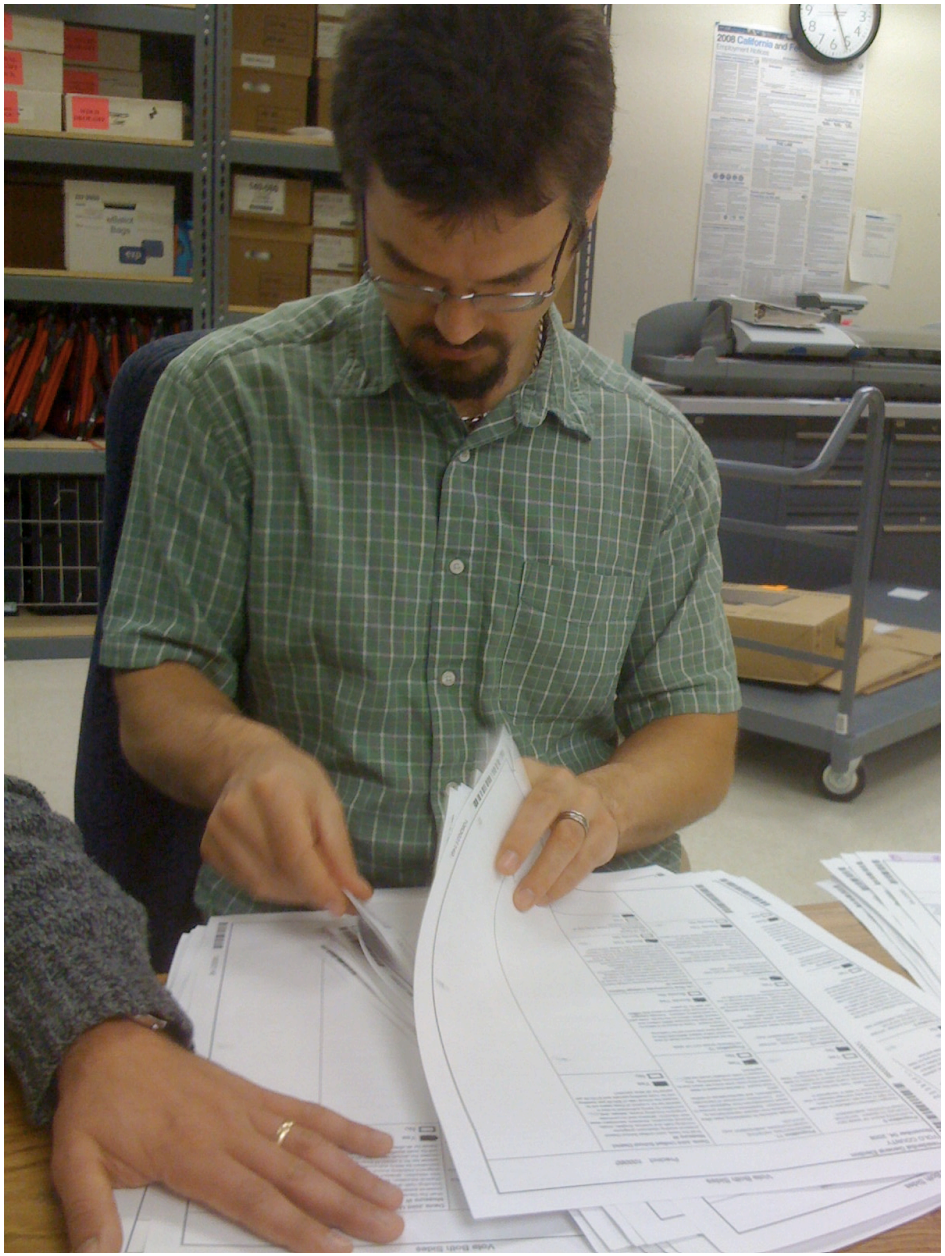
Any procedure designed, adopted, and implemented by the audit team shall be implemented to ensure with at least 99% statistical power that for each federal, gubernatorial or other Statewide election held in the State, a 100% manual recount of the voter-verifiable paper records would not alter the electoral outcome reported by the audit . . .

[procedures] shall be based upon scientifically reasonable assumptions . . . including but not limited to: the possibility that within any election district up to 20% of the total votes cast may have been counted for a candidate or ballot position other than the one intended by the voters . . .

Say what?

