## Problem Set 3

Instructor: Prof. Yun S. Song
Due: February 14, 2013, in the beginning of class.

1. Consider a biased coin that shows heads with probability $p$ and tails with probability $1-p$.
(a) Suppose Alice tosses the coin exactly 100 times. Let " $H=i$ " denote the event of observing $i$ heads and " $T=j$ " the event of observing $j$ tails. For $0 \leq i, j \leq 100$, find $\mathbb{P}(H=i, T=j)$, $\mathbb{P}(H=i)$, and $\mathbb{P}(T=j)$. Are " $H=i$ " and " $T=j$ " independent events?
(b) Suppose Alice is given a random number generator that returns a non-negative integer $k$ with probability $e^{-\lambda} \lambda^{k} / k$ !, where $\lambda>0$ (i.e., it follows a Poisson distribution with parameter $\lambda)$. Alice uses the random number generator to obtain a single non-negative random integer and tosses the biased coin exactly that many times. As before, let " $H=i$ " denote the event of observing $i$ heads and " $T=j$ " the event of observing $j$ tails. For non-negative integers $i, j$, find $\mathbb{P}(H=i, T=j), \mathbb{P}(H=i)$, and $\mathbb{P}(T=j)$. Are " $H=i$ " and " $T=j$ " independent events?
2. Do the following problems from the textbook: $2.2(17) ; 2.4(6,8) ; 2.5(4,8)$
