Urning Voter Confidence

Department of Mathematics
Reed College
Portland, OR
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Philip B. Stark
Department of Statistics
University of California, Berkeley
statistics.berkeley.edu/∼stark

Abstract:Automated counting processes make errors. How can we determine whether the apparent winner of an election really won? Could the margin be due to error–machine error, programming error, processing error, voter error or even deliberate fraud? Post-election audits–manual tallies of the votes in a random sample of precincts– are designed to answer that question. Election auditing is complex and there is tension between the need for efficiency and the need for transparency. Confirming an election outcome can be couched as a statistical hypothesis test. The null hypothesis is that the apparent winner is not the true winner. If the data allow us to reject that hypothesis with a small P-value, we have high confidence in the election outcome. That is, we have high confidence if, on the assumption that anybody other than the apparent winner really won, the chance is tiny that the observed miscount in the sampled precincts would be as small is it was observed to be. How does confidence depend on the sample size, the margin, the sizes of precincts in the race, and the observed miscount? One method that seems politically and computationally feasible is modelled on–you guessed it–drawing colored marbles from an urn.

Outline

- Voting systems: punchcard, optically scanned, DRE (VVPAT)
- Errors: voter, system, election worker, fraud
- Examples: Ohio 2004, San Diego County 2004, Napa County 2004
- "Software independence" and auditing
- Laws: California, Massachusetts, Federal
- How to commit election fraud (if you must)
- Confidence and the urn model
- References

Voting Systems

Punchcard & lever systems. Discouraged by *Help America Vote Act* of 2002

Optically scanned ballots: "bubble in" like a Scantron form. Produces auditable paper trail. Voter intent vs. machine scan.

Direct-recording Electronic (DRE): touchscreens, etc. VVPATs. Felten group, TTBR. CA SoS Bowen de-certified.

Ohio 2004

Published 5 November 2004 by the Associated Press
Machine Error Gives Bush Thousands of Extra Ohio Votes
by John McCarthy

COLUMBUS, Ohio – An error with an electronic voting system gave President Bush 3,893 extra votes in suburban Columbus, elections officials said. Franklin County's unofficial results had Bush receiving 4,258 votes to Democrat John Kerry's 260 votes in a precinct in Gahanna. Records show only 638 voters cast ballots in that precinct. Bush's total should have been recorded as 365.

San Diego County 2004

Published 23 April 2004 by San Diego Union Tribune

State panel says Diebold glitches tainted primary

By Bill Ainsworth

SACRAMENTO A state panel has recommended banning the use of 15,000 Diebold touch-screen voting machines used in the March primary in San Diego and three other counties, saying malfunctioning computers kept people from casting ballots.

. . .

According to a recent report by the California Secretary of State's Office, the failure prevented nearly half of the county's precincts from opening on time, a higher number than some previous estimates. The report has been sent to the Attorney General's Office for a possible criminal investigation.

. . .

Members of the panel chastised Diebold, contending, among other things, that the company:

Misled state officials about the prospects of the voting machines' receiving federal approval.

Installed uncertified software in machines in 17 counties.

Downplayed security concerns that had been raised in four separate studies.

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San Diego County hasn't yet paid Diebold the $31 million for the 10,200 machines that it used in March.

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Early this month, the county revealed that 2,821 optically scanned absentee ballots were miscounted because of a glitch in the computer tabulation system, also supplied by Diebold.

Napa County Presidential Primary 2004

Published 12 March 2004 by Wired News

Lost E-Votes Could Flip Napa Race

by Kim Zetter

Napa County in Northern California said on Friday that electronic voting machines used in the March presidential primary failed to record votes on some of its paper ballots, which will force the county to re-scan over 11,000 ballots and possibly change the outcome of some close local races.

. . . Napa Registrar of Voters John Tuteur said they discovered the problem on Thursday while conducting a manual recount of 1 percent of precincts, . . . they discovered that the machine wasn't recording certain votes.

. . . the machine was calibrated to detect carbon-based ink, but not dye-based ink commonly used in gel pens, . . . a Sequoia technician ran test ballots through the machine to calibrate its reading sensitivity, but failed to test for gel ink.

Software Independence

Want a system so that even if the devil (or Diebold or Sequoia or Microsoft or . . .) is the programmer, you can trust the result.