2008 Yolo County, CA Measure W Audit






Voted Ballots
Hefty


TALLY SHEET
NUMBER OF VOTES CAST FOR EACH CANDIDATE
SIGN CERTIFICATE ON FRONT COVER
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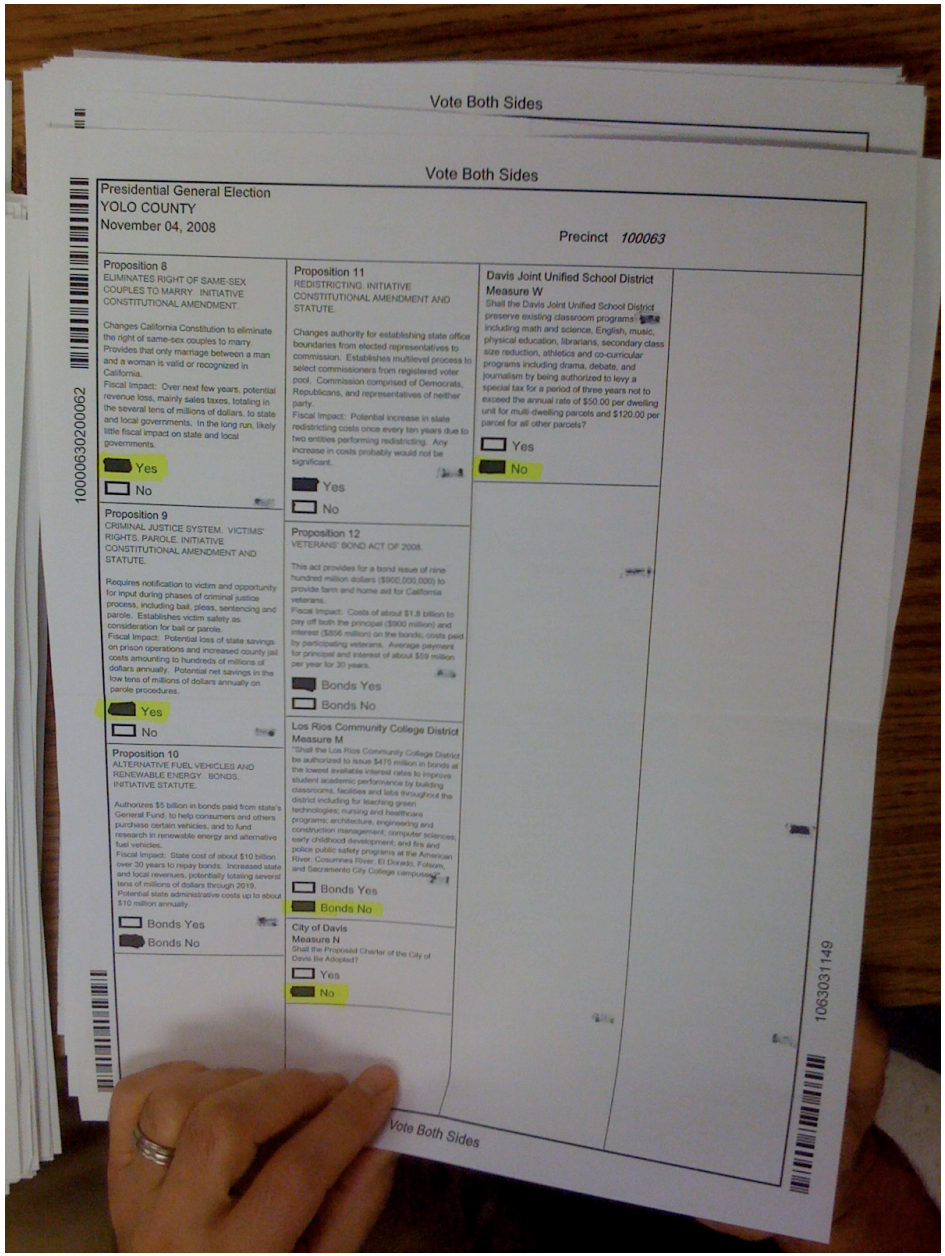
Vote Both Sides
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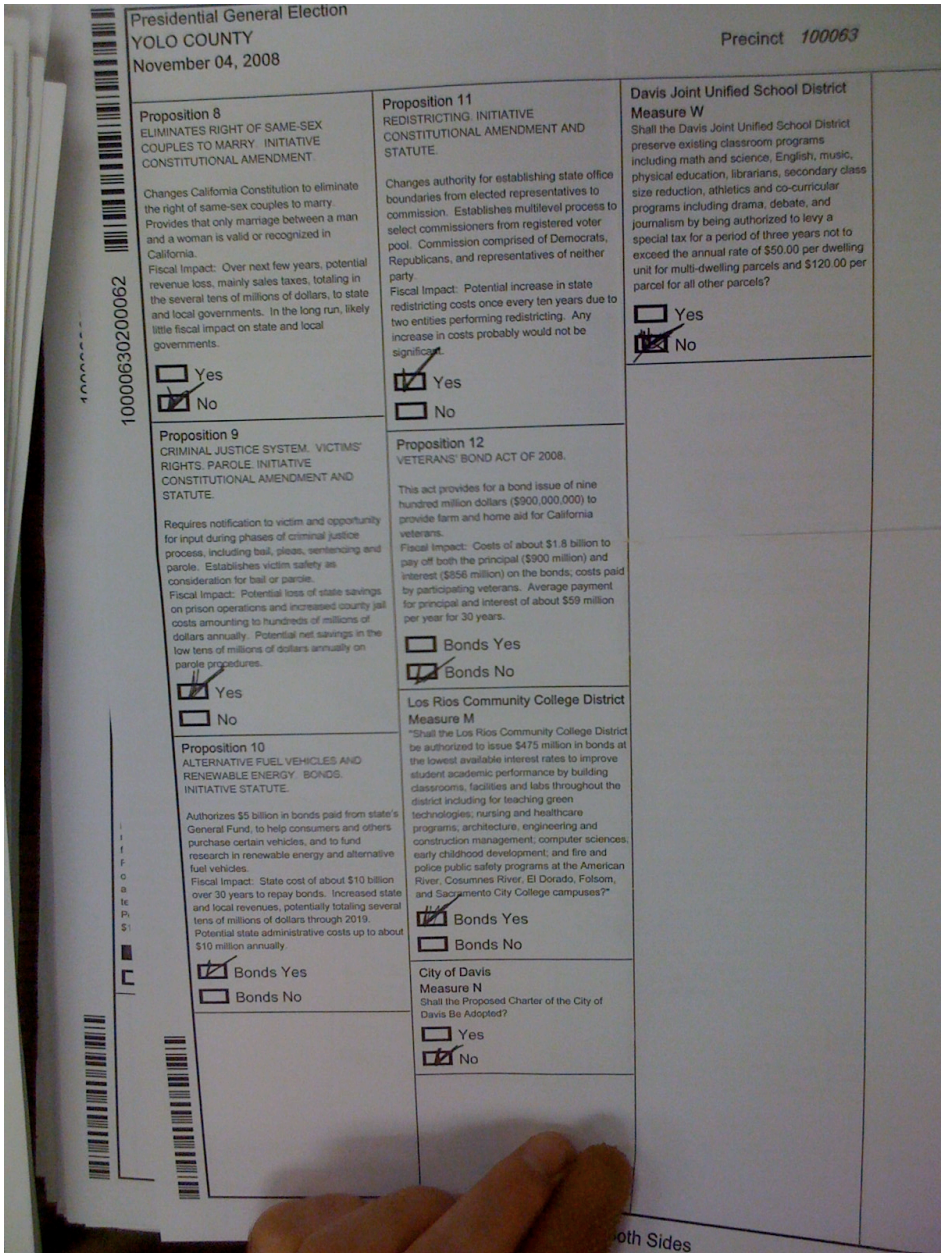
Presidential General Election
YOLO COUNTY
November 04, 2008
Precinct 100063
Proposition 8
ELIMINATES RIGHT OF SAME-SEX COUPLES TO MARRY. INITIATIVE CONSTITUTIONAL AMENDMENT.
Changes California Constitution to eliminate the right of same-sex couples to marry. Provides that only marriage between a man and a woman is valid or recognized in California.
Fiscal Impact: Over next few years, potential revenue loss, mainly sales taxes, totaling in the several tens of millions of dollars, to state and local governments. In the long run, likely little fiscal impact on state and local governments.
Yes
No
Proposition 9
CRIMINAL JUSTICE SYSTEM. VICTIMS' RIGHTS. PAROLE. INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE.
Requires notification to victim and opportunity for input during phases of criminal justice process, including bail, pleas, sentencing and parole. Establishes victim safety as consideration for bail or parole.
Fiscal Impact: Potential loss of state savings on prison operations and increased county jail costs amounting to hundreds of millions of dollars annually. Potential net savings in the low tens of millions of dollars annually on parole procedures.
Yes
No
Proposition 10
