

13. Let the joint probability density function of X and Y be given by

$$f(x, y) = \begin{cases} 2e^{-(x+2y)} & \text{if } x \geq 0, \quad y \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X^2Y)$.

14. Let X and Y be two independent random variables with the same probability density function given by

$$f(x) = \begin{cases} e^{-x} & \text{if } 0 < x < \infty \\ 0 & \text{elsewhere.} \end{cases}$$

Show that g , the probability density function of X/Y , is given by

$$g(t) = \begin{cases} \frac{1}{(1+t)^2} & \text{if } 0 < t < \infty \\ 0 & t \leq 0. \end{cases}$$

15. Let X and Y be independent exponential random variables both with mean 1. Find $E[\max(X, Y)]$.
16. Let X and Y be independent random points from the interval $(-1, 1)$. Find $E[\max(X, Y)]$.
17. Let X and Y be independent random points from the interval $(0, 1)$. Find the probability density function of the random variable XY .
18. A point is selected at random from the disk

$$R = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 \leq 1\}.$$

Let X be the x -coordinate and Y be the y -coordinate of the point selected. Determine if X and Y are independent random variables.

19. Six brothers and sisters who are all either under 10 or in their early teens are having dinner with their parents and four grandparents. Their mother unintentionally feeds the entire family (including herself) a type of poisonous mushrooms that makes 20% of the adults and 30% of the children sick. What is the probability that more adults than children get sick?
20. The lifetimes of mufflers manufactured by company A are random with the following density function:

$$f(x) = \begin{cases} \frac{1}{6}e^{-x/6} & \text{if } x > 0 \\ 0 & \text{elsewhere.} \end{cases}$$