Post-election audits are key

But the voting system must produce an auditable record. (e.g., optical scan ballot or possibly VVPAT)

California Elections Code §15360

(a) During the official canvass of every election in which a voting system is used, the official conducting the election shall conduct a public manual tally of the ballots tabulated by those devices, including absent voters' ballots, cast in 1 percent of the precincts chosen at random by the elections official. If 1 percent of the precincts is less than one whole precinct, the tally shall be conducted in one precinct chosen at random by the elections official.

In addition to the 1 percent manual tally, the elections official shall, for each race not included in the initial group of precincts, count one additional precinct. The manual tally shall apply only to the race not previously counted.

Additional precincts for the manual tally may be selected at the discretion of the elections official.

(b) If absentee ballots are cast on a direct recording electronic voting system at the office of an elections official or at a satellite location of the office of an elections official pursuant to Section 3018, the official conducting the election shall either include those ballots in the manual tally conducted pursuant to subdivision (a) or conduct a public manual tally of those ballots cast on no fewer than 1 percent of all the direct recording

electronic voting machines used in that election chosen at random by the elections official.

(c) The elections official shall use either a random number generator or other method specified in regulations that shall be adopted by the Secretary of State to randomly choose the initial precincts or direct recording electronic voting machines subject to the public manual tally.

(d) The manual tally shall be a public process, with the official conducting the election providing at least a five-day public notice of the time and place of the manual tally and of the time and place of the selection of the precincts to be tallied prior to conducting the tally and selection.

(e) The official conducting the election shall include a report on the results of the 1 percent manual tally in the certification of the official canvass of the vote. This report shall identify any discrepancies between the machine count and the manual tally and a description of how each of these discrepancies was resolved. In resolving any discrepancy involving a vote recorded by means of a punchcard voting system or by electronic or electromechanical vote tabulating devices, the voter verified paper audit trail shall govern if there is a discrepancy between it and the electronic record.

Proposed Massachusetts Law H671

SECTION 1. Chapter 54 of the general laws, as appearing in the 2004 Official Edition, is hereby amended by inserting after section 109 the following section:-

Section 109A. Audits of election results

Following each primary, general or special election, a random manual, hand-count audit of the votes for each federal office, state-wide ballot question and state-wide office shall be conducted in not less than five percent of the precincts in the Commonwealth.

The precincts to be audited shall be chosen at random using a manual, non-computerized drawing, supervised by the state secretary. The time and place of the drawing shall be announced in advance. The drawing shall be open to representatives of each political party and to the public. The drawing shall take place not more than twenty-four hours after the close of the polls.

The time and place of the audits in each precinct to be audited shall be publicly announced in advance and the audits shall be performed in full public view. The audit shall be conducted pursuant to the procedures for hand-counts of ballots in Section 105 of Chapter 54, except for the following provisions: 1) audit teams shall be trained to conduct the audits in the precincts, 2) a number of audit teams sufficient to complete the

audit in five days shall be trained and ready for deployment around the state on the first through the fifth day after the election, 3) these teams shall consist of four members: one member shall read votes, one member shall observe the reader of the votes, one member shall record votes on an official sheet, and one member shall observe the recorder, 4) the team shall include members of different parties, 5) team members may switch tasks as long as the observers are of a different party than the person they are observing at any given time, 6) two additional observers-at-large of different parties shall also be present at each precinct, and 7) each precinct audit will be presided over by the city or town clerk, or his designee, who shall record the tallies from each packet of ballots on an official master tally sheet when the counting of that packet is completed, observed by the two observers-at-large.

For each audited precinct, the tallies of the votes for each candidate and ballot question as determined in the audit shall be recorded and reported to the state secretary within five days after the election. The state secretary shall make these results available to the Legislatures Committee on Election Laws with sufficient time for a statewide audit to be conducted, as provided for in this section. Each precinct shall also report to the state secretary the model of the electronic or mechanical voting or counting device, if any, used to determine the original vote tally.

Upon receipt of the results of the manual audit count from the audited precincts, the state secretary shall calculate the total number of votes

for each candidate and ballot question as recorded in the manual audit count and shall compare this total to the sum of the originally reported votes for each candidate and ballot question in the audited precincts.

Whenever the vote total obtained from the manual count:
1) Differs by more than one percent from the originally reported vote tally for any given candidate or ballot question, or
2) Indicates a different prevailing candidate or outcome, either passage or defeat, of one or more ballot questions than the originally reported tally, the following actions shall take place:
1) a statewide hand-counted audit shall be conducted for that contest, to be completed and reported within sufficient time to meet state and federal election deadlines for performing recounts and certification,
2) the discrepancy between the hand-counted audit and the original vote tally shall be analyzed to ascertain the cause of the discrepancy. The state secretary shall oversee the analysis and shall publish and make available online the findings within 180 days.