ALTERNATIVE FUEL VEHICLES AND RENEWABLE ENERGY. BONDS. INITIATIVE STATUTE.
Authorizes $5 billion in bonds paid from state's General Fund, to help consumers and others purchase certain vehicles, and to fund research in renewable energy and alternative fuel vehicles.
Fiscal Impact: State cost of about $10 billion over 30 years to repay bonds. Increased state and local revenues, potentially totaling several tens of millions of dollars through 2019. Potential state administrative costs up to about $10 million annually.
Bonds Yes
Bonds No
Proposition 11
REDISTRICTING. INITIATIVE CONSTITUTIONAL AMENDMENT AND STATUTE.
Changes authority for establishing state office boundaries from elected representatives to commission. Establishes multilevel process to select commissioners from registered voter pool. Commission comprised of Democrats, Republicans, and representatives of neither party.
Fiscal Impact: Potential increase in state redistricting costs once every ten years due to two entities performing redistricting. Any increase in costs probably would not be significant.
Yes
No
Proposition 12
VETERANS' BOND ACT OF 2008.
This act provides for a bond issue of nine hundred million dollars ($900,000,000) to provide farm and home aid for California veterans.
Fiscal Impact: Costs of about $1.8 billion to pay off both the principal ($900 million) and interest ($856 million) on the bonds; costs paid by participating veterans. Average payment for principal and interest of about $59 million per year for 30 years.
Bonds Yes
Bonds No
Los Rios Community College District Measure M
