CAST: Canvass Audits by Sampling and Testing.

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Philip B. Stark
Department of Statistics
University of California, Berkeley
statistics.berkeley.edu/∼stark

Vote Counting

- Counts are subject to various kinds of error.
- Counting errors $\Rightarrow$ risk of naming the wrong winner.

Audit to deter & detect fraud
monitor/improve equipment, procedures, & software
ensure total error too small to change the outcome

Explanation of the error not important for verifying outcome

Statistical Audits

Can *limit* and *quantify* that risk.

Could guarantee that, if the election outcome is wrong (e.g., because of fraud or a hardware or software bug) there's a 99% chance the audit will call for a complete recount.

Count as little as possible whenever a full recount would show the same winner.

Need complete, accurate audit trail. Chain of custody; compliance vs. materiality.

Step 1: pick minimum chance β of catching an incorrect semi-official result

Typically matter of legislation or administrative rule.

To limit auditing burden, can choose β smaller for small races than for countywide or statewide races

Step 2: pick the maximum number of stages S

If the audit cannot confirm the semi-official outcome after the first stage of sampling, draw a second sample, etc.

If the audit cannot confirm the semi-official outcome at stage S (because the audit has found too much error at every stage), count all votes by hand.

Step 3: select subtotals comprising batches and strata

Define batches of votes; partition batches into strata.

Fewer votes per batch, generally smaller audit effort (when outcome is correct).

—Semi-official counts for each batch in a stratum *must be published* prior to sampling from stratum.

—Need upper bound on # valid votes in each batch for any candidate.

Every batch in exactly one stratum.

P: total # batches of ballots across all strata

C: total # strata.

B_c: # batches in stratum c, $c = 1, \ldots, C$.

Step 4: find upper bounds on the number of votes per candidate per batch

Need maximum # votes each candidate or position could possibly get, batch by batch.

Can use voter registrations, pollbooks, or an accounting of ballots.

E.g., if accounting of ballots shows b_p ballots voted in precinct p, # votes per candidate in precinct p is $\leq b_p$.

Step 5: initialize variables

Set $s = 1$; s is the current stage of the audit.

P_s is # as-yet-unaudited batches at stage s.

If there has been no targeted auditing so far, then $P_1 = P$.

Step 6: calculate all pairwise margins

For each semi-official winner w and loser ℓ, calculate the margin of victory in votes:

$$V_{w\ell} = \text{ (votes for winner } w) \; - \; \text{(votes for loser } \ell). \qquad (1)$$

based on semi-official results for the P_s batches not yet audited, and audit results for the $P - P_s$ batches already audited.

If any $V_{w\ell} \leq 0$, audit already found so much error that the list of winners has changed: abort the audit and count all votes manually.

Step 7: find upper bounds on the maximum overstatement of pairwise margins

Candidate w is any semi-official winner.

Candidate ℓ is any semi-official loser.

For each batch p not yet audited, compute

$$u_{pw\ell} = \frac{\text{votes for candidate } w \text{ in batch } p}{V_{w\ell}} - \frac{\text{votes for candidate } \ell \text{ in batch } p}{V_{w\ell}} + \frac{b_p}{V_{w\ell}},$$

according to the semi-official results for batch p.

Compute $u_p = \max_{w,\ell} u_{pw\ell}$.

u_p is most that error counting the votes in batch p could have overstated any margin, expressed as a fraction of that margin, adjusted for errors already discovered.

Step 8: targeted audits

If there are a few unaudited batches p for which u_p is much larger than the rest, auditing those batches deliberately can reduce substantially the sample size required in the random audit to follow.

If additional batches are selected for targeted audit, count them and return to step 6.

Step 9: select the desired threshold for "escalation"

Set the error "tolerance" $t \in [0, 1)$

If audit finds any margin is overstated by t or more, audit progresses to the next stage.

The smaller t is, the smaller the sample size at each stage—but also generally the greater the chance that the audit will progress to the next stage.

If $t = 0$, the audit will go to the next stage if the current sample has even one discrepancy that overstates any margin.

Step 10: find sample sizes for the next random sample

Define $\beta_0 = \beta^{1/S}$; $\tilde{u}_p = u_p - \min(t, u_p)$.

$T =$ sum of $\tilde{u}_p$ for all as-yet-unaudited batches p.

1. For not-yet-audited batches, start with the largest value of $\tilde{u}_p$; add successively smaller values until sum $\geq 1 - T$. q is # terms in the sum.

2. Find the smallest integer n such that
$$\left(\frac{P_s - q}{P_s}\right)^n \leq 1 - \beta_0. \tag{2}$$

3. Sample size n_c for stratum c is smallest whole number $\geq$
$$n \times \frac{\#\text{unaudited batches in stratum } c}{P_s}. \tag{3}$$

Step 11: draw the next sample and count votes

Select batches using transparent, mechanical, verifiable source of randomness, such as fair 10-sided dice.

Computer-generated “pseudo-random” numbers *not* appropriate: essentially impossible for public to verify the selection is fair.

For $c = 1, \ldots, C$, draw a random sample of n_c batches from the as-yet-unaudited batches in stratum c.

Count the votes for each candidate in each sample batch by hand.

Step 12: calculate the maximum pairwise overstatement

For each batch p just audited, calculate

$$
\begin{aligned}
e_{pw\ell} \;=\; & \text{((reported votes for } w - \text{reported votes for } \ell) \\
& - \text{(audited votes for } w - \text{audited votes for } \ell))\,/V_{w\ell}
\end{aligned}
$$

for all pairs (w, ℓ) of semi-official winners w and losers ℓ.

Define

$$t_s = \max_p \max_{w,\ell} e_{pw\ell} \tag{4}$$

over all batches p audited during stage s.

Step 13: certify, perform a full count, or proceed to the next step

If $t_s \leq t$ certify the election and stop.

If $t_s > t$, and we are at stage $s = S$, count all votes by hand.

Otherwise, add one to s; perform targeted auditing ad lib; set P_s to be the number of batches not yet audited; go back to step 6.

Logistical Issues

Must commit to counts in each stratum before sampling from stratum.

Staging/stratification: cross-county, absentee, provisional, etc.

Other sampling schemes: lessons from financial auditing

Confidence level? Vary by race? Random sample of races? Legislative input needed.

Data, data, data: need Preliminary Statement of Vote in machine-readable form.

Marin County, CA experiment

Confirmed outcome of February 2008 vote for Measure A (required 2/3 majority) at confidence $\beta = 75\%$.

Cost \$0.35 per audited ballot, including salaries, transportation, etc.

Plan to expand test to 5 California counties in November 2008.