When a manual count is conducted, the sum of the votes from each audited precinct for each candidate and ballot question shall be calculated for each model of electronic or mechanical voting or counting device and compared to the sum of the originally reported tallies from those machines in the audited precincts. Whenever the difference between the hand-counted audit and the originally reported tally is more than one percent for any particular machine model, the discrepancy between

the hand-counted audit and the original vote tally shall be analyzed to ascertain the cause of the discrepancy. The state secretary shall oversee the analysis and shall publish and make available online the findings within 180 days.

Whenever the difference between the vote total obtained from the manual count in any given precinct differs by more than 3 percent from the vote count originally reported by that precinct, the following actions shall take place:
1) a full hand-count shall be conducted in the city or town where the said precinct is located, and
2) the discrepancy between the hand-counted audit and the original vote tally shall be analyzed to ascertain the cause of the discrepancy. The state secretary shall oversee the analysis and shall publish and make available online the findings within 180 days.

If there is a discrepancy between a manual count and originally reported tallies, where the original results were obtained using electronic equipment, the manual count of the official paper ballots shall be the official vote of record.

The results of audits and hand-counts, as well as the corresponding data for the originally reported tallies, shall be made publicly available on a precinct-by-precinct basis both in hardcopy and in electronic file format.

H.R. 811 (Proposed): Voter Confidence and Increased Accessibility Act of 2007 (Holt)

. . .

SEC. 322. NUMBER OF BALLOTS COUNTED UNDER AUDIT.
(a) In General- Except as provided in subsection (b), the number of voter-verified paper ballots which will be subject to a hand count administered by the Election Audit Board of a State under this subtitle with respect to an election shall be determined as follows:

(1) In the event that the unofficial count as described in section 323(a)(1) reveals that the margin of victory between the two candidates receiving the largest number of votes in the election is less than 1 percent of the total votes cast in that election, the hand counts of the voter-verified paper ballots shall occur in 10 percent of all precincts (or equivalent locations) in the Congressional district involved (in the case of an election for the House of Representatives) or the State (in the case of any other election for Federal office).

(2) In the event that the unofficial count as described in section 323(a)(1) reveals that the margin of victory between the two candidates receiving the largest number of votes in the election is greater than or equal to 1 percent but less than 2 percent of the total votes cast in that election, the hand counts of the voter-verified paper ballots shall occur in 5 percent of all precincts (or equivalent locations) in the Congressional district involved

(in the case of an election for the House of Representatives) or the State (in the case of any other election for Federal office).

(3) In the event that the unofficial count as described in section 323(a)(1) reveals that the margin of victory between the two candidates receiving the largest number of votes in the election is equal to or greater than 2 percent of the total votes cast in that election, the hand counts of the voter-verified paper ballots shall occur in 3 percent of all precincts (or equivalent locations) in the Congressional district involved (in the case of an election for the House of Representatives) or the State (in the case of any other election for Federal office).

(b) Use of Alternative Mechanism- Notwithstanding subsection (a), a State may adopt and apply an alternative mechanism to determine the number of voter-verified paper ballots which will be subject to the hand counts required under this subtitle with respect to an election, so long as the National Institute of Standards and Technology determines that the alternative mechanism will be at least as effective in ensuring the accuracy of the election results and as transparent as the procedure under subsection (a).

How to commit election fraud (if you must)

- make sure the election uses DREs w/o VVPATs; hack the software.

- if the election uses DREs w/ VVPATS, hack the software and spoil the VVPATs with "household chemicals" (TTBR report)

- if you know that the audit will be based on whether any errors are found in a simple random sample, hide the fraud in as few precincts as possible.

So what?

California law, etc., (will) require an audit of a random sample of precincts.

How do we use the audit data to decide whether the election went wrong? What if the audit finds error? (It will.)

Hypothesis test: Want to test the null hypothesis that the election went wrong, at significance level 1%.
If can reject, conclude election went right (or something unlikely happened).

Complete rules

Need to specify:

- how big a sample to take initially
- what to do if the audit finds error. Escalate? When to stop?
- when to certify the election, and when to recount all ballots

Secretary Bowen called for audits based on statistical confidence that the election went the right way.

Currently, no such method (but I'll tell you about one anyway)

Confidence in what?