"Shall the Los Rios Community College District be authorized to issue $475 million in bonds at the lowest available interest rates to improve student academic performance by building classrooms, facilities and labs throughout the district including for teaching green technologies; nursing and healthcare programs; architecture, engineering and construction management; computer sciences; early childhood development; and fire and police public safety programs at the American River, Cosumnes River, El Dorado, Folsom, and Sacramento City College campuses?"
Bonds Yes
Bonds No
City of Davis
Measure N
Shall the Proposed Charter of the City of Davis Be Adopted?
Yes
No
Davis Joint Unified School District
Measure W
Shall the Davis Joint Unified School District preserve existing classroom programs including math and science, English, music, physical education, librarians, secondary class size reduction, athletics and co-curricular programs including drama, debate, and journalism by being authorized to levy a special tax for a period of three years not to exceed the annual rate of $50.00 per dwelling unit for multi-dwelling parcels and $120.00 per parcel for all other parcels?
Yes
No
10000630200062
1063031149
Vote Both Sides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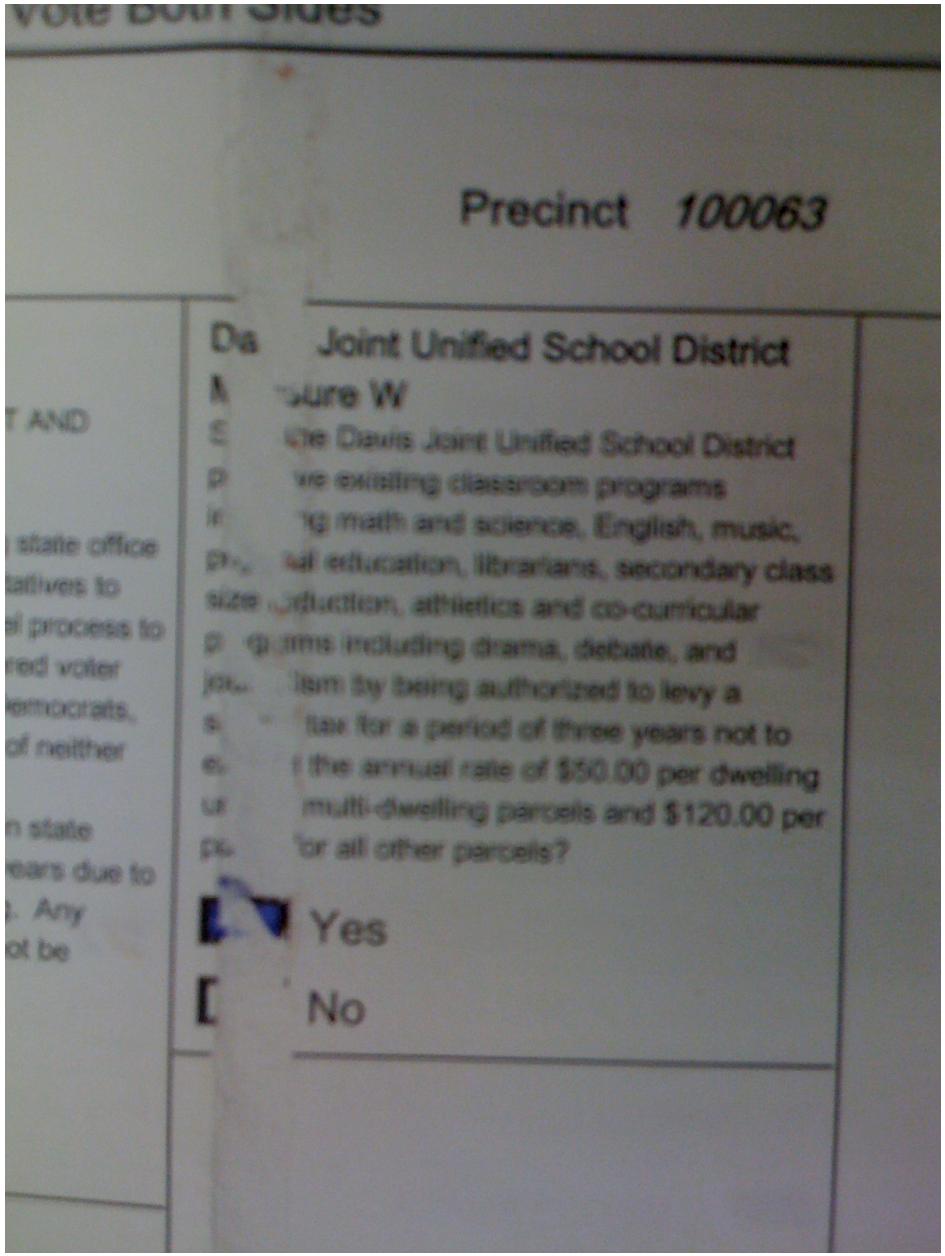
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Precinct 100063
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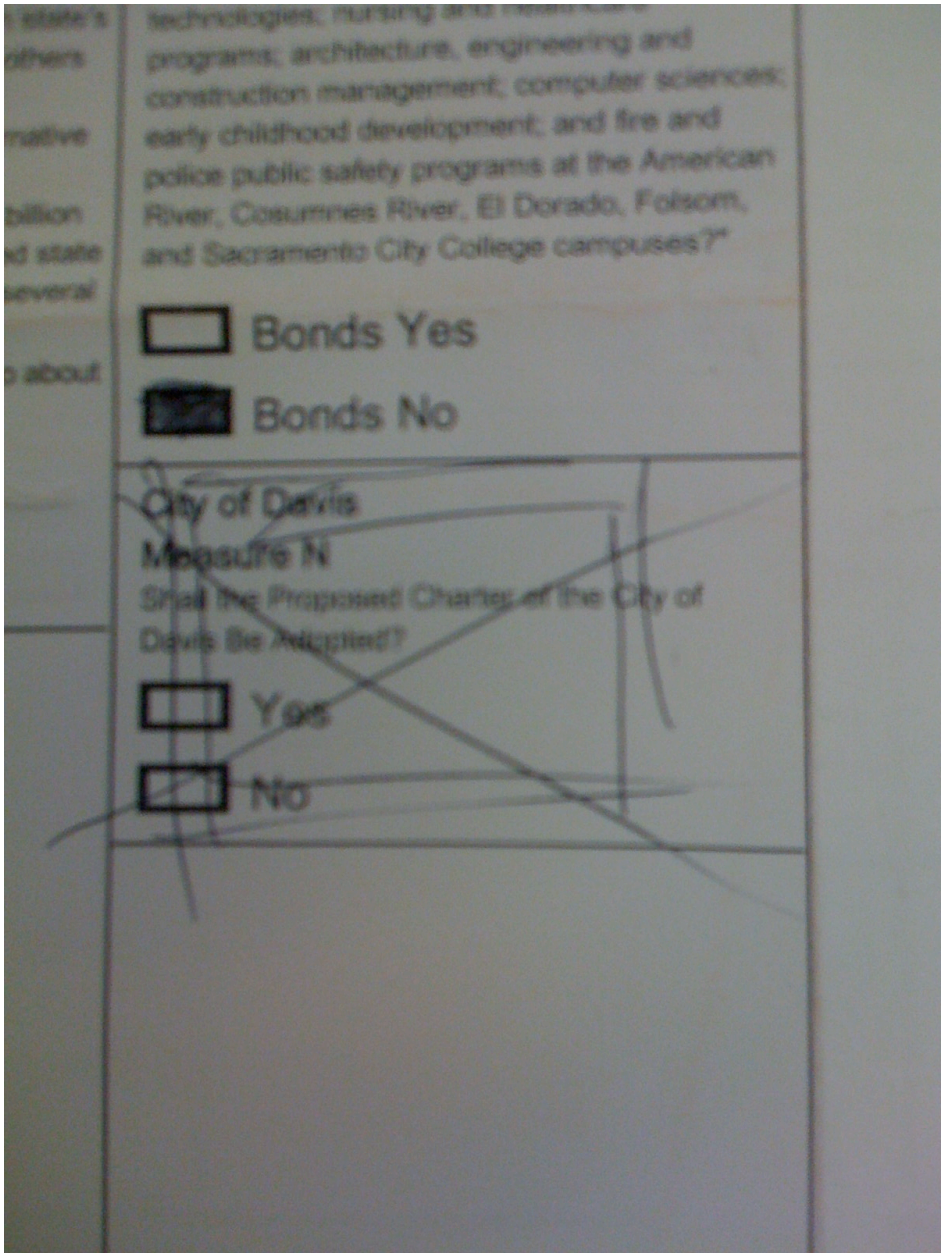
Yes
No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Selecting batches at random

