Stat 134 (P2): Concepts of Probability, UC Berkeley, Spring 2013

# Problem Set 11

Instructor: Prof. Yun S. Song Due: April 25, 2013

**Show all your work to receive full credit.**

1. [9 Points] Do the following problems from the textbook: 6.2 (2,4), 6.3 (2)

2. [5 Points] Consider a Poisson process with rate $\lambda$. Let $W_1$ denote the waiting time to the first arrival, and, for $i > 1$, let $W_i$ denote the waiting time between the $(i-1)$th and the $i$th arrivals. For $1 \leq k \leq n$, find the conditional density of $T_k = W_1 + \cdots + W_k$ given that there are $n$ arrivals in the time interval $(0, 1)$. What is the name of this density? Determine its parameters.

3. [10 Points] Consider a sequence of independent Bernoulli trials, each with $\mathbb{P}(\text{success}) = p$ and $\mathbb{P}(\text{failure}) = 1 - p$. A "run of $s$ successes" is defined as $s$ consecutive trials that result in successes. A "run of $f$ failures" is similarly defined.

   (a) [5 Points] Find the probability that a run of $s$ successes occurs before a run of $f$ failures.

   (b) [5 Points] Find the expected number of trials until a run of $s$ successes is obtained for the first time.

4. [15 Points] Let $X_1, \ldots, X_n$ be independent and uniformly distributed on the interval $[0, a]$, and let $L_1, \ldots, L_{n+1}$ denote the associated gap sizes. For $i < j$, define $M_{i,j} = \min\{L_i, L_{i+1}, \ldots, L_j\}$.

   (a) [5 Points] Find $\mathbb{P}(M_{2,n+1} > x)$.

   (b) [5 Points] Find $\mathbb{P}(M_{2,n} > x \mid X_{(n)} = t)$.

   (c) [5 Points] For $x, t \in [0, a]$, find $\mathbb{P}(X_{(1)} \leq x \mid X_1 = t)$, where $X_{(1)}$ is the first order statistic of $X_1, \ldots, X_n$. This problem illustrates that the conditional distribution of a continuous random variable can be discontinuous.