
EXAMPLE **B** Let us calculate how much better optimal allocation is than proportional allocation for the population of hospitals. From Theorem C and Corollary A, we have

$$\mathrm{Var}(\overline{X}_{sp}) = \mathrm{Var}(\overline{X}_{so}) + \frac{1}{n}\sum W_l(\sigma_l - \bar{\sigma})^2$$

Therefore,

$$\begin{aligned}\frac{\mathrm{Var}(\overline{X}_{sp})}{\mathrm{Var}(\overline{X}_{so})} &= 1 + \frac{\frac{1}{n}\sum W_l(\sigma_l - \bar{\sigma})^2}{\mathrm{Var}(\overline{X}_{so})} \\ &= 1 + \frac{\sum W_l(\sigma_l - \bar{\sigma})^2}{(\sum W_l\sigma_l)^2} \\ &= 1 + .218\end{aligned}$$

Thus, under proportional allocation, the variance of the mean is about 20% larger than it is under optimal allocation. ■

We can also compare the variance under simple random sampling with the variance under proportional allocation. The variance under simple random sampling is, neglecting the finite population correction,

$$\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$$

In order to compare this equation with that for the variance under proportional allocation, we need a relationship between the overall population variance, σ^2, and the strata variances, σ_l^2. The overall population variance may be expressed as

$$\sigma^2 = \frac{1}{N}\sum_{l=1}^{L}\sum_{i=1}^{N_l}(x_{il} - \mu)^2$$

Also,

$$\begin{aligned}(x_{il} - \mu)^2 &= [(x_{il} - \mu_l) + (\mu_l - \mu)]^2 \\ &= (x_{il} - \mu_l)^2 + 2(x_{il} - \mu_l)(\mu_l - \mu) + (\mu_l - \mu)^2\end{aligned}$$

When both sides of this last equation are summed over l, the middle term on the right-hand side becomes zero since $N_l\mu_l = \sum_{l=1}^{N_l} x_{il}$, so we have

$$\begin{aligned}\sum_{i=1}^{N_l}(x_{il} - \mu)^2 &= \sum_{i=1}^{n_l}(x_{il} - \mu_l)^2 + N_l(\mu_l - \mu)^2 \\ &= N_l\sigma_l^2 + N_l(\mu_l - \mu)^2\end{aligned}$$

Dividing both sides by N and summing over l, we have

$$\sigma^2 = \sum_{l=1}^{L} W_l\sigma_l^2 + \sum_{l=1}^{L} W_l(\mu_l - \mu)^2$$