Software pseudo-random number generators:
not transparent, hackable.

One ticket per precinct:
hard to verify; hard to mix (Vietnam draft).

2008 Oregon law uses one ticket per precinct; allows selections before publishing election results.

10-sided dice (Marin County) [Roll 1] [Roll 2]

Ping-pong balls (Alameda County) [Static] [Tumbling]

Alameda has 1204 precincts. Pick 1s digit, 10s, 100s.
If result is between 205 and 999, stop.
Else, remove 2–9 & pick 1000s digit.

Unintended consequences?

Logistic tradeoff

Errors and time for dice rolling, look-up tables (especially for PPS sampling).

Hybrid selection (Marin & Santa Cruz audits)

Roll 10-sided dice to get a 6-digit seed.
Use "good" open source PRNG to generate a sequence of numbers from the seed in a reproducible way.
(Used the Mersenne Twister implemented in R)

General principles

Margin small $\Rightarrow$ less error required to produce it erroneously.

Sample small $\Rightarrow$ can be likely that sample will find few or no errors, even if machines named the wrong winner.

No look, no see: absence of evidence is not evidence of absence.

Smaller margins $\Rightarrow$ lower confidence.

Smaller samples $\Rightarrow$ lower confidence.

Larger discrepancies in sample $\Rightarrow$ lower confidence.

Sample big (compared with margin) $\Rightarrow$ likely to see big discrepancies in the sample if machines named wrong winner.

Rigorous statistical audit

> If it's very likely that the audit would have found larger discrepancies than it did find, had the machines named the wrong winner, confirm the outcome.
>
> Otherwise, keep counting.

If the outcome is confirmed, either the correct winner was named, or something very unlikely happened.

Equivalent: confirm if sequential nonparametric lower confidence bound for the margin is ≥ 0.

Complete procedure says:

- how many batches to audit initially
- given the discrepancies in the audit sample, whether to confirm the outcome or expand the audit
- eventually declares "outcome confirmed" or "full recount."
- change of full recount if outcome is wrong is at least 99%, e.g.

Only one basic approach so far does that.