Existing methods focus on detection, not testing:

What is the smallest sample size n such that if there is enough miscount to alter the outcome, the chance the sample will reveal at least one error is 99% or more?

The right question:

Given the miscount observed in the audit sample, what is our confidence that the right person was named the winner? (Also need to account for stratification.)

("confidence" used informally: 100% - chance of error)

Assumptions

1. The race is a winner-take-all contest.

2. The number of ballots cast in each precinct is known with certainty.

3. The true number of votes cast in each precinct for each candidate is unknown, as are the true number of invalid ballots and the true number of ballots that do not record a vote for any candidate (undervotes).

4. Any kind of vote can be miscounted as any other kind of vote. For example, a vote for one candidate could be counted as a vote for a different candidate, an undervote or an invalid ballot.

5. A hand tally of a precinct reveals the exact miscount. There is no way to introduce or to hide miscounts retroactively, and hand counts are not subject to error. (When the hand count does not match the machine count, the hand count is typically repeated until the counters are confident that the problem is with the machine count. The hand count is the legal gold standard.)

6. Precincts are selected at random for post-election audit by simple random sampling.

The Plan in broad brush

1. Select a test statistic.

2. Select a sampling design and an increasing sequence of sample sizes (n_s). Select a sequence of significance levels that give a level-α test overall.(E.g., $\alpha_s \equiv \alpha/2^s$, $s = 1, \ldots$.)

3. Set $s = 1$. Set the initial sample to be the empty set.

4. Augment the current sample by a (possibly stratified) random sample so that it contains n_s precincts in all.

5. Tally the votes in the new precincts by hand.

6. Calculate the value of the test statistic for the entire sample.

7. Calculate the maximum P-value for the test statistic over all ways of allocating error among the precincts that would result in a different election outcome.

8. If the maximum P-value is less than α_s, certify the election. Otherwise, increment s and return to step 4, unless all N precincts have now been hand tallied.

C	# counties with precincts in the race
C	the integers $\{1, \ldots, C\}$
K	# candidates in the race
N_c	# of precincts in the race in county c
$N \equiv \sum_{c=1}^{C} N_c$	total precincts in the race
$\mathcal{N} = \{1, \ldots, N\}$	the integers $\{1, \ldots, N\}$
v_{kp}	# votes reported for candidate k in precinct p
v_{0p}	# undervotes and invalid ballots reported for precinct p
$v_p = (v_{kp})_{k=0}^{K}$	vector of reported votes in precinct p
$V_k \equiv \sum_{p=1}^{N} v_{kp}$	# votes reported for candidate k.
M	apparent margin in the race, in votes
a_{kp}	true # votes for candidate k in precinct p
$a_p = (a_{kp})_{k=0}^{K}$	true vector of votes for precinct p
$b_p \equiv \sum_{k=0}^{K} v_{kp} = \sum_{k=0}^{K} a_{kp}$	# ballots cast in precinct p
$B \equiv \sum_{p=1}^{N} b_p$	total ballots cast in the race
$e_p = \frac{1}{2} \sum_{k=0}^{K} \lvert v_{kp} - a_{kp} \rvert$	vote discrepancy in precinct p
$E = \sum_{p=1}^{N} e_p$	total discrepancy in the race
u_p	a priori upper bound on e_p. $u_p \leq \sum_{k=0}^{K} v_{kp} - \min_k v_{kp}$
w_p	relative weight for miscount in precinct p.
$\mathcal{J}_n^*$	a random sample of n elements of $\mathcal{N}$

Marginal notes

Can't tell which ballots were miscounted from totals.

Discrepancy in the counts in precinct p

$$e_p \equiv \frac{1}{2} \sum_{k=0}^{K} |v_{kp} - a_{kp}| \leq \sum_{k=0}^{K} v_{kp} - \min_k v_{kp}. \qquad (1)$$

To make false margin M, need $E \equiv \sum_{p=1}^{N} e_p \geq M/2$.

Moving $M/2$ votes from the apparent runner-up wipes out margin.

The Test

Suffices to test the hypothesis $E \geq M/2$. If reject at significance level α, either race is OK or something with chance $\leq \alpha$ happened.

Test statistic: weighted sample maximum.

$$\tau_w(\mathcal{J}_n^*) = \max_{p \in \mathcal{J}_n^*}(e_p/w_p). \tag{2}$$

Reject if τ_w is "small enough."

