Problems:
Q 1 Ferguson Chapter II Section 2.6 Q 6 (a)
Solution
\[
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]
The first and seventh rows are dominated by the second and sixth respectively.

\[
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1
\end{pmatrix}
\]
Columns 3 and 5 are dominated by columns 1 and 7 respectively.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]
Row 3 is dominated by row 1.

\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]
Column 3 is dominated by column 2.
The remaining matrix is diagonal. Using the formula for diagonal matrices we have that the value is \( \frac{1}{4} \) and the optimal strategies are \( x = y = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) \). In the original game this translates to strategies of

\[
x = (0, \frac{1}{4}, \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{4}, 0), \quad y = (\frac{1}{4}, \frac{1}{4}, 0, 0, 0, \frac{1}{4}, \frac{1}{4})
\]

Q 2 Ferguson Chapter II Section 2.6 Q 9

The payoff matrix is

\[
\begin{pmatrix}
0 & 1 & 2 \\
1 & 0 & 1 \\
2 & 1 & 0
\end{pmatrix}
\]

The second row is dominated by the average of the first and third rows.

\[
\begin{pmatrix}
0 & 1 & 2 \\
2 & 1 & 0
\end{pmatrix}
\]

The second column is dominated by the average of the first and third rows

\[
\begin{pmatrix}
0 & 2 \\
2 & 0
\end{pmatrix}
\]

By symmetry between the two states the optimal strategies are \( x = y = (\frac{1}{2}, \frac{1}{2}) \) and the value is 1. The optimal strategies in the original game are \( x = y = (\frac{1}{2}, 0, \frac{1}{2}) \).

Q 3 Ferguson Chapter II Section 3.7 Q 2 (a) The diagonal term is a saddle point so the value is 0.

(b) If \( d_i > 0 \) and \( d_j < 0 \) then \((i, j)\) is a saddle point so the value is 0.

(c) If all the diagonal entries are negative then the value is \((d_1^{-1} + \ldots + d_n^{-1})^{-1}\).

Q 4 Ferguson Chapter II Section 3.7 Q 4

The payoff matrix is

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
\frac{1}{2} & 1 & 0 & 0 \\
\frac{1}{2} & \frac{1}{2} & 1 & 0 \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{pmatrix}
\]

We get the system of linear equations

\[
x_1 + \frac{1}{2}(x_2 + x_3 + x_4) = v \\
x_2 + \frac{1}{2}(x_3 + x_4) = v \\
x_3 + \frac{1}{2}x_4 = v \\
x_4 = v \\
x_1 + x_2 + x_3 + x_4 = 1
\]

Solving these we get \( x_4 = v, \) \( x_3 = v - \frac{1}{2}x_4 = v/2, \) \( x_2 = v - \frac{1}{2}(x_3 + x_4) = v/4 \) and \( x_1 = v - \frac{1}{2}(x_2 + x_3 + x_4) = v/8 \). Then \( 1 = x_1 + x_2 + x_3 + x_4 = \frac{15}{8}v \) so \( v = \frac{8}{15} \) and \( x = (\frac{1}{15}, \frac{2}{15}, \frac{4}{15}, \frac{8}{15}) \). Solving similarly for \( y \) we have that \( y = (\frac{8}{15}, \frac{4}{15}, \frac{2}{15}, \frac{1}{15}) \).