Notation

f	# winners (vote for f)
P	# audit batches in the contest
K	# candidates in contest, after "pooling"
$\mathcal{W}$	indices of the f apparent winners
$\mathcal{L}$	indices of the $K - f$ apparent losers
a_{kp}	actual vote for candidate k in batch p
$A_k \equiv \sum_p a_{kp}$	actual total vote for candidate k
$A_{w\ell} \equiv A_w - A_\ell$	actual margin of candidate w over candidate ℓ
b_p	upper bound on a_{kp}
v_{kp}	reported vote for candidate k in batch p
$V_k \equiv \sum_p v_{kp}$	total vote reported for candidate k
$V_{w\ell} \equiv V_w - V_\ell$	apparent margin of candidate w over candidate ℓ

Sufficient condition for correct outcome:

The apparent winners are the actual winners if

$$\min_{w \in \mathcal{W}, \ell \in \mathcal{L}} A_{w\ell} > 0. \tag{1}$$

Define

$$e_{w\ell p} \equiv \frac{(v_{wp} - v_{\ell p}) - (a_{wp} - a_{\ell p})}{V_{w\ell}}. \tag{2}$$

Outcome must be right unless

$$\sum_{p=1}^{P} e_{w\ell p} \geq 1 \text{ for some } w \in \mathcal{W},\ \ell \in \mathcal{L}. \tag{3}$$

Maximum relative overstatement of pairwise margins (MRO) in batch p:

$$e_p \equiv \max_{w \in \mathcal{W}, \ell \in \mathcal{L}} e_{w\ell p}. \tag{4}$$

Bounding the error in each batch

$$\max_{w\in\mathcal{W},\ell\in\mathcal{L}} \sum_{p=1}^{P} e_{w\ell p} \leq \sum_{p=1}^{P} \max_{w\in\mathcal{W},\ell\in\mathcal{L}} e_{w\ell p} = \sum_{p=1}^{P} e_p. \tag{5}$$

b_p: bound on a_{kp} from pollbooks, # registered voters, ballot accounting, etc.

$$e_p \leq \max_{w\in\mathcal{W},\ell\in\mathcal{L}} \frac{v_{wp} - v_{\ell p} + b_p}{V_{w\ell}} \equiv u_p. \tag{6}$$

Total of margin overstatements

$$E \equiv \sum_{p=1}^{P} e_p. \tag{7}$$

Electoral outcome must be right if $E < 1$.

The whole shebang

1. Pick the min chance β of full manual count when result is wrong

2. Pick the max # of stages S, "escalation probabilities" $\beta_1, \beta_2, \ldots, \beta_S$ s.t. $\pi_s \beta_s = \beta$.

3. Select subtotals that comprise batches, & strata.
   B_c is # batches in stratum c, $c = 1, \ldots, C$.

4. Fnd upper bounds b_p on the number of votes per candidate per batch from voter registrations, pollbooks, or an accounting of ballots.

5. Set $s = 1$ (stage).
   $P_s = P$ (un-audited batches at stage s).

6. Find pairwise margins:

$$V_{w\ell} = \text{ (votes for winner } w) \; - \; \text{(votes for loser } \ell). \quad (8)$$

   Use the semi-official results for the P_s batches that have not yet been audited, and the audit results for the $P - P_s$ audited batches.

   If $\min_{w \in \mathcal{W}, \ell \in \mathcal{L}} V_{w\ell} \leq 0$, the list of winners has changed. Abort the audit and count all the votes by hand.

7. For each batch p that has not been audited, compute

$$u_p \equiv \max_{w \in \mathcal{W}, \ell \in \mathcal{L}} \frac{V_{wp} - V_{\ell p} + b_p}{V_{w\ell}}. \tag{9}$$

8. If there are a few un-audited batches p with especially large u_p audit them and return to step 6.

9. Set the tolerable level of error, $t \in [0, 1)$. If any margin is overstated by t or more, the audit will progress to the next stage.

10. Find the incremental sample sizes.

    For the P_s batches p not yet audited, define:
    $t_p \equiv \min(t, u_p)$; $T \equiv \sum_p T_p$; $\tilde{u}_p = u_p - t_p$.

    (a) Starting with the largest value of $\tilde{u}_p$, add successively smaller values of $\tilde{u}_p$ just until the sum of those values is $\geq 1 - T$. q is # terms in the sum.

    (b) Find the smallest whole number n such that

$$\left(\frac{P_s - q}{P_s}\right)^n \leq 1 - \beta_s. \tag{10}$$

    (c) Sample size n_c for stratum c is

$$n_c \equiv \left\lceil n \times \frac{\#\text{unaudited batches in stratum } c}{P_s} \right\rceil. \tag{11}$$

$$n^* = n_1 + n_2 + \cdots + n_C \geq n. \tag{12}$$

11. Select batches using a transparent, verifiable source of randomness, such as fair 10-sided dice. For each stratum $c = 1, \ldots, C$, draw n_c batches from the as-yet-unaudited batches in stratum c, count votes by hand.

