Stat 134 (P2): Concepts of Probability, UC Berkeley, Spring 2013

## Problem Set 9

Instructor: Prof. Yun S. Song
Due: April 11, 2013

## Show all your work to receive full credit.

1. [15 Points] Do the following problems from the textbook: $5.1(2,4,6), 5.2(10,12)$
2. [5 Points] Let $X$ and $Y$ denote continuous random variables with joint probability density

$$
f(x, y)= \begin{cases}3 e^{-2 x-y}, & \text { if } 0<x<y<\infty \\ 0, & \text { otherwise }\end{cases}
$$

(a) [2 Points] Find the marginal probability density functions of $X$ and $Y$.
(b) [1 Points] Are $X$ and $Y$ independent? Explain.
(c) $[2$ Points $]$ Find $\mathbb{E}(X Y)$.
3. [12 Points] Let $X_{1}, \ldots, X_{n}$ be independent and uniformly distributed on the interval [ $0, a$ ], and let $L_{1}, \ldots, L_{n+1}$ denote the associated gaps, as discussed in class.
(a) [3 Points] Find the joint probability density of the order statistics $X_{(i)}$ and $X_{(j)}$, where $1 \leq i<j \leq n$.
(b) [3 Points] Find the probability density of the spread $Y=X_{(n)}-X_{(1)}$ and the expectation of $Y$.
(c) [6 Points] (Challenge problem) Let $L_{(1)}$ denote the first order statistic of $L_{1}, \ldots, L_{n+1}$. Find the probability density of $L_{(1)}$.

