Stat 134 (P2): Concepts of Probability, UC Berkeley, Spring 2013

Problem Set 8

Instructor: Prof. Yun S. Song — Due: April 4, 2013

Show all your work to receive full credit.

1. [9 Points] Do the following problems from the textbook: 4.4 (10), 4.5 (4, 6)

2. [15 Points] Han Solo gets a new laser gun for his birthday and decides to test it out. He goes to a shooting range that has an infinitely long straight wall, with coordinate system $x \in (-\infty, +\infty)$ along the wall, parallel to the floor. Han Solo goes to position x_0, stands at distance d from the wall, and then fires the laser gun horizontally in a uniformly random direction toward the wall. Let X denote the x-coordinate of the point on the wall where the laser hits.

   (a) [3 Points] Find the density function $f_X(x)$ of X.

   (b) [3 Points] Does $\mathbb{E}(X)$ exist? If so, find it. If not, explain why it does not exist.

   (c) [3 Points] Define $Y = \frac{d^2}{d^2+(X-x_0)^2}$ and find the density function $f_Y(y)$ of Y.

   (d) [3 Points] Find the cumulative distribution function $F(y)$ of Y.

   (e) [3 Points] Find $\mathbb{E}(Y)$ and $\mathrm{Var}(Y)$.

3. [6 Points] Recall the airport check-in desk problem discussed in class: Your airline has two counters at the check-in desk. You arrive there at the same time as two other customers X and Y, who immediately take the two counters. You will wait and take whichever of the two counters becomes available first. Assume that service times for different customers are independent and are all exponentially distributed with the same parameter λ.

   (a) [3 Points] What is the expected time (time of waiting in line plus the time of service) until you get your ticket?

   (b) [3 Points] What is the expected total time until all three customers get their tickets?

4. [3 Points] Let $X_1, \ldots, X_n$ be independent and uniformly distributed on the unit open interval $(0, 1)$, and let $X_{(1)}, \ldots, X_{(n)}$ denote the associated order statistics. For fixed k, find the limiting density of $nX_{(k)}$ as $n \to \infty$. This limiting density function corresponds to that of a probability distribution familiar to you. What are its name and parameters? Explain why this convergence result is intuitive.