12. For each of the n^* batches p audited in this stage, find

$$e_{w\ell p} = \frac{v_{wp} - v_{\ell p} - (a_{wp} - a_{\ell p})}{V_{w\ell}} \tag{13}$$

for all pairs (w, ℓ), $w \in \mathcal{W}$, $\ell \in \mathcal{L}$. There are $n^* \times w \times \ell$ of those values.

$$t_s \equiv \max_{p,w,\ell} e_{w\ell p}. \tag{14}$$

13. If $t_s \leq t$, certify the election and stop. If $t_s > t$ and $s = S$, count all the votes by hand. Otherwise, increment s; perform any desired targeted auditing; set P_s tto # batches not yet audited; return to step 6.

Alternative: sampling proportional to size

Define the *taint* of precinct p:

$$\tau_p \equiv e_p / u_p \leq 1. \tag{15}$$

Total of upper bounds on the overstatement of margins:

$$U \equiv \sum_p u_p. \tag{16}$$

Total overstatement of pairwise margins:

$$E \equiv \sum_p e_p \equiv \sum_p u_p \tau_p. \tag{17}$$

Suppose values τ_p are distinct. Define the random variable X to have probability distribution

$$\mathbb{P}\{X = \tau_p\} = u_p/U. \tag{18}$$

Can construct X by drawing precinct p with probability u_p/U & auditing the selected precinct.

$$\begin{aligned} \mathbb{E}X &\equiv \sum_p \tau_p \frac{u_p}{U} \\ &= \sum_p \frac{e_p}{u_p} \frac{u_p}{U} \\ &= E/U. \end{aligned} \tag{19}$$

A nonparametric upper confidence bound for $\mathbb{E}X$ gives an upper confidence bound for E.

Can certify if, with high confidence, $\mathbb{E}X < 1$.

Approaches

Massart-Dvoretsky-Kiefer-Wolfowitz inequality. (Move mass $\sqrt{\ln(\alpha)/2n}$ from left tail to 1.)

Hoeffding's Inequality (since $\mathbb{P}\{X \in [0,1]\} = 1$).

Multinomial bounds. (Trinomial bound joint work with Luke Miratrix.)

Sampling without replacement? (Serfling? Generalizations of MDKW?)

Stratification?

Advantages of PPS

Common in financial auditing.

More efficient: look harder where more error could hide.

For Marin Measure B, cut sample size from 22 to 12 (but the sampled batches are on average larger).

For Santa Cruz County Supervisor, District 1, cut sample size from 40 to 16.

Disadvantages of PPS

Harder to explain to non-geeks.

Harder to implement without software (need look-up tables).

Sampling probabilities different for the same batches for different races.

Yolo County Measure W, November 2008

Davis school bond. Required simple majority. Used SRS.

batches	yes	no	undervote	overvote	margin
114	25,297	8,118	3,001	2	17,179

Took $t = 14/17,179$ (confirm if no batch has more than 14 overstatement errors).

Assumed batches with $u_p - t < 0.009$ were entirely in error; sampled from remaining 95.

At least 20 would have to have margin overstatements bigger than t for the outcome to be wrong $\rightarrow$ sample size $n = 6$.

Counted about 2,500 ballots by hand on 17 November 2008. 1 extra "yes" and 1 extra "no."

Hypothetical example: cartoon of U.S. House Race

2 stages.

400 precincts split across 2 counties: 300 and 100.

Stratify by mode of voting (in-precinct or by mail) and county: 4 strata, sizes 300, 300, 100 and 100 batches

3 candidates, overvotes, undervotes.

t corresponds to 3 vote overstatement of margin of victory.

Audit batches equal size (255 votes), equal numbers of reported votes including 13 votes reported for candidate 3, 2 overvotes and 3 undervotes.

Worst-case erroneous escalation: all "tainted" in one stratum. (Random taint much less likely to trigger escalation.)

(1) V_{12}	(2) β	(3) β_1	(4) β_2	(5) n	(6) n^*	(7) f	(8) $\gamma_{0.01}$	(9) $\omega_{0.01}$	(10) $\gamma_{0.005}$	(11) $\omega_{0.005}$	(12) n_2^*
5.2%	75%	76.0%	98.9%	37	38	4.75%	34.7%	23.8%	18.8%	7.2%	108
		86.6%	86.6%	51	54	6.75%	45.3%	17.7%	25.5%	4.9%	50
	90%	91.0%	98.9%	61	62	7.75%	50.0%	34.9%	28.7%	11.3%	108
		94.9%	94.9%	76	78	9.75%	58.3%	31.2%	34.8%	9.5%	68
10.0%	75%	76.0%	98.9%	18	20	2.50%	22.3%	9.3%	11.6%	2.4%	54
		86.6%	86.6%	25	28	3.50%	28.7%	6.3%	15.3%	1.5%	26
	90%	91.0%	98.9%	29	30	3.75%	28.7%	12.1%	15.3%	3.1%	54
		94.9%	94.9%	36	38	4.75%	34.7%	11.2%	18.8%	2.8%	36
19.6%	75%	76.0%	98.9%	9	12	1.50%	15.4%	4.0%	7.9%	1.0%	28
		86.6%	86.6%	13	14	1.75%	15.4%	2.1%	7.9%	0.5%	14
	90%	91.0%	98.9%	15	16	2.00%	15.4%	4.0%	7.9%	1.0%	30
		94.9%	94.9%	18	20	2.50%	22.3%	4.5%	11.6%	1.1%	20

