Problems:
From the text: Section 6.13 Q 1, Section 6.15 Q 30, 49

Q 1 On a graph $G = (V,E)$ and for $\lambda > 0$ define a distribution on independent sets $\sigma \in \{0,1\}^V$ as follows:

$$P[\sigma] = \frac{1}{Z} \lambda^{\sum_{i \in V} \sigma_i} I(\sigma \text{ is an independent set})$$

Describe a step of the Gibbs sampler. What can you say about this distribution when $\lambda$ is very large. Describe how you might use this to find the largest independent sets in the graph. Can you suggest a reason why this may not be an efficient Markov chain?

Q 2 On a graph $G = (V,E)$ and functions $\psi_i : \{0,1\} \rightarrow \mathbb{R}, \phi_{i,j} : \{0,1\}^2 \rightarrow \mathbb{R}$ we define a probability distribution on $\sigma \in \{0,1\}^V$ as follows:

$$P[\sigma] = \frac{1}{Z} \exp \left[ \sum_{i \in V} \psi_i(\sigma_i) + \sum_{(i,j) \in E} \phi_{i,j}(\sigma_i, \sigma_j) \right]$$

Describe a step of the Gibbs sampler for this distribution.