Kalman recursions.

Proof. (Text.)

\[
I_t = Y_t - P_t Y_t
\]

orthogonal

\[
= Y_t - G_t \hat{X}_t
\]

\[
= G_t (X_t - \hat{X}_t) + w_t
\]

\[
P(\cdot | \ldots, Y_t)
\]

\[
= P(\cdot | \ldots, Y_{t-1}) + P(\cdot | I_t)
\]
\[ x_{t+1} = P_t (x_t) \]
\[ = P_{t-1} (x_{t+1}) + P (x_{t+1} | I_t) \]
\[ \Delta_t = E_{\pi_t} I_t' = G_t \Omega_t G_t' + R_t \]
\[ \Theta_t = E_{\pi_{t+1}} I_t' = F_t \Omega_t G_t' \]
\[ P(x_{t+1} | I_t) = \Theta_t \Delta_t^{-1} I_t \]
\[ P_{t-1} (x_{t+1}) = P_{t-1} (F X_t + V_t) \]
\[ = F_{t-1} X_t \]

Giving (8.4.1)
For (8.4.2)

\[
\Omega_{t+1} = E\left[(\hat{X}_{t+1} - \hat{X}_t)(\hat{X}_{t+1} - \hat{X}_t)\right]
\]

= \left( E(\hat{X}_{t+1} \hat{X}_t) - E(\hat{X}_{t+1})E(\hat{X}_t) \right) \tag{\circ}

\begin{align*}
\{ & \quad E(U) = E(E(U|A)) \\
    & \quad \text{Var}(U) = E\text{Var}(U|A) + \text{Var}(E(U|A)) \}
\end{align*}

\(\circ = E_+ E(\hat{X}_t \hat{X}_t) F'_+ \Omega t_+ \Omega t_+ \tag{8.4.2} \)

- \quad E_+ E(\hat{X}_t \hat{X}_t) F'_+ \quad \Omega t_+ \Omega t_+ \quad \Theta \quad \Theta' \quad \tag{8.4.2} \)