Show all your work to receive full credit.

1. [15 Points] Do the following problems from the textbook: 5.1 (2, 4, 6), 5.2 (10, 12)

2. [5 Points] Let $X$ and $Y$ denote continuous random variables with joint probability density

$$f(x, y) = \begin{cases} 3e^{-2x-y}, & \text{if } 0 < x < y < \infty, \\ 0, & \text{otherwise}. \end{cases}$$

(a) [2 Points] Find the marginal probability density functions of $X$ and $Y$.

(b) [1 Points] Are $X$ and $Y$ independent? Explain.

(c) [2 Points] Find $E(XY)$.

3. [12 Points] Let $X_1, \ldots, X_n$ be independent and uniformly distributed on the interval $[0, a]$, and let $L_1, \ldots, L_{n+1}$ denote the associated gaps, as discussed in class.

(a) [3 Points] Find the joint probability density of the order statistics $X_{(i)}$ and $X_{(j)}$, where $1 \leq i < j \leq n$.

(b) [3 Points] Find the probability density of the spread $Y = X_{(n)} - X_{(1)}$ and the expectation of $Y$.

(c) [6 Points] (Challenge problem) Let $L_{(1)}$ denote the first order statistic of $L_1, \ldots, L_{n+1}$. Find the probability density of $L_{(1)}$. 