Ballot-polling Risk-limiting Audits in Two Pages (±1)

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Purpose: Risk-limiting audit: Large chance of a full hand count of the paper if the machine-count outcome is wrong. Minimize number of ballots inspected if the machine-count outcome is right. Risk is the largest chance that a wrong outcome won’t be corrected by a full hand count.

Requirements: Paper audit trail, ballot manifest that explains how ballots are stored, dice, pencil, paper.

Advantages: Virtually no set-up costs, requires nothing of voting system, preserves voter anonymity, counting burden low unless margin is very small, like an opinion poll

Disadvantages: Does not check tabulation, only winners

Historical workload: Among 255 state presidential contests between 1992 and 2008, the median expected sample size to confirm the plurality winner in each state using BRAVO was 307 ballots (per state).


Tools for selecting ballots at random using dice and a ballot manifest are at http://statistics.berkeley.edu/~stark/Vote/auditTools.htm
Workload estimate: Two Candidates, 10% Risk Limit

<table>
<thead>
<tr>
<th>Winner’s True Share</th>
<th>Ballots drawn</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>median</td>
<td>90th percentile</td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>70%</td>
<td>22</td>
<td>60</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>65%</td>
<td>38</td>
<td>108</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td>84</td>
<td>244</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>58%</td>
<td>131</td>
<td>381</td>
<td>184</td>
<td></td>
</tr>
<tr>
<td>55%</td>
<td>332</td>
<td>974</td>
<td>469</td>
<td></td>
</tr>
<tr>
<td>54%</td>
<td>518</td>
<td>1,520</td>
<td>730</td>
<td></td>
</tr>
<tr>
<td>53%</td>
<td>914</td>
<td>2,700</td>
<td>1,294</td>
<td></td>
</tr>
<tr>
<td>52%</td>
<td>2,051</td>
<td>6,053</td>
<td>2,900</td>
<td></td>
</tr>
<tr>
<td>51%</td>
<td>8,157</td>
<td>24,149</td>
<td>11,556</td>
<td></td>
</tr>
<tr>
<td>50.5%</td>
<td>32,547</td>
<td>96,411</td>
<td>46,126</td>
<td></td>
</tr>
</tbody>
</table>

Procedure for 10% risk limit, one contest, majority winner:

1. Set $T = 1$. $s$ is winner’s share of the valid votes according to the vote tabulation system.

2. Select a ballot at random.

3. If the ballot shows a valid vote for the reported winner, multiply $T$ by $2s$.

4. If the ballot shows a valid vote for anyone else, multiply $T$ by $2(1 - s)$.

5. If $T > 10$, stop the audit: Reported outcome stands.
   Otherwise, if we want to perform a full hand count at this point, do. If not, return to step 2.
Arbitrary number of contests and winners: For each contest under audit, consider all pairs \((w, \ell)\) of winners and losers. Let \(s_{w\ell}\) be the fraction of votes \(w\) was reported to have received among ballots reported to show a vote for \(w\) or \(\ell\) or both. For instance, suppose Alice, Bob, Candy, and Dan are candidates in a school board contest with two winners, in which voters were allowed to vote for up to two candidates. Alice reportedly received 80\%, Bob 60\%, Candy 25\% and Dan 20\%. Then there are four (winner, loser) pairs: (Alice, Candy), (Alice, Dan), (Bob, Candy), and (Bob, Dan). The corresponding values of \(s\) are

\[
\begin{align*}
s_{\text{Alice Candy}} &= \frac{80\%}{(80\% + 25\%)} = 76.2\%, \\
s_{\text{Alice Dan}} &= \frac{80\%}{(80\% + 20\%)} = 80\%, \\
s_{\text{Bob Candy}} &= \frac{60\%}{(60\% + 25\%)} = 70.6\%, \\
s_{\text{Bob Dan}} &= \frac{60\%}{(60\% + 20\%)} = 75\%.
\end{align*}
\]

Full procedure for 10\% risk limit:

1. Set \(T_{w\ell} = 1\) for all (winner, loser) pairs \((w, \ell)\) in each audited contest.
2. Select a ballot at random.
3. If the ballot shows a valid vote for a reported winner \(w\) in some audited contest, then for each loser \(\ell\) in that contest that did not receive a valid vote on that ballot, multiply \(T_{w\ell}\) by \(2s_{w\ell}\). Repeat for all such \(w\) and for all audited contests on the ballot.
4. If the ballot shows a valid vote for a reported loser \(\ell\) in some audited contest, then for each winner in that contest that did not receive a valid vote on that ballot, multiply \(T_{w\ell}\) by \(2(1 - s_{w\ell})\). Repeat for all such \(\ell\) and for all audited contests on the ballot.
5. If any \(T_{w\ell} \geq 10\), do not update that \(T_{w\ell}\) again, even if we draw more ballots.
6. If all \(T_{w\ell}\) are at least 10, stop the audit: The reported results stand. Otherwise, if we want to perform a full hand count at this point, do. If not, return to step 2.