If the sampling fractions within all strata are small,

$$\text{Var}(\overline{X}_s) \approx \sum_{l=1}^{L} \frac{W_l^2 \sigma_l^2}{n_l}$$

EXAMPLE A We again consider the population of hospitals. As we did in the discussion of ratio estimates, we assume that the number of beds in each hospital is known but that the number of discharges is not. We will try to make use of this knowledge by stratifying the hospitals according to the number of beds. Let stratum A consist of the 98 smallest hospitals, stratum B of the 98 next larger, stratum C of the 98 next larger, and stratum D of the 99 largest. The following table shows the results of this stratification of hospitals by size:

Stratum	N_l	W_l	μ_l	σ_l
A	98	.249	182.9	103.4
B	98	.249	526.5	204.8
C	98	.249	956.3	243.5
D	99	.251	1591.2	419.2

Suppose that we use a sample of total size n and let

$$n_1 = n_2 = n_3 = n_4 = \frac{n}{4}$$

so that we have equal sample sizes in each stratum. Then, from Theorem B, neglecting the finite population corrections and using the numerical values in the preceding table, we have

$$\begin{aligned}\text{Var}(\overline{X}_s) &= \sum_{l=1}^{4} \frac{W_l^2 \sigma_l^2}{n_1} \\ &= \frac{4}{n} \sum_{l=1}^{4} W_l^2 \sigma_l^2 \\ &= \frac{72{,}042.6}{n}\end{aligned}$$

and

$$\sigma_{\overline{X}_s} = \frac{268.4}{\sqrt{n}}$$

The standard deviation of the mean of a simple random sample is

$$\sigma_{\overline{X}} = \frac{587.7}{\sqrt{n}}$$

Comparing the two standard deviations, we see that a tremendous gain in precision has resulted from the stratification. The ratio of the variances is .20; thus a stratified estimate based on a total sample size of $n/5$ is as precise as a simple random sample of size n. The reduction in variance due to stratification is comparable to that achieved