Statistics 135 Spring 2004.

Homework #10 - Due 5/5.

1. From chapter 12: 3, 7, 19 (do not do non-parametric test, but do all possible t-tests with appropriate Bonferroni adjustment.)

2. Suppose data is generated by the 'zero-intercept' simple linear regression model given by
$$y_i = \beta_1 x_i + \epsilon_i.$$
Here, $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2), i = 1, \ldots, n$.

   (a) Derive the MLE for β_1.

   (b) Derive the least squares estimate of β_1 using the matrix approach. Be sure to identify the involved vectors. Does your result agree with part (a)?

   (c) Find $\sigma^2_{\hat{\beta}_1}$.

   (d) Argue that the residuals $e_i = (y_i - \hat{y}_i)$ follow a normal distribution with mean zero. Do not apply Thm 5.

   (e) For the following data $(x_i, y_i) = \{(1,1), (2,1), (3,4), (4,4)\}$ estimate β_1 and $\sigma^2_{\hat{\beta}_1}$. Does the fitted LS line pass through the origin? Verify.

   (f) Using the results of part (e), test the hypothesis that there is no linear relationship between X and Y. Use $\alpha = .05$. (Be sure to get the *degrees of freedom* correct!)

3. For the multiple linear regression setting.

   (a) Prove Thm 9 (d).

   (b) With $\mathbf{P}$ defined as in Thm 9, show $\mathbf{PX} = \mathbf{X}$.

   (c) With $\hat{\beta}$ the LS estimate, show that $\mathbf{e}'\mathbf{X}\hat{\beta} = \mathbf{0}$. What does this othogonality mean?

4. Select and work a simple linear regression problem of your choosing from the book. In addition to the book problem, make sure to include plots (of $\mathbf{X}, \mathbf{Y}$ as well as residual plots.)