Ballot-polling Risk-limiting Audits in Two Pages (± 1)

Philip B. Stark
Department of Statistics
University of California, Berkeley

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Purpose: Risk-limiting audit: Large chance of a full hand count of the paper if the machine-count outcome is wrong. Minimize number of ballots inspected if the machine-count outcome is right. *Risk* is the largest chance that a wrong outcome won't be corrected by a full hand count.

Requirements: Paper audit trail, *ballot manifest* that explains how ballots are stored, dice, pencil, paper.

Advantages: Virtually no set-up costs, requires nothing of voting system, preserves voter anonymity, counting burden low unless margin is very small, like an opinion poll

Disadvantages: Does not check tabulation, only winners

Historical workload: Among 255 state presidential contests between 1992 and 2008, the median expected sample size to confirm the plurality winner in each state using BRAVO was 307 ballots (per state).

Reference: Lindeman, M., P.B. Stark, and V.S. Yates, 2012. BRAVO: Ballot-polling Risk-Limiting Audits to Verify Outcomes. *2012 Electronic Voting Technology Workshop/Workshop on Trustworthy Elections (EVT/WOTE '12).* https://www.usenix.org/system/files/conference/evtwote12/evtwote12-final27.pdf

Tools for selecting ballots at random using dice and a ballot manifest are at `http://statistics.berkeley.edu/~stark/Vote/auditTools.htm`

Workload estimate: Two Candidates, 10% Risk Limit

Winner's	Ballots drawn		
True Share	median	90^{th} percentile	Mean
70%	22	60	30
65%	38	108	53
60%	84	244	119
58%	131	381	184
55%	332	974	469
54%	518	1,520	730
53%	914	2,700	1,294
52%	2,051	6,053	2,900
51%	8,157	24,149	11,556
50.5%	32,547	96,411	46,126

Procedure for 10% risk limit, one contest, majority winner:

1. Set $T = 1$. s is winner's share of the valid votes according to the vote tabulation system.

2. Select a ballot at random.

3. If the ballot shows a valid vote for the reported winner, multiply T by

$$2s.$$

4. If the ballot shows a valid vote for anyone else, multiply T by

$$2(1 - s).$$

5. If $T > 10$, stop the audit: Reported outcome stands.
   Otherwise, if we want to perform a full hand count at this point, do.
   If not, return to step 2.

Arbitrary number of contests and winners: For each contest under audit, consider all pairs (w, ℓ) of winners and losers. Let $s_{w\ell}$ be the fraction of votes w was reported to have received among ballots reported to show a vote for w or ℓ or both. For instance, suppose Alice, Bob, Candy, and Dan are candidates in a school board contest with two winners, in which voters were allowed to vote for up to two candidates. Alice reportedly received 80%, Bob 60%, Candy 25% and Dan 20%. Then there are four (winner, loser) pairs: (Alice, Candy), (Alice, Dan), (Bob, Candy), and (Bob, Dan). The corresponding values of s are

$$s_{\text{Alice Candy}} = 80\%/(80\% + 25\%) = 76.2\%,$$
$$s_{\text{Alice Dan}} = 80\%/(80\% + 20\%) = 80\%,$$
$$s_{\text{Bob Candy}} = 60\%/(60\% + 25\%) = 70.6\%,$$
$$s_{\text{Bob Dan}} = 60\%/(60\% + 20\%) = 75\%.$$

Full procedure for 10% risk limit:

1. Set $T_{w\ell} = 1$ for all (winner, loser) pairs (w, ℓ) in each audited contest.
2. Select a ballot at random.
3. If the ballot shows a valid vote for a reported winner w in some audited contest, then for each loser ℓ in that contest that did not receive a valid vote on that ballot, multiply $T_{w\ell}$ by $2s_{w\ell}$. Repeat for all such w and for all audited contests on the ballot.
4. If the ballot shows a valid vote for a reported loser ℓ in some audited contest, then for each winner in that contest that did not receive a valid vote on that ballot, multiply $T_{w\ell}$ by $2(1 - s_{w\ell})$. Repeat for all such ℓ and for all audited contests on the ballot.
5. If any $T_{w\ell} \geq 10$, do not update that $T_{w\ell}$ again, even if we draw more ballots.
6. If all $T_{w\ell}$ are at least 10, stop the audit: The reported results stand. Otherwise, if we want to perform a full hand count at this point, do. If not, return to step 2.