(1) Margin between candidates 1 and 2. 5.2%: 125 votes for candidate 1 and 112 for candidate 2 in each batch; 10.0%: 131 versus 106; 19.6%: 143 versus 94. (2) Min chance of full count if the outcome is wrong. (3) Min chance audit goes from stage 1 to stage 2 if the outcome is wrong. (4) Min chance audit goes from stage 2 to a full count if the outcome is wrong, if it gets to stage 2. (5) Stage 1 sample size before adjusting for stratification. (6) Stage 1 sample size adjusted for stratification. (7) $n^*/800 \times 100\%$. (8) Max chance audit gets to stage 2 if 1% of audit batches overstate V_{12} by more than 3 votes. (9) Max chance of a full count if 1% of audit batches overstate V_{12} by more than 3 votes, and the stage 1 net error is zero. (10) Same as (8), but 0.5% of batches have large overstatements of V_{12}. (11) Same as (9), but 0.5% of batches have large overstatements of V_{12}. (12) Stage 2 sample size if the net error in stage 1 is zero.

Logistical issues: stratification, etc.

Samples for different counties drawn independently: stratified.

VBM and absentee ballots not counted right away.

Makes sense to start with a uniform sampling rate, then escalate as necessary.

Can test separately in each stratum for proportional share of M.

Reject overall hypothesis if all reject; conservative.

OR, P-value for proportional sample $\leq$ P-value for unstratified sample w/ replacement.

PPS Sampling.

November 2006 Minnesota U.S. Senate Race

MN requires:
Counties with <50,000 registered voters audit ≥ 2 precincts;
counties with 50,000–100,000 registered voters audit ≥ 3;
counties with $\geq$100,000 registered voters audit ≥ 4.

≥ 1 precinct audited in each county must have $\geq$150 votes cast.

$C = 87$ counties, $P = 4,123$ precincts, $n = 202$ audited.
Audited precincts had from 2–2,393 ballots cast.

Voters	under& invalid	Fitzgerald Indep	Kennedy Repub	Klobuchar D/F/L	Cavlan Green	Powers Constit	Write-ins
2,217,818	15,099	71,194	835,653	1,278,849	10,714	5,408	901
$V_{w\ell}$	N/A	1,207,655	443,196	N/A	1,268,135	1,273,441	1,277,948

Pool Cavlan, Powers, write-ins: pseudo-candidate apparently lost to Klobuchar by 1,261,773 votes; $K = 4$.
$\max_p u_p = 0.0097$; $\max_p e_p = 4.5 \times 10^{-6}$; $q = 166$.

Conservative P-value

Pretend sample was drawn with replacement from all 4,123 precincts, but that only 78 precincts were drawn, as if the population sampled using the minimum sampling fraction among counties

1.9% sample ($n = 78$) w/ replacement	SRS ($n = 202$) w/o replacement
4.05%	0.02%

Sharper treatment of stratification (with Mike Higgins) decreases conservative P-value to 1.9%.

5 February 2008 Marin County Measure A

First election ever audited to attain target level of confidence in the result.

Audited to attain 75% confidence that a full manual recount would find the same outcome.

Required 2/3 majority to pass. Margin 298 votes.

Stratified random sample: 6 polling-place counts, 6 VBM counts.

Marin Measure A data

precinct	registered	type	ballots	yes	no	bound	audited
2001	1326	IP	391	278	101	286	yes
		VBM	657	438	193	456	no
2004	893	IP	284	204	66	214	yes
		VBM	389	257	116	268	yes
2010	6	VBM	6	4	2	4	no
2012	740	IP	218	167	43	173	yes
		VBM	342	242	89	250	no
2014	983	IP	299	214	75	221	no
		VBM	420	306	95	319	yes
2015	905	IP	217	167	44	171	yes
		VBM	483	332	131	346	yes
2019	1048	IP	295	215	70	222	yes
		VBM	567	395	160	403	yes
2101	923	IP	265	169	79	181	no
		VBM	439	275	133	296	yes
2102	900	IP	223	144	68	152	yes
		VBM	410	233	142	257	yes
All	7724	PRO	252	176	54	191	no

Marin Measure A audit timeline

Milestone	Date
Election day	5 February
Polling place results available	7 February
Random selection of polling place precincts	14 February
VBM results available	20 February
Random selection of VBM precincts	20 February
Hand tally complete	20 February
Provisional ballot results available	29 February
Computations complete	3 March

Costs:

$1,501, including salaries and benefits for 4 people tallying the count, a supervisor, support staff to print reports, resolve discrepancies, transport ballots and locate and retrieve VBM ballots from the batches in which they were counted.

$0.35 per ballot audited. $1\frac{3}{4}$ days.

Other stuff

Auditing more than one contest using PPS sampling?

False Discovery Rate.

Small races? Lower confidence? Only audit random sample of races?

Sharper treatment of stratification (with Mike Higgins)

Auditing entire ballots, not contests.

Recap

- Vote counting is not perfect; errors can affect outcomes
- Auditing laws that address the problem fall short
- There's a way to fix them using Statistics
- It seems practical/workable in examples

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More voting-related links: http://statistics.berkeley.edu/~stark/Vote/index.htm