Stat153 Midterm Exam 1. (October 7, 2010)

Name:...

Student ID:...

This is an open-book exam: you can use any material you like. Exam papers will be handed out at 12:40, the exam will go from 12:45 to 1:55. There are six questions. You may answer all six if you wish; your grade will consist of the total from the best five questions. Each part of each question has a percentage written next to it - the percentage of the grade that it constitutes.

1. (20% total) Consider the time series

$$X_t = -2t + W_t + 0.5W_{t-1},$$

where $W_t \sim N(0, \sigma^2)$.

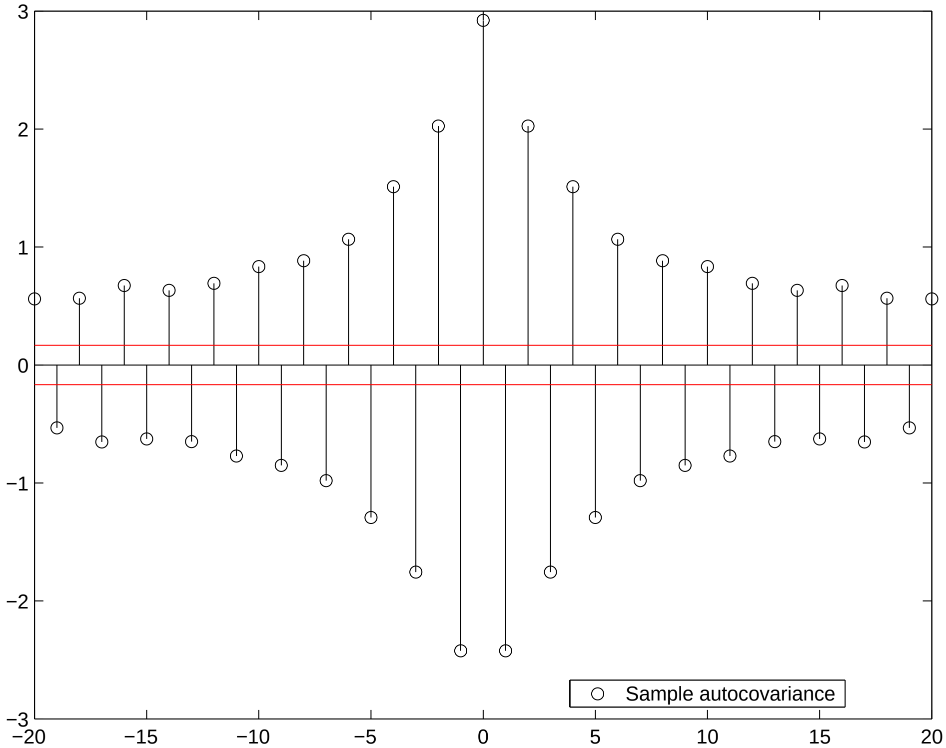
   (a) (10%) What are the mean function and autocovariance function of this time series? Is this time series stationary? Justify your answer.