More notation

For $x, y \in \mathbb{R}^N$ and $\mathcal{J} \subset \mathcal{N}$, define

$$\mu_{\mathcal{J}}(x) \equiv \max_{p \in \mathcal{J}} |x_p| \tag{3}$$

$$\sigma_{\mathcal{J}}(x) \equiv \sum_{p \in \mathcal{J}} x_p. \tag{4}$$

$$(x \wedge y)_p = \min(x_p, y_p), \quad p \in \mathcal{N}, \tag{5}$$

$$(x \vee y)_p = \max(x_p, y_p), \quad p \in \mathcal{N} \tag{6}$$

$$(x/y)_p = x_p/y_p, \quad p \in \mathcal{N}, \quad y_p \neq 0. \tag{7}$$

$$\mathcal{X} = \mathcal{X}(u, M) \equiv \{x \in \mathbb{R}^N : x \leq u \text{ and } \sigma_{\mathcal{N}}(x) \geq M/2\}, \tag{8}$$

Maximum P-value

of the hypothesis $E \geq M/2$ if we observe $\tau_w(\mathcal{J}_n^*) = m$.

$$\pi(m) = \pi(m; n, u, w, M) \equiv \max_{x \in \mathcal{X}(u,M)} \mathbb{P}_x\{\mu_{\mathcal{J}_n^*}(x/w) \leq m\}, \quad (9)$$

Let $q = q(m, u, w, M)$ be the largest integer for which

$$\sigma_{\mathcal{J}_q^-}(u \wedge mw) + \sigma_{\mathcal{N} \setminus \mathcal{J}_q^-}(u) \geq M/2. \quad (10)$$

Then

$$\pi(m; n, u, w, M) = \begin{cases} 0 & q < n \\ \frac{\binom{q}{n}}{\binom{N}{n}} & q \geq n \end{cases} \quad (11)$$

Urn model!

How many tainted marbles in the urn? Find q iteratively: Set $\mathcal{J} = \mathcal{N}$.

1. If $\#\mathcal{J} < n$ or $\sigma_{\mathcal{J}}(u \wedge mw) + \sigma_{\mathcal{N}\setminus\mathcal{J}}(t) \geq M/2$, $q = \#\mathcal{J}$. .

2. Otherwise, let $p \equiv \arg\max_{j\in\mathcal{J}}[u - (u \wedge mw)]$. Remove p from $\mathcal{J}$ and return to step 1.

The whole shebang

1. select significance level α and a sequence (α_s) so that sequential tests at significance levels $\alpha_1, \alpha_2, \ldots$ give an overall significance $\leq \alpha$.

2. select positive precinct weights $w = (w_p)_{p \in \mathcal{N}}$. E.g. $w_p = b_p$.

3. select a rule for increasing the sample size n_s for stage s in the event that the sample at stage $s-1$ does not let us reject the hypothesis that the error $E \geq M/2$. Need $n_s - n_{s-1} \geq 1$.

4. select a multiplier m_1 that specifies our tolerance for error in the initial sample: want to reject the hypothesis that the overall error $E \geq M/2$, if for every precinct p in the initial sample, $e_p \leq m_1 w_p$.

5. choose a set of error bounds $(u_p)_{p \in \mathcal{N}}$,

6. compute the apparent margin M from the reported vote counts in each precinct, $\{v_p\}_{p \in \mathcal{N}}$

7. determine the initial sample size n_1 so that $\pi(m_1; n_1, u, w, M) \leq \alpha_1$.

8. set $s = 1$, $n_0 = 0$ and $\mathcal{J}_0 = \emptyset$.

9. draw a random sample $\mathcal{J}^*_{n_s - n_{s-1}}$ of size $n_s - n_{s-1}$ from $\mathcal{N} \setminus \mathcal{J}_{s-1}$. Set $\mathcal{J}_s = \mathcal{J}_{s-1} \cup \mathcal{J}^*_{n_s - n_{s-1}}$. Calculate $\tau_w(\mathcal{J}_s)$.

10. if $\pi(\tau_w(\mathcal{J}_s); n_s, u, w, M) \leq \alpha_s$, certify the election and stop. Otherwise, increment s.

11. if $n_s < N$, return to step 9. Otherwise, audit any precincts not yet in the sample. Certify the election if the outcome was correct.

Stratified samples

Can test separately in each stratum $c \in \mathsf{C}$ for $E_c \geq (M/2)(B_c/B)$.

Reject overall hypothesis if all C are rejected.

Composite test is conservative.

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