HW5

1 ①

$$A = \begin{array}{c|cc|c} & & & \text{row min} \\ & -1 & -3 & -3 \\ & -2 & 2 & -2 \\ \text{col max} & -1 & 2 & \end{array} \Rightarrow \text{No saddle point}$$

Let the optimal strategy for player I be $\binom{p}{1-p}$

$\Rightarrow -p - 2(1-p) = -3p + 2(1-p)$ (using equalising strategies)

$\Rightarrow -p + 2p - 2 = -3p + 2 - 2p$

$6p = 4 \qquad p = 4/6 = 2/3, \ 1-p = 1/3$

Similarly, Let $(q, 1-q)^T$ be optimal for II under pure strategies ei from player I,

$-q - 3(1-q) = -2q + 2(1-q)$

$-q - 3 + 3q = -2q + 2 - 2q$

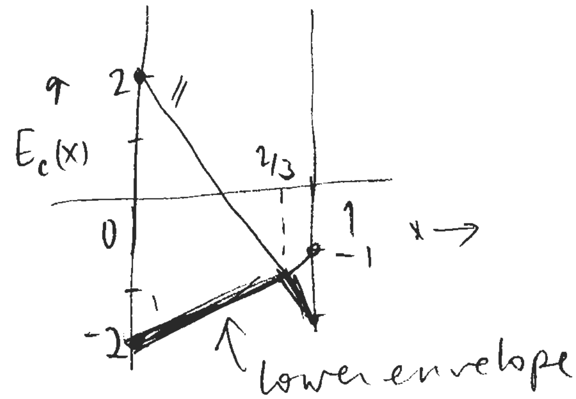
$6q = 5, \quad q = 5/6 \qquad (5/6, 1/6)$

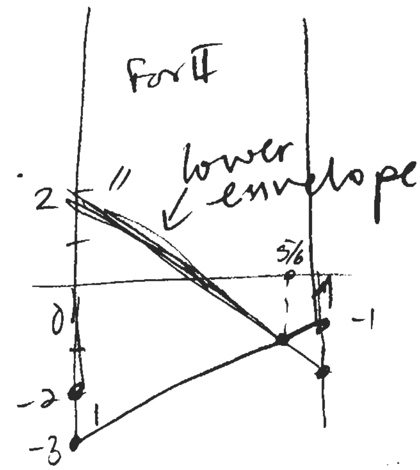
$$V = -\frac{2}{3} - 2\left(\frac{1}{3}\right) = -\frac{4}{3}$$

$$\text{or } V = -\frac{5}{6} - \frac{3}{6} = -\frac{8}{6} = -\frac{4}{3}$$

Alternatively: Graphical method

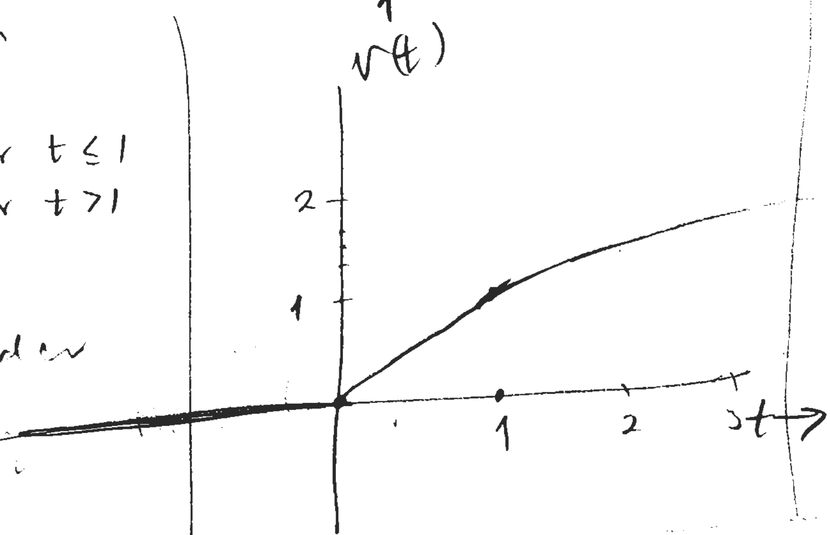
$$A = \begin{array}{c|cc} & y & 1-y \\ \hline x & -1 & -3 \\ 1-x & -2 & 2 \end{array}$$





