Statistics 135 Fall 2003.

FINAL EXAM. Name: Student ID:

Instructions: **Show your work! Good luck!**

1. Suppose $X_i \overset{iid}{\sim} f(x|p), i = 1, \ldots, n$. Here, $f(x|p) = \binom{m}{x} p^x (1-p)^{m-x}$.

   (a) (6pts) Derive the maximum likelihood estimate (MLE) of p.

   (b) (6pts) Derive the asymptotic variance of the MLE.

2. (4pts) The figure below depicts a QQ-plot for data $X_i \sim F(x)$ and $Y_i \sim G(y)$.

   Which relationship do you think is the most likely between $F(x)$ and $G(y)$? Pick the correct answer.

   (a) $G(y) = F(y)$

   (b) $G(y) = F(y - a)$, some $a < 0$.

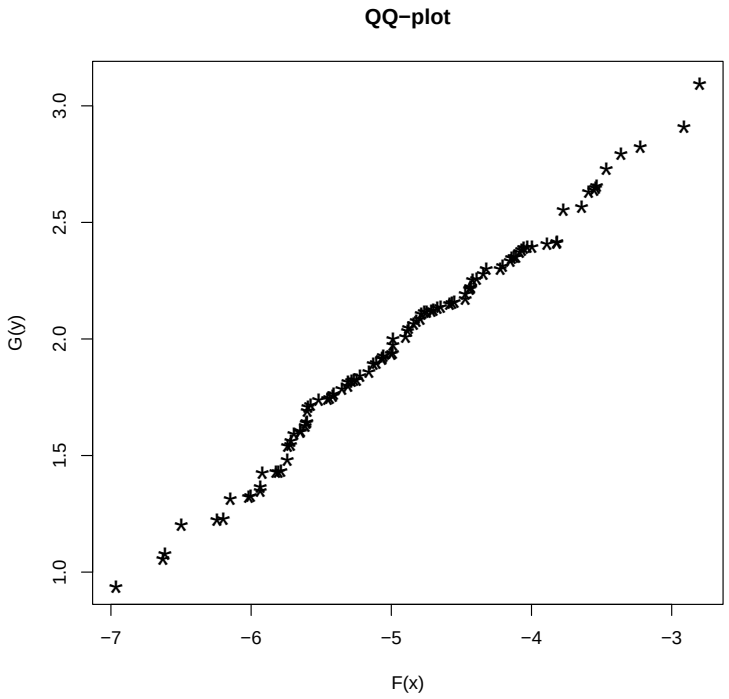
   (c) $G(y) = F(\frac{y}{b})$, some $b > 0$.

   (d) $G(y) = F(\frac{y-a}{b})$, some $a < 0, b < 0$.

   (e) $G(y) = F(\frac{y+a}{b})$, some $a > 0, b > 0$.

   (f) $G(y) = F(\frac{y-a}{b})$, some $a < 0, b > 0$.

   (g) None of the above.



3. Hout et al. (1981) reports on a survey addressing couples satisfaction with marital life. One of the question items asked "Sex is fun for me and my partner", to which the following two responses were available: (a) occasionally or somewhat often, or (b) very often or always. The following table summarizes the responses of the survey:

	Wife's rating	
Husband's rating	Occasionally, Somewhat often	Very often, always
Occasionally, Somewhat often	24	19
Very often, always	16	36

(a) (2pts) Are the husband's and wife's responses associated? This is *not* a "Fisher's exact" setup; but is it a test of homogeneity or a test of independence? Explain *briefly* in words and be sure to state your hypotheses in terms of your decision.

Ho:

Ha:

(b) (6pts) Test your hypotheses at $\alpha = .10$.

Critical value:
Brief conclusion in English:

(c) (4pts) What is the estimate of the conditional probability that the husband is very satisfied given that the wife is similarly satisfied? Compare to the conditional probability that the husband is very satisfied given that the wife is not satisfied. (You don't have to interpret).

4. The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results were:

Circuit type	Response time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

(a) (7pts) Test the hypothesis that the circuit types have the same response time.

(b) (3pts) Which assumptions must be met for the test in part (a) to be valid?

5. The following data give the diffusion time (hours) of a silicon wafer used in manufacturing integrated circuits and the resulting sheet resistance of transfer:

Diffusion time	.56	1.1	1.58	2	2.45
Sheet resistance	83.7	90	90.2	94.2	91.6

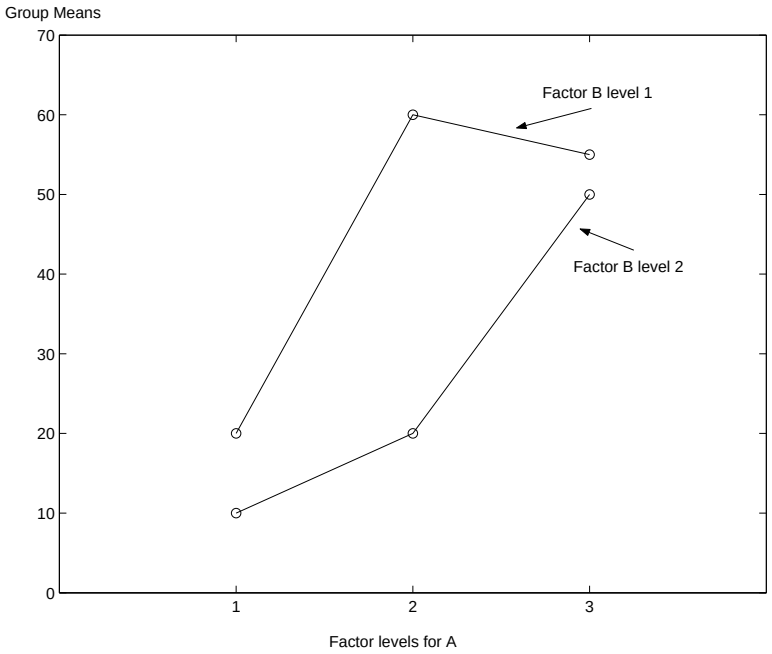
(a) (6pts) Find the least squares line predicting resistance from diffusion time.

(b) (4pts) Test if there is a linear relationship between diffusion time and sheet resistance at $\alpha = .05$. Be sure to specify your hypotheses below.

Ho : vs. Ha : .

Critical value:
Conclusion (circle one): Reject Fail to reject

6. (4pts) The figure below depicts the observed means in a two-way Anova experiment. Factor A has three levels, and factor B has two levels, yielding a total of six experimental groups. Study the graph and tell if there seems to be an interaction between the factors A and B. Motivate your answer!



7. Suppose observations $\{y_i, x_i\}, i = 1, \ldots, n$, are generated by the model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, where $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2)$.

   (a) (6pts) Assuming that β_1 and σ^2 are both known constants, derive the maximum likelihood estimate of β_0.

   (b) (6pts) Assuming that β_1 and σ^2 are both known constants, derive the (generalized) likelihood ratio test for testing

   $$\text{Ho} : \beta_0 = 0 \quad \text{vs.} \quad \text{Ha} : \beta_0 \neq 0.$$

   Be sure to indicate *exactly* what your test statistic is, and and for what values of the statistic (i.e., small or large) the null hypothesis is rejected.

(c) (4pts) Indicate what distribution your test statistic has, and find a critical value corresponding to $\alpha = .1$.