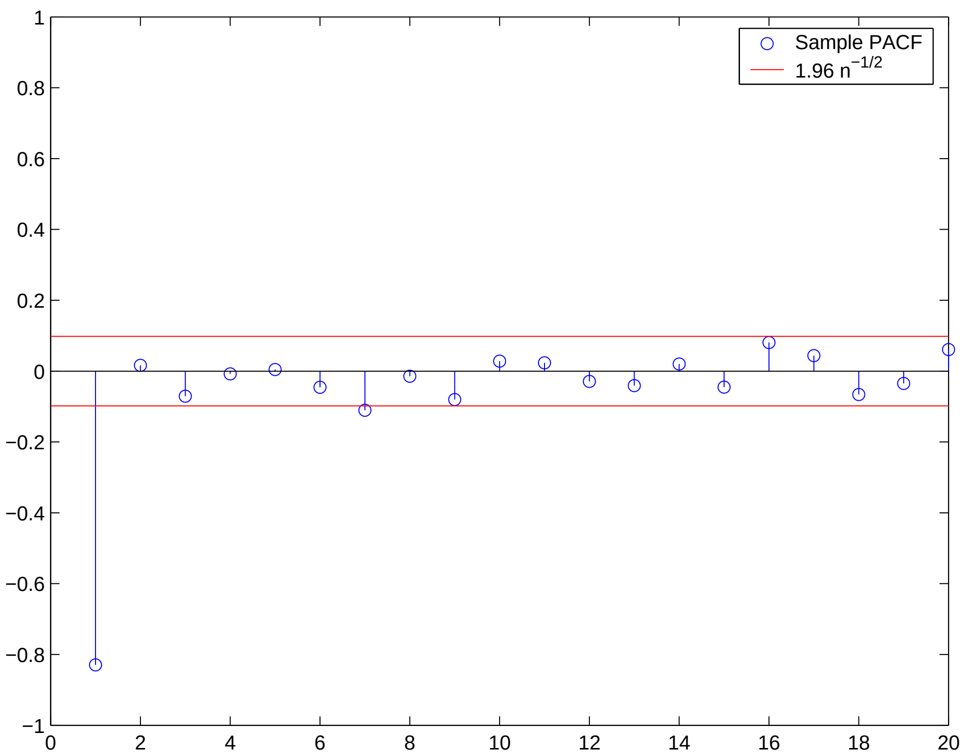
(b) (10%) Consider the first differences of the time series above, that is, consider

$$Y_t = \nabla X_t = X_t - X_{t-1}.$$

What are the mean function and autocovariance function of this time series? Is this time series stationary? Justify your answer.

2. (20% total) Let $x_1, x_2, \ldots, x_T$ be a realization of a stationary time series. Here are the sample autocovariance and the sample partial autocorrelation functions.





(a) (10%) We wish to model this time series. Would you suggest an AR(p) or an MA(q) model? What should its order be? Why?

(b) (10%) What is the variance $\mathsf{Var}(X_t)$?

3. (20% total) Consider the following ARMA model

$$X_t = X_{t-1} - 0.25X_{t-2} + W_t - 0.25W_{t-1},$$

where $W_t \sim N(0, 1)$.

(a) (10%) This model is causal. Why?

(b) (10%) Compute the coefficients ψ_j of the causal representation of this time series,

$$X_t = W_t + \sum_{j=1}^{\infty} \psi_j W_{t-j}.$$

4. (20% total) Consider the ARMA model of the previous question:

$$X_t = X_{t-1} - 0.25X_{t-2} + W_t - 0.25W_{t-1}.$$

(a) (10%) This model is also invertible. Why?

(b) (10%) Compute the coefficients π_j of the invertible representation of this time series,

$$W_t = X_t + \sum_{j=1}^{\infty} \pi_j X_{t-j}.$$

5. (20% total) Consider the following AR(3) process

$$X_t = 0.2X_{t-1} - 0.2X_{t-2} + 0.6X_{t-3} + W_t,$$

where $W_t \sim N(0, \sigma_w^2)$.

(a) (10%) We wish to forecast X_{11} using the following values:

$$\begin{array}{lllll} x_1 = 3.1, & x_2 = 0.93, & x_3 = -2.3, & x_4 = 3.2, & x_5 = 1.3, \\ x_6 = -2.5, & x_7 = 1.6, & x_8 = 3.0, & x_9 = -3.5, & x_{10} = -0.74. \end{array}$$

Use the best linear predictor to compute this forecast.

(b) (10%) If $\sigma_w^2 = 1$, give a 95% prediction interval for the value X_{11}.

6. (20% total) Suppose that we have 1500 observations $x_1, x_2, \ldots, x_{1500}$ of a time series, and we have chosen to estimate an AR(2) model,

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + W_t,$$

where $W_t \sim WN(0, \sigma_w^2)$. We have computed the following sample autocovariances.

$$\hat{\gamma}(0) = 5.0, \quad \hat{\gamma}(1) = 0.0, \quad \hat{\gamma}(2) = 2.5, \quad \hat{\gamma}(3) = 0.0, \quad \hat{\gamma}(4) = 1.0.$$

(a) (10%) Estimate the coefficients ϕ_1, ϕ_2, and the noise variance σ_w^2.

(b) (10%) Give an approximate 95% confidence interval for the coefficient ϕ_2.