$\dfrac{2}{\text{II}-14}$ $A = \begin{pmatrix} 0 & 2 \\ t & 1 \end{pmatrix}$ row min: 0; $\begin{cases} t & \text{for } t \le 1 \\ 1 & \text{for } t > 1 \end{cases}$

col max $\begin{cases} t, & t \ge 0 \\ 0, & t < 0 \end{cases}$ $\quad 2$



⇒ we have to consider 3 cases:

(i) $t < 0$

(ii) $0 \le t \le 1$

(iii) $t > 1$

Case (i)

$t < 0$ $\quad \begin{pmatrix} 0 & 2 \\ t & 1 \end{pmatrix}$ row min: 0, t

col max: 0, 2

⇒ 0 is a saddle point & $v(t) = 0$

Case (ii) $0 \le t \le 1$

$\begin{pmatrix} 0 & 2 \\ t & 1 \end{pmatrix}$ row min: 0, t

col max: t, 2

⇒ t is a saddle point & $v(t) = t$

Case iii $t > 1$

$\begin{pmatrix} 0 & 2 \\ t & 1 \end{pmatrix}$ row min: 0, 1

col max: t, 2

no saddle point

$t(1-p) = 2p + 1 - p$

$\Rightarrow p = \dfrac{t-1}{t+1}$

$v(t) = \dfrac{2t}{t+1}$

$\lim_{t \to \infty} \dfrac{2t}{t+1} = \dfrac{2}{1+1/t} = 2$

3

3.2
pg 70

$$A = \begin{pmatrix} 0 & 9 & \cancel{1} & \cancel{1} \\ 5 & 0 & \cancel{6} & \cancel{7} \\ 2 & 4 & \cancel{3} & \cancel{3} \end{pmatrix} \to \begin{pmatrix} 0 & 9 \\ 5 & 0 \\ 2 & 4 \end{pmatrix} \to \begin{pmatrix} 0 & 9 \\ 5 & 0 \end{pmatrix}$$

Note that col 3 & col 4 dominate col 1, so they can be discarded.

Now note that $\frac{1}{2}(a_{11}+a_{21}) \geq 2$ & $\frac{1}{2}(a_{12}+a_{22}) \geq 4$
$\Rightarrow$ a convex combination of rows 1 & 2 dominates row 3

Now we can solve this 2x2 matrix to get

$$p = \begin{pmatrix} 5/14 \\ 9/14 \end{pmatrix} = q, \quad v = 45/14$$

4

3.3
pg 70

There are two possible ways to solve this problem.

(1.) Use a payoff mtx of probabilities $\begin{pmatrix} 3/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/2 \end{pmatrix}$
where Player I wants to maximise his chances of destroying the item.
Solve to get $p = (2/11, 6/11, 3/11)^T = q$ & $v = \frac{6}{44}$.
Note $v < \frac{1}{2}$ & it is a probability, so game favors PII.

(2) Suppose PI wins \$1 for destroying item & loses \$1 for failing. Then payoff matrix $= \begin{pmatrix} 1/2 & -1 & -1 \\ -1 & -1/2 & -1 \\ -1 & -1 & 0 \end{pmatrix}$
where the entries are his expected gains under the given probabilities ($a_{11} = (\frac{3}{4})(1) + (\frac{1}{4})(-1) = \frac{1}{2}$, $a_{12} = (1)(-1)$ etc.)
Solving this game gives the same optimal strategy & $v = -8/11$, so favors PII