STAT 134 (P2): CONCEPTS OF PROBABILITY, UC BERKELEY, SPRING 2013

Problem Set 9

Instructor: Prof. Yun S. Song

Due: April 11, 2013

Show all your work to receive full credit.

- 1. [15 Points] Do the following problems from the textbook: 5.1 (2, 4, 6), 5.2 (10, 12)
- 2. [5 Points] Let X and Y denote continuous random variables with joint probability density

$$f(x,y) = \begin{cases} 3e^{-2x-y}, & \text{if } 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) [2 Points] Find the marginal probability density functions of X and Y.
- (b) [1 Points] Are X and Y independent? Explain.
- (c) [2 Points] Find $\mathbb{E}(XY)$.
- 3. [12 Points] Let X_1, \ldots, X_n be independent and uniformly distributed on the interval [0, a], and let L_1, \ldots, L_{n+1} denote the associated gaps, as discussed in class.
 - (a) [3 Points] Find the joint probability density of the order statistics $X_{(i)}$ and $X_{(j)}$, where $1 \le i < j \le n$.
 - (b) [3 Points] Find the probability density of the spread $Y = X_{(n)} X_{(1)}$ and the expectation of Y.
 - (c) [6 Points] (Challenge problem) Let $L_{(1)}$ denote the first order statistic of L_1, \ldots, L_{n+1} . Find the probability density of $L